**Republic of Iraq**
**Ministry of Higher Education and Scientific Research**
**University of Babylon**
**College of Engineering**
**Department of Civil Engineering**

# STOCHASTIC MODELS OF MONTHLY FLOW OF GREATER ZAB RIVER

A Thesis
Submitted to The College of Engineering of The University of Babylon in Partial Fulfillment of The Requirements for The Degree of Master of Science in Water Resources Engineering

BY

**Anees Kadhum Idrees Al-Saadi**

(H. Dip. Civil Engineering 1999)

Supervised By

**Asst. Prof. Dr. Salah Tawfeek Ali Al-Baz'az**

**Asst. Prof. Kadhim Naief Kadhim Al-Ta'ee**

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**بِسْمِ اللهِ الرَّحْمنِ الرَّحِيمِ**

**{وَاللّهُ أنزَلَ مِنَ السَّمَاءِ مَاءً فَأَحْيَا بِهِ الأَرْضَ بَعْدَ مَوْتِهَا إِنَّ فِي ذَلِكَ لآيَةً لِّقَوْمٍ يَسْمَعُونَ}**

**صَدَقَ اللّهُ الْعَظِيمِ**

**(سورة النحل - الايه 65)**

# Dedication

- To my teacher who terminated his lifetime to be the lighted to my life way….
- To my family: My dear parents, my brothers, and my sisters ……...
- To my wife and my sons .......…………………………………………
- To all people who love me and all that I love deeply….…………...

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# Abstract

The Generation of synthetic river flow data is important in the planning, design and operation of water resources systems. In this study, two different methods of time series analysis are applied to monthly flow of Greater Zab River, which is one tributaries of Tigris River. Firstly, PARMA model is applied to monthly flow for the period from October 1933 to September 1992. Box-Cox transformation is used for each season in the model as it is considered the best transformation to normalize and standardize the data. Periodic autocorrelation function (PeACF) and periodic partial autocorrelation function (PePACF) are used for identification of the appropriate models for each season. The conditional maximum likelihood method is used to estimate the parameters of the models. Eight models of this type are considered for each season, i.e. (PARMA(1,0), PARMA(2,0), PARMA(0,1) PARMA(0,2) PARMA(1,1) PARMA(2,2) PARMA(1,2) PARMA(2,1)). However, the PARMA (1, 0) model is selected as the best model for each season by using AIC and SIC since this model gives minimum value of AIC and SIC compared with other above models. Portmanteau lack of fit test is used for diagnostic checking. The PARMA (1, 0) model is then used to find the future values for monthly flow for ten years from 1993 to 2002.

Secondly, SARIMA models for the same data of monthly flow of Greater Zab River from 1933 to 1992 are examined. The natural logarithmic method is used to normalize data and then first order simple and seasonal differencing is used. Autocorrelation function (ACF) and partial autocorrelation function (PACF) are used for model identification and the method of unconditional maximum likelihood is used to estimate the parameters of the models. Four models of this type are examined which are SARIMA$(0,1,1)(0,1,1)_{12}$, SARIMA$(0,1,2)(0,1,1)_{12}$ , SARIMA$(1,1,0)(1,1,0)_{12}$, SARIMA$(2,1,0)(1,1,0)_{12}$ and SARIMA $(0,1,1)(0,1,1)_{12}$ since they correspond to the minimum value of AIC and SIC. Portmanteau lack of fit test is used as diagnostic checking. All of these models are used to find the future values of monthly flow for ten years from 1993 to 2002.

The future values of monthly flow as obtained using each of the two methods are compared with the historical values for the period from 1993 to 2002 by using the minimum forecasted error. The results showed that the SARIMA$(0,1,1)(0,1,1)_{12}$ is better than PARMA(1,0) in representing the original data for the Greater Zab river, for this reason we used the SARIMA$(0,1,1)(0,1,1)_{12}$ model to find the future values of monthly flow for ten years from 2003 to 2012.

# Supervisors Certificate

We certify that preparation of this thesis entitled "***Stochastic Models of Monthly Flow of Greater Zab River***", was prepared by " ***Anees Kadhum Idrees AL- Saadi***" under our supervision at University of Babylon in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering in field of specialization is ***Water Resources Engineering.***

Signature:

**Name: Asst. Prof. Dr. Salah Tawfeek Ali Al-Baz'az**

**(Supervisor)**

Date: / /

Signature:

**Name: Asst. Prof. Kadhim Naief Kadhim Al-Ta'ee**

**(Supervisor)**

Date: / /

## Examining committee Certificate

We certify as an examine committee that we have read this thesis entitled *"Stochastic Models of Monthly Flow of Greater Zab River"* and examined the student *"Anees Kadhum Idrees AL-Saadi"*, in its content and what related to it, and that in our opinion it meets the standard of a thesis for the degree of Master of Science in Civil Engineering in field of specialization is ***Water Resources Engineering***.

Signature:
Name: **Asst. Prof. Dr. Jawad Kadhim Abood**
(Member)
Date: 2 / 3 /2010

Signature: A.H.K. Al-Shukur 2.3.2010
Name: **Asst. Prof. Dr. Abdul-Hassan K. Al-Shukur**
(Member)
Date: 2 / 3 /2010

Signature: Salah Tawfeek
Name: **Asst. Prof. Dr. Salah Tawfeek Ali Al-Baz'az**
(Supervisor)
Date: 4/ 4/2010

Signature: K. N. Attaee
Name: **Asst. Prof. Kadhim Naief Kadhim Al-Ta'ee**
(Supervisor)
Date: 2/ 3 /2010

Signature: K.R. Abeed
Name: **Asst. Prof. Dr. Kareem Radhi Abaid Al-Murshidi**
(chairman)
Date: 2 / 3 /2010

Approval of Civil Engineering Department

Signature: AYA Ali

**Name: Prof. Dr. Ammar Yasir Ali**
Head of the Civil Engineering Department.
Date: 2 / 3 /2010

Approval of Deanery of the College of Engineering.

Signature: Salah Tawfeek

**Name: Asst. Prof. Dr. Salah Tawfeek Ali Al-Baz'az**
Dean of the College of Engineering.
Date: 4 / 4 /2010

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# List of Symbols

| Symbol | Definition |
|---|---|
| ACF | Autocorrelation function |
| AR(p) | Autoregressive model of order (p) |
| ARIMA (p,d,q) | Autoregressive integrated moving average model of order (p) , (q) and degree of differencing (d) |
| $a_t$ | The shock process (independent part of stochastic component) or white noise process. |
| $a_{r,m}$ | Periodic white noise process at year (r) and month (m). |
| AIC | Akaike information criterion |
| $AIC_m$ | Periodic Akaike information criterion at season m. |
| $c_k$ | Autocovariance coefficient at lag k |
| $c_o$ | Autocovariance coefficient at lag zero |
| $C_k$ , | Coefficients of kurtosis (non periodic and periodic) |
| $C_s$ , $C_s)_m$ | Coefficients of skewness (non periodic and periodic) |
| D | Degree of seasonal differencing (for seasonality removal). |
| d | Maximum number of differencing necessary to make the time series stationary in the mean. |
| $F_t$ | Forecasted data at time t. |
| $F_{i,j}$ | Future series at year (i) and month (j) |
| $J_t$ | Jump component at time t. |
| k | Number of lags. |
| M | Maximum lag number. |
| MARE% | Mean absolute relative error. |
| MAE | Mean absolute error. |
| MSE | Mean square error. |
| N | No. of observations |

| | |
|---|---|
| PARMA (p,q) | Periodic autoregressive moving average model of order (p),(q). |
| PAR(p) | Periodic autoregressive model of order (p). |
| PMA(q) | Periodic moving average model of order (q). |
| p | Order of PAR (p) or PARMA (p, 0) model. |
| PACF | Partial autocorrelation function |
| PeACF | Periodic autocorrelation function |
| PePACF | Periodic partial autocorrelation function |
| $P_t$ | Periodic or seasonal component at time t. |
| q | Order of PMA(p) or PARMA(0, q) model |
| $Q_1$ | Box-Pierce chi-square statistics. |
| $Q_2$ | Box-Ljung statistics. |
| $Q_3$ | Li- Mcloed statistics. |
| $r_k$ | Autocorrelation coefficient at lag k. |
| $r_{k,m}$ | Periodic autocorrelation coefficient at lag k and season m. |
| $r_k^m(a_{r,m})$ | Periodic autocorrelation coefficient of the residual component ($a_{r,m}$). |
| $r_k(a_t)$ | Autocorrelation coefficient of the residual component (at). |
| SARIMA (p,d,q) | Multiplicative seasonal autoregressive integrated moving average model of order (p), (q) and degree of differencing (d). |
| S | Seasonal or periodic cycle such as 6, 12, 24 months, etc. |
| $Sd_m$ | Seasonal standard deviation at season m. |
| $S^2_a$ | Sampled Variance of residual. |
| SIC | Schwarz information criterion. |
| $SIC_m$ | Periodic Schwarz information criterion at season m. |
| SS | Minimum sum of squares |
| $S_m$ | Periodic minimum sum of squares at season m. |
| $T_t$ | Trend component at time t. |

| | |
|---|---|
| $U_{\varepsilon/2}$ | Deviate exceeded by a proportion 0.68 or 1.96. |
| $W_t$ | Stochastic component at time t |
| $W_{i,j}$ | Stochastic component at year i, and month j. |
| $X_t$ | Time series observation at time t |
| $X_{r,m}$ | Original series at month (m) and year (r) |
| $\overline{X}_r$ | Annual mean at year r |
| $\overline{X}_m$ | Seasonal mean at season m |
| $Y_t$ | Stationary time series |
| $Y_{i,j}$ | Normality series at month (j) and year (i) |
| $Z_t$ | Normalized time series. |
| $\hat{Z}_t(\ell)$ | Forecast series at origin time t and lead time $\ell$. |
| $\hat{Z}_{r,m}(\ell)$ | Forecast series at year r and month m and lead time $\ell$. |
| $Z_{t+\ell} \pm$ | Upper and Lower probability limits |
| $\alpha$ | Level of significance. |
| $\phi_{i,m}$ | Periodic autoregressive parameter at season m. |
| $\theta_{j,m}$ | Periodic moving average parameter at season m. |
| $\phi,\Phi,\theta,\Theta$ | Parameters of the SARIMA models. |
| $\phi(\beta)$ | Autoregressive operator of order p. |
| $\theta(\beta)$ | Moving average operator of order q. |
| $\Theta_Q(\beta^S)$ | Seasonal autoregressive operator of order Q. |
| $\Phi_P(\beta^S)$ | Seasonal moving average operator of order P. |
| $\beta^j$ | Backward shift operator of order j. |
| $\beta^S$ | Seasonal backward shift operator of order S. |
| $\sigma_a$ | Residual standard deviation. |
| $\sigma_a^2$ | Variance of the white noise process (at). |
| $(\sigma_a{}^2)_m$ | Periodic residuals variance at season m. |
| $\sigma_j$ | The standard deviation at season j |

| | |
|---|---|
| $\mu_j$ | The mean at season j |
| $\sigma_{k,m}$ | Periodic Autocovariance coefficients at lag k and season m. |
| $\sigma_{0,m}$ | Periodic Autocovariance coefficients at lag zero and season m. |
| $\nabla^d$ | Backward difference of order d. |
| $\nabla_s^D$ | Seasonal backward difference of order D and season S. |
| $\nabla_s$ | Seasonal difference at season s |
| $\lambda$ | Power transformation constant |
| $\chi^2_\xi(k)$ | The significant point expected by a proportion $\xi$ of the $\chi^2$ distribution , having k degree of freedom |
| $\ell$ | Lead time forecasts. |

# List of Abbreviations and Acronyms

| | |
|---|---|
| ACF | Autocorrelation function. |
| AIC | Akaike information criterion. |
| AICc | Bias corrected Akaik information criterion. |
| Apr. | April. |
| AR | Autoregressive model. |
| ARI | Autoregressive integrated model. |
| ARIMA | Autoregressive integrated moving average model. |
| ARMA | Autoregressive moving average model. |
| ASCE | American Society for Civil Engineering. |
| Aug. | August. |
| BIC | Bayesian information criterion. |
| Cont. | Continuous. |
| Dec. | December. |
| DIR1 | Direction 1. |
| DIR2 | Direction 2. |
| LS | Least square = sum of squares. |
| FARIMA | Fractionally Differenced Autoregressive Integrated Moving Average model. |
| Feb. | February. |
| GA | Genetic Algorithm. |
| IMA | Integrate moving average model. |
| Jan. | January. |
| Jul. | July. |
| ℓn | Natural Logarithm. |
| L.L | Lower Probability Limit. |
| MA | Moving average model. |
| MAE | Mean Absolute Error. |

| | |
|---|---|
| MAPE | Mean Absolute Percentage Error. |
| Mar. | March. |
| MARE% | Mean absolute relative error |
| MLE | Maximum Likelihood Estimate. |
| MPEVII | Mean Percent Error. |
| MSE | Mean Square Error. |
| Nov. | November. |
| Oct. | October. |
| PACF | Partial autocorrelation function. |
| PeACF | Periodic autocorrelation function. |
| PePACF | Periodic Partial autocorrelation function. |
| PeAR | Periodic Autoregressive. |
| PeMA | Periodic moving average. |
| RACF | Residual Autocorrelation function. |
| RMSE | Root mean square error. |
| SAR | Seasonal autoregressive model. |
| SARI | Seasonal autoregressive integrated model. |
| SARIMA | Seasonal autoregressive integrated moving average model. |
| SARMA | Seasonal autoregressive moving average model. |
| SBC | Schwarz Bayesian criterion. |
| SIC | Schwarz information criterion. |
| Sep. | September. |
| SIMA | Seasonal Integrated moving average model. |
| SMA | Seasonal moving average model. |
| SS | Sum of squares. |
| U.L | Upper Probability Limit. |

# Chapter One

# INTRODUCTION

## 1.1 Prelude

Time series analysis and modeling is an important tool in hydrology and water resources. It is used for building mathematical models to generate synthetic hydrologic records, to determine the likelihood of extreme events, to forecast hydrologic events, to detect trends and shifts in hydrologic records, and to fill in missing data and extend records. Synthetic river flow series are useful for determining the dimensions of hydraulic works, for flood and drought studies, for optimal operation of reservoir systems, for determining the risk of failure of dependable capacities of hydroelectric systems, for planning capacity expansion of water supply and irrigation systems, and for many other purposes (Salas *et. al*, 1985; Salas, 1993). For example, hydrologic drought properties (severity and duration) of various return periods are needed to assess the degree to which a water supply system will be able to cope with future droughts and, accordingly, to plan alternative water supply strategies(Tesfaye, 2005).

A model that describes the probability structure of a sequence of observations is called a "*stochastic process*". A time series of N successive observations $Z_t$= ($Z_1$,$Z_2$, ...,$Z_n$) is regarded as a sample realization from an infinite population of such samples, which could have been generated by the process. Time series consists of deterministic and nondeterministic part. The Deterministic part is represented by jump, trend, and seasonal components while the nondeterministic part is represented by the stochastic component. This Stochastic component contains a dependent and independent part (Box and Jenkins, 1976). An important feature of time series is the stationarity. Stationarity of time series means that statistical properties are the same over time, i.e. such a time series fluctuates around a fixed mean value (Caldwell, 2006). The time series is nonseasonal or seasonal and nonstationarity or stationarity, the dependent part of the stochastic component can be represented by stochastic models. In the case nonseasonal and stationarity, stochastic models are represented by autoregressive (AR), moving average (MA), and autoregressive moving average (ARMA) models. Another classes of stochastic models which represent nonseasonal nonstationarity time series are autoregressive integrated (ARI), integrated moving average (IMA), and autoregressive integrated moving average (ARIMA) models. Seasonal and stationarity models can be represented by seasonal autoregressive (SAR), seasonal moving average (SMA), and seasonal autoregressive moving average (SARMA) models ,while seasonal and nonstationarity time series models represent by seasonal autoregressive integrated (SARI), seasonal integrated moving average (SIMA), seasonal autoregressive Integrated moving average (SARIMA) models, periodic autoregressive (PAR), periodic moving average (PMA) and periodic autoregressive moving average models. These models in all types may be for a single site, multisite, univariate, or multivariate stochastic process (Box and Jenkins, 1976).

The statistical characteristics of hydrologic series are important deciding factors in the selection of the type of model. For example, in most cases known in nature, river flows have significant periodic behavior in the mean, standard deviation and skewness. In addition to these periodicities, they show a time correlation structure that may be either constant or periodic. Such serial dependence or autocorrelation in river flow series usually arises from the effect of storage, such as surface, soil, and ground storages, which cause the water to remain in the system through subsequent time periods. The common procedure in modeling such periodic river flow series is first to standardize or filter the series and then fit an appropriate stationary stochastic model to the reduced series (Salas et al., 1980; Salas, 1993; Chen and Rao, 2002). However, standardizing or filtering most river flow series may not yield stationary residuals due to periodic autocorrelations. In these cases, the resulting model is misspecified (Tiao and Grupe, 1980).

Univariate stochastic models are widely used and in these models variables are separated from each other. Other models are used in the generation of multivariate synthetic sequences, where the sequences may pertain stations, seasons, or combination of stations and seasons which are called multivariate models. Single site models deal with the variables at a certain point (the correlation exists in time only), while multisite models deal with the correlations in both time and space. (Matalas, 1967, quoted Al-Ta'ee, 2009).

## 1.2 Objectives of the Present Study

The main objectives of the present study are:

1. Applying periodic autoregressive moving average (PARMA) model and Fitting Box-Jenkins multiplicative seasonal models (SARIMA) to represent monthly flow data for Zab River.
2. Comparing the result of the two models which are mentioned above and select the best model for forecasting ten years from 2003 to 2012. These forecasted values are computed based on the 95 percent probability limits.

## 1.3 Organization of the Thesis

The present thesis consists of seven chapters:

- Chapter One (Introduction) includes a general introduction, objectives of time-series analysis, characteristics of hydrological time series, description of the study area and data, objectives of the present study and organization of the research.

- Chapter Two (Review of Literature) presents a brief description of previous studies on stochastic models.
- Chapter Three (Theoretical Background for stochastic method) is devoted to define components of time series, stationarity and normality of time series. Autocorrelation and periodic autocorrelation analysis, removal of periodic or seasonal component, building PARMA and SARIMA models and methods of forecasting are also described.
- Chapter Four (Application of PARMA Model and Fitting SARIMA model) applies PARMA models and Fitting SARIMA model, the monthly flow data of the Greater Zab river is normalized and standardized
- Chapter five (Forecasting) uses $PARM_{A12}$ (1, 0) model and SARIMA model to forecast future values of flow.
- Chapter six presents Conclusions and Recommendations.

# Chapter Two

# LITERATURE REVIEW

## 2. Introduction

Time series analysis is a major tool in forecasting of hydrological elements. Some of available reviews of stochastic models in various regions of world as well as Iraq are presented here in historical sequence as shown in the following subsections.

## 2.1 Studies on Time Series from Other Countries

**Lohani and Wang, (1987)** used AR(1) model to represent 12 monthly water quality parameters taken at three stations on Chung Kang river in Taiwan. Five years of data (1976–1980) were used for basic analysis and the data in the sixth year (1981) were used for the comparison of the results forecasted from the time series model. Different types of transformation were used such as square root, logarithmic, and power transformation. Autocorrelation function (ACF) and partial autocorrelation function (PACF) were used for model identification. Maximum likelihood estimate (MLE) was used to estimate the model parameters. Akaik information criterion (AIC) was used as diagnostic checking. It was found that the AR(1) model was the suitable model for these data.

**Mcleod, et. al., (1987)** combined river flow forecasts by taking two case studies, the usefulness of combination techniques for improving forecasts was examined. In the first study, a transfer function–noise (TFN) model, a periodic autoregressive (PAR) model, and a conceptual model were employed to forecast quarter-monthly river flows. In the second study, PAR models and seasonal autoregressive Integrated moving average (SARIMA) models were used to forecast monthly river flow. Combining the individual forecasts from these two statistical time series models did not result in significantly better forecasts. The PAR model appeared to be a more suitable model for monthly river flows.

**Jayawardena and Lai, (1989)** used AR(1) model, AR(2) model, and a higher order autoregressive moving average (ARMA (5, 8)) model to represent 21 years (1965-1985) of mean monthly water quality data in the Guangzhou reach of the Pearl River in the southern China. Eighteen years of data was used for model development while the model performance was compared with the data for the remaining three years. The turning point test and Kendall's rank correlation test were used for the detection of the trend component and it was removed by fitting a regression equations. The autocorrelation and spectral analysis were used for detection the periodicity. The stochastic component of the series was obtained by subtracting the periodic mean and it was standardized by dividing the periodic standard deviations. The comparison between observed and generated series were found to be satisfactory.

**Bell, (1993)** compared seasonal ARIMA models as presented in Box and Jenkins (1970) with ARIMA component (structural) models as presented in Harvey (1989). Both models were augmented as appropriate with the same regression variables to account for calendar effects, level shifts, and additive outliers. The comparison showed that all ARIMA component (structural) models tended to provide a poor fit to the sort of time series seasonally adjusted by the Census Bureau.

**Smadi, (1994)** compared three estimation methods, namely the method of moments, method of conditional LS, and method of conditional ML, theoretically and also through a simulation study for three different PAR models and investigated their behaviors for various cases. He showed that all methods produce estimates which are close to the actual values of the parameters and conditional LS estimates and conditional ML estimates of the PAR parameters are same; and also identical to moment estimates except for some initial seasons in which some observations are lost. Finally, he concluded that the conditional LS estimates are superior to moment estimation in terms of bias and mean squared error.

**Ula and Smadi, (1997)** showed that periodic covariance stationarity conditions for univariate and multivariate PARMA processes could be reduced to eigenvalue problems which are analytically easier to deal with. The $\omega$-span lumping over all $\omega$ periods and the $p$-span lumping of the $p$ th order PAR process over $p$ periods are investigated respectively and it is shown that the $p$ span lumping provides the periodic stationarity conditions in an analytically simpler form as compared to the $\omega$-span lumping in case of $p < \omega$ . Whatever the AR order $p^*$ of the lumped vector is, the stationarity conditions can be reduced to an eigenvalue problem.

**Ooms and Franses, (1998)** proposed a new periodic model where long memory characteristics at yearly lags, so called seasonal long memory, varied from month to month to the Fraser river in Netherlands. The model specification was motivated by examining sample periodic autocorrelation functions for monthly river flows at long yearly lags. Based on simple time series plots and periodic sample autocorrelations, they documented that monthly river flow data displayed long memory. In addition to pronounced seasonality. Gaussian Maximum Likelihood month by month was used to estimate the model.

**Anderson, et.al., (1999)** developed the innovations algorithm for periodically stationary processes and showed how to obtain estimates for the parameters of the PARMA model.

**Basawa and Lund, (1999)** studied the asymptotic properties of parameter estimates for casual and invertible PARMA models and derived a general limit result for PARMA parameter estimates with a moving average component.

**Lund and Basawa, (1999)** studied analysis problems with periodically correlated (PC) time series. Frequency domain test to detect periodicities in the autocovariance structure of an observed series was presented. A PARMA model was introduced as useful class of PC time series models. Comparisons to seasonal Box-Jenkins models was made. Moment, Maximum likelihood and estimation equation techniques of estimation was considered. Limit distributions of parameter estimates was discussed.

**Trajkovic, (1999)** used three models to represent 15 years (1961-1975) of monthly reference crop evapotranspiration in the area of Nis, Yugoslavia. Ten years (1961-1970) were used for basic analysis and the other five years (1971-1975) were used for the comparison of the results obtaining from three models. These models were (i) two simple mathematical models (yearly differencing (YD) model and monthly average (MAV) model) and (ii) SARIMA model of order (0, 0, 0)×(1, 1, 1)12. The application of YD and MAV models was successful because of the domination of seasonal component in the evapotranspiration time series. The minimum value of each mean square error (MSE), maximum absolute error (MXE), mean absolute error (MAE), and mean percent error (MPEVII) for July was used for the comparison between the results from the three models from 1971 to 1975. Then, SARIMA found a very effective and reliable prediction model.

**Ahmad, et. al., (2001)** used three different approaches of stochastic modelling: (i) multiplicative ARIMA model, (ii) deseasonalized model using Fourier series technique, and (iii) set of Thomas–Fiering model for various months to model 10 years (1981-1990) of monthly mean water quality data consisting of six water quality parameters at ten typical locations along the river Ganges in India. The multiplicative ARIMA models having both nonseasonal and seasonal components were, in general, identified as appropriate models. In the deseasonalize modeling approach, the lower order ARIMA models were found appropriate for the stochastic components. The set of Thomas– Fiering models were formed for each month for all water quality parameters. These models were used to forecast the future values. The error estimates of forecasts, which were computed by root mean square error (RMSE) and mean absolute percentage error (MAPE), from the three approaches were compared to identify the most suitable approach for the reliable forecast. The deseasonalize modeling approach was recommended for forecasting of water quality parameters.

**Lehmann and Rode, (2001)** studied the analysis of weekly data samples from the river Elbe at Magdeburg in German for 13 years period from 1984 to 1996 to investigate the changes in metabolism and water quality in this river since the German reunification in 1990. Autoregressive component models and ARIMA models were used to reveal the improvement of water quality due to reduction of waste water emissions since 1990 and to determine the long term and seasonal behavior of important water quality variables. ARIMA model was identified by ACF and PACF plots. The maximum likelihood parameter estimation procedure was used to estimate the ARIMA model parameters and the diagnostic check was done. The trend and seasonal behavior of almost water quality indicators differed significantly in the periods before and after 1990.

**Ula and Smadi, (2003)** studied the identification of periodic moving-average (PMA) models by using periodic autocorrelation function

(PeACF) and considered the identification of PAR models by using periodic partial autocorrelation function (PePACF).

**Kihoro, et. al., (2004)** compared the performance of artificial neural networks (ANN) model and seasonal ARIMA model in forecasting of seasonal monthly time series. Three data sets were used for this study: (i) 12 years of monthly totals in thousands of international airline passengers from January 1949 to December 1960, (ii) 13 years monthly totals in thousands of world tourists visiting Kenya from January 1971 to December 1983, and (iii) 20 years of monthly mean air temperature at Nottingham Castle from January 1920 to December 1939. For each data set a seasonal ARIMA model was fitted to the first N-12 values after appropriate transformation of the raw data ( natural log for airline data, divided by range for tourists data, and none for air temperature data). The best model was found by inspecting Akaik information criterion (AIC), bias corrected Akaik information criterion (AICc), and Bayesian information criterion (BIC) for the minimum. Box-Ljung statistics (Qc) was used to check the diagonesis of models

**Kurunc, et .al., (2005)** evaluated the forecasting performance of two modeling methods, ARIMA and Thomas–Fiering method, for selected water quality constituents and stream flow of the Yesilirmak River at Durucasu monitoring station in Turkey. For this purpose 18 years (1984–2001) of monthly mean water data and monthly stream flow were used to obtain the best model. 13 years (1984–1996) were used for analysis and 5 years (1997-2001) were used for the comparison of observed to generated data. The model with the minimum Schwarz Bayesian criterion (SBC) value was selected as the best model. The results of forecast accuracy were measured by root mean square error (RMSE) and mean absolute error (MAE) for two approaches. Thomas–

Fiering model presented more reliable forecasting of water quality constituents and streamflow than ARIMA model.

**Mahpol, (2005)** introduced new approach in making a forecast by combining the Box-Jenkins methodology for SARIMA model and Genetic Algorithm (GA). Data used in this study were collected from the year 1996 until year 2003 that had been classified into total monthly electricity generated in kWh unit. This study proposed the possibility of using GA's approach as one of the unique forecasting method. It also represented a preliminary work in this research and practices of GA.

**Tesfaye, (2005)** studied the modeling of the periodic hydrological time series in general and river flows in particular. The innovations algorithm was used to obtain estimates of certain model parameters. Asymptotic distribution for the innovations estimates and model parameter were used with general technique for identifying the periodic autoregressive moving average (PARMA) models. Monthly river flow data for faster River in British Colombia and the Salt River Arizora were used in a detailed simulation study to test the effectiveness of the innovation estimation procedures and asymptotic as a method for PARMA model identification, to illustrate residual modeling procedure and to prove the ability to generate realistic synthetic river flow. The asymptotic distribution of the discrete Fourier transform of the innovation estimates and PARMA model parameters was developed to obtain a parsimonious model. The applicability of these methods was demonstrated by using weakly river flow data for the Fraser River.

**Yurekli, et. al., (2005)** tested the residuals from the ARIMA models fitted to monthly stream flow data by alternative methods for three gauging stations located on the Cekerek Stream watershed in Turky. Independence analysis of the residuals was examined by using the Ljung-

Box Q statistic, runs test, and turning point test. The selected parsimony model for each data set among the ARIMA models fulfilled the diagnostic checks considering the Schwarz Bayesian criterion (SBC). The simple linear regression approach was applied to explain the association between the observed and predicted monthly data sequences. The results from the regression analysis supported the existence of a statistically significant linear relationship between the observed and predicted data.

**Nochai and Nochai, (2006)** used Box-Jenkins technique to develop model for three types of oil palm price of Thailand in three types as farm price, wholesale price, and pure oil price for the period of five years (2000–2004). Ljung-Box Q statistic was used to check the diagnosis of model. Finding of the appropriate ARIMA model for forecasting in three types of oil palm price was done according to the minimum of mean absolute percentage error (MAPE). ARIMA model for forecasting farm price of oil palm was found ARIMA (2, 1, 0), ARIMA model for forecasting wholesale price of oil palm was found ARIMA (1, 0, 1) or ARMA(1, 1), and ARIMA model for forecasting pure oil price of oil palm was found ARIMA (3, 0, 0) or AR(3).

**Sabry, et. al., (2007)** used Box and Jenkins technique to predict daily traffic volume of 14 years (1990-2003) of average annual, average monthly, and average weekly daily traffic volume on a street in both directions (DIR1 and DIR2) for Tanta–Mansoura, Egypt. Years from 1990 to 2002 were used in building model. The resulting models were used to forecast traffic volumes for year 2003. The forecasted traffic volumes were then compared with the actual traffic volumes in 2003. The best estimated model for forecasting average annual daily traffic volume had been found the ARIMA (1, 0, 0) model for DIR1 and ARIMA (3, 0, 0) model for both DIR2 and TOTAL. The best estimated model for forecasting average monthly daily traffic volume had been found

SARIMA (3, 0, 0)×(3, 0, 0)12 model for DIR1, SARIMA (3, 0, 0)×(2, 0, 0)12 model for DIR2, and SARIMA (2, 0, 0)×(2, 0, 0)12 model for TOTAL. While in forecasting average weekly daily traffic volume, it had been found that the best estimate model is the SARIMA (2, 0, 0)×(2, 0, 0)52 model for each DIR1, DIR2, and TOTAL. SARIMA model was considered the best in forecasting short terms such as average monthly and average weekly daily traffic but was not accurate in forecasting long periods such as average annual daily traffic.

**Gautier, (2007)** used periodic models, where the parameters vary periodically with time, estimation of PARMA models was considered. Least squares were used to estimate of parameters. PARMA model was fitted with intercepts to the observed series can be asymptotically more efficient than fitting a PARMA model without intercepts to the mean-corrected series. Applications of PARMA models to a real series dealing with French motorway traffic. A comparison with seasonal ARIMA models was also presented.

## 2.2 Previous Applications of Time Series to Data from Iraq

**Shaker, (1986)** used singlesite AR(1) model, ARIMA(1, 0, 1) model, and multisite autoregressive model (Matalas model) for four flow stations on the Tigris river in Iraq. These models were used for daily stream flow of Tigris river at Mosul, Fatha, Baghdad, and Kut station for the period from 1936 to 1982. T-test was used for detecting nonhomogeneity and split sample method was used for removing it. The homogeneous series was normalized using a power transformation. The periodicity was detected by the correlogram technique and removed by harmonic analysis. The ARIMA (1, 0, 1) model was found to be the most attractive one and the comparison between generated and observed data was found to be satisfactory.

**Al–Husseini, (2000)** used AR(1) model, MA(1) model, and ARMA(1, 1) model as univariate models and first order multivariate model (Matalas model) to fit stochastic component of eight years (1992-1998) of monthly mean water quality parameter at Hilla station, middle Euphrates region of Iraq. Nonhomogeneity in mean and standard deviation was detected by t- testing and removing through using split sample method. Periodicity was detected by correlogram technique and removed by Fourier analysis. The ACF and PACF ware used for model identifications. Univariate stochastic model was proved good in forecasting the concentrations of all parameters except hardness which was represented by Matalas model. The comparison was found to be satisfactory.

**Mahmood, (2000)** applied AR(1) model, AR(2) model, and ARIMA(1, 1, 1) model to analyze monthly time series of water quality data for 10 years (1991–1998) which included ten parameters on the Euphrates river at kufa city, Iraq. Eight years were used for basic analysis and the other two years were used for the comparison of the results. Common log was used in the series normalization and ACF and PACF were used for model identification. The seasonal component was not existed in these types of observations. The comparison was found to be satisfactory.

**Al-Tikriti, (2001)** used singlesite AR(1) model to model seven parameters of average weekly water quality data for 14 years (1984–1997) in discharges at two stations on Euphrates river (Hindiya and Samawa station) in Iraq. Ten years of data (1984–1993) were used for building the stochastic model and the last four years (1994–1997) were used for the checking of model acceptability. Nonhomogeneity was removed by using split sample method. The periodicity was detected by correlogram technique and removed by harmonic analysis. The single site autoregressive model with first order (AR(1)) was found convenient for the parameter in the two stations while multisite model was not used because of the weak correlation between the two stations. The comparison of the statistical parameters of these records and with generated series was found to be acceptable.

**Al-Mousawi, (2003)** applied two stochastic singlesite autoregressive models with first order (AR(1)) and multisite model with first order (AR(1) Matalas model) to model eight hydrochemical parameters of monthly water quality data (raw water and water supply) for 15 years (1987–2001) in discharges at four stations (Hindya, Hilla, Hssien, and Hashimiya) on Hilla river (branch of Euphrates river in Iraq). Twelve years of data (1987-1998) were used for basic analysis and model building and the remaining three years (1999- 2001) were used to check the models acceptability. Nonhomogeneity was detected by suitable statistical tests and removed by split sample method. Power transformation was used to transform data to normal distribution. Periodicity was detected by correlogram technique and removed by harmonic analysis. Multisile model was used to study the cross correlations between the stations. This model was found to be adequate to describe the process. The singlesite first order autoregressive model (AR(1)) was found to be adequate for all stations. The comparison of the statistical parameters of observed and generated series was found to be satisfactory.

**Abed, (2007)** used two stochastic methods, ARMA models and SARIMA models (Box-Jenkins seasonal models), in analyzing seven parameters of monthly water quality data for 13 years (1992–2004) in discharges at three stations (Hindya, Kufa, and Diwaniya station) on the middle Euphrates region of Iraq. Ten years of data (1992-2001) were used for basic analysis and the last three years (2002-2004) were used for comparison between observed data and forecasted data in each method to choose the suitable method. The guide of the choice from these two methods is the minimum mean absolute relative error (MARE).In building ARMA models, the split sample method was used to test nonhomogeneity of mean and standard deviation and linear regression equation for both annual means and standard deviation was used to remove nonhomogeneity (trend component). Different types of transformations were used such as Box-Cox, square root, and logarithmic transformation. Autocorrelation analysis was used to test periodicity or seasonality of data and harmonic method was used to removed seasonality. ACF and PACF were used for model identification and the conditional likelihood function was used to choose the best model according to minimum sum of squares. Akaik information criterion (AIC), Portmanteau lack of fit, and residual autocorrelation function (RACF) were used as diagnostic checking. In building Box-Jenkins seasonal (SARIMA) model, the natural logarithm (ln) was used to normalize data. First order simple and first order seasonal differencing was used. ACF and PACF were used for model identification and the unconditional likelihood function was used to choose the best model according to minimum sum of squares. Portmanteau lack of fit test was used as diagnostic checking.

**Al-Ta'ee, (2009)** used three methods of analysis, to record rainfall and evaporation from Hilla and other seven neighbouring meteorological observing stations. Firstly, Thiessen's polygon method was used to calculate monthly mean rainfall for Babylon Governorate. The same polygons were used to calculate monthly mean evaporation for Babylon. Secondly, the Penman method was modified using Hilla record of evaporation to calibrate the three equations that were used by Penman.

New equations which were applicable to Hilla were different from those originally suggested by Penman for England that was presented so that computed evaporation was very close to the observed one. Thirdly, a Box-Jenkins seasonal model (SARIMA model) was applied to Hilla records of monthly evaporation and monthly rainfall which consists of twenty-nine years of records from 1978 to 2006. The natural logarithm (ln) was used to normalize data. Then, first order simple and seasonal differencing was used. Autocorrelation function (ACF) and partial autocorrelation function (PACF) was used for model identification. Unconditional likelihood function was used to choose the best model corresponding to minimum sum of squares. Portmanteau lack of fit test was used as diagnostic checking.

# Chapter Three

# STOCHASTIC METHODS

## 3.1 Components of Time Series

Time series as a sequence of observations ordered by a time parameter, time series may be measured continuously or discretely, if the set is continuous, the time series is said to be continuous. If the set is discrete, the time series is said to be discrete. (Yaffee and McGee, 1999). If future values of a time series are exactly determined by some mathematical function, the time series is said to be deterministic. If the future values can be described only in terms of probability distribution, the time series is said to be nondeterministic or simply a statistical time series (Box and Jenkins, 1976).

Time series may consist of four components depending on the type of variable and the average time interval. These components may exist in monthly time series, which may be formulated by:

$X_t = J_t + T_t + P_t + W_t$, ……………………………………………………………(3-1)

where $X_t$ is the time series observations at time t (= 1, 2, 3, …, N), $J_t$ is the Jump component, $T_t$ is the trend component, $P_t$ is the periodic or seasonal component, $W_t$ is the stochastic component, and N is the number of Observations. These components are shown in Figure 3.1.

When the components are nonlinearly related, the relationship (Equation 3-1) can often be made linear by taking logarithms (Jayawardena and Lai, 1989). Jump, trend, and periodic components represent the deterministic part of the process while the stochastic component represents the nondeterministic part. Therefore, the first three components should be detected and identified by suitable formulations and decomposed from the stochastic component.

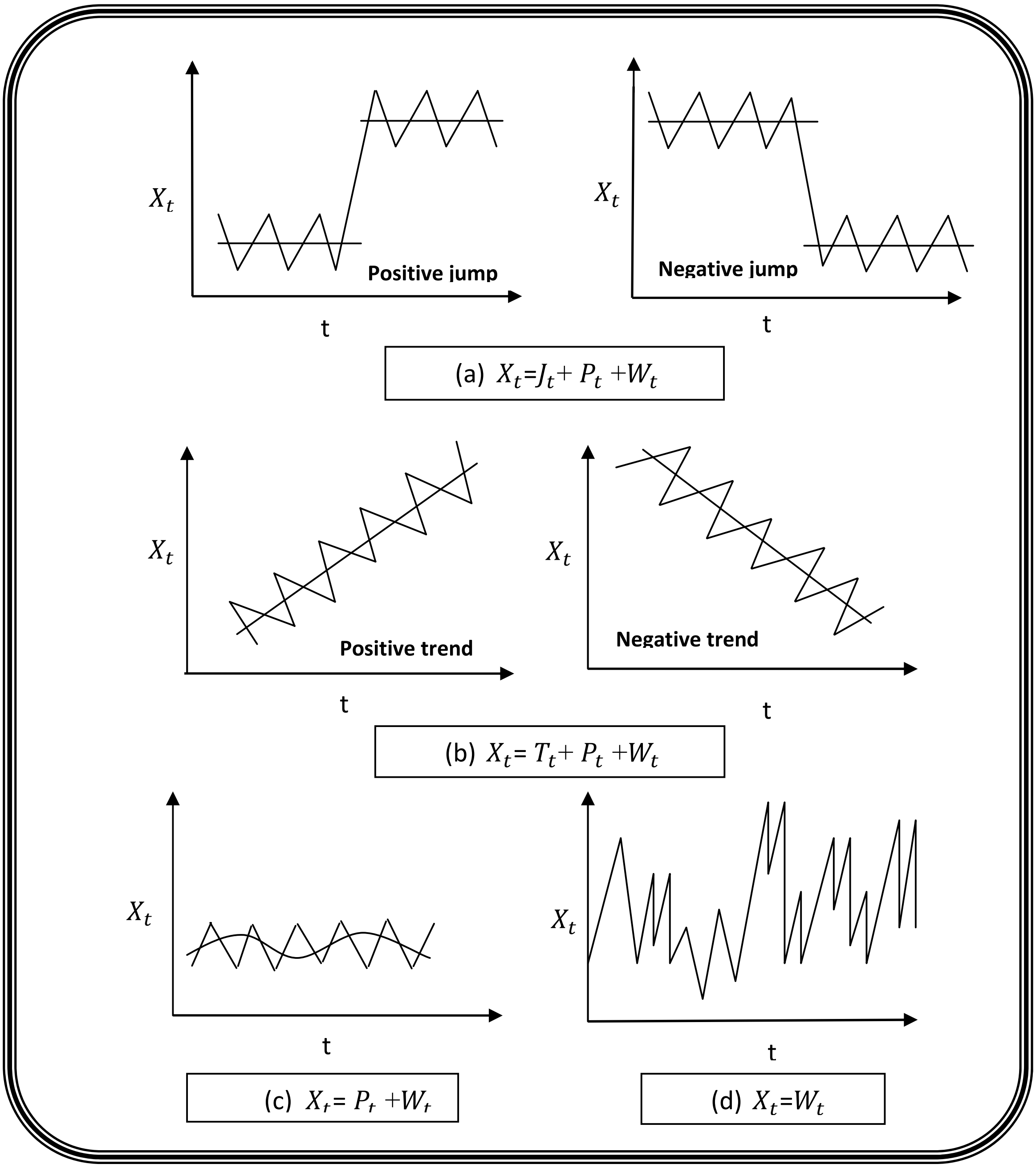



**Figure 3.1: Components of Hydrological Time Series (Shaker,1985)**

## 3.2 Stationarity

The property stationarity of a random process is related to the mean value and variance of observation data, both of which should be constant over time and the covariance between the observations $X_t$ and $X_{t-d}$ should only depend on the distance between the two observations and does not change over time (Palit and Popovic, 2005). This implies that no trend or jump exists in the series. Hence, the classification of a time series as stationary or nonstationary is not merely for descriptive purpose. Available statistical methods are designed to analyze stationary time series. Therefore, if the time series is defined to be nonstationary, it must be transformed into stationary (Matalas, 1967). The nonstationarity will frequently be apparent in the time series plot of the series. (Cryer and Chan, 2008).

Testing of nonstationarity can be made by autocorrelation analysis. If autocorrelation function starts high and declines slowly, then the series is nonstationary. Therefore, the differencing must be done as suggested by Box and Jenkins (Batchelor, 2004; Haghaparast, 2007).

In Box and Jenkins model the differencing technique is used to make the mean of time series as constant as possible and its degree of the regressive equation equal to zero. Thus, the stationary time series is obtained by taking appropriate number of differences for the original time series ($X_t$) as follows:

$$Y_t = \nabla X_t = X_t - X_{t-1} \text{ , ..........} \quad (3\text{-}2)$$

or

$$Y_t = X_t - \beta X_t$$

$$= (1-\beta)X_t \quad \text{,..........} \quad (3\text{-}3)$$

where $\nabla$ is the backward difference of order one, $Y_t$ is stationary time series and $\beta$ is the backward shift operator of order one. Generally,

$$\beta^j X_t = X_{t-j}$$

where $\beta^j$ is the backward shift operator of order j.

The second order difference gives the following new time series:

$$Y_t = \nabla^2 X_t = \nabla(\nabla X_t)$$

$$= \nabla(X_t - X_{t-1}) = \nabla X_t - \nabla X_{t-1}$$

$$= X_t - X_{t-1} - X_{t-1} + X_{t-2} = X_t - 2X_{t-1} + X_{t-2}$$

$$= (1 - 2\beta + \beta^2)\, X_t$$

$$= (1 - \beta)^2\, X_t \, , \qquad \text{(3-4)}$$

In general, if (d) differences are taken from the original time series, the new time series can be written as follows:

$$Y_t = \nabla^d X_t = (1 - \beta)^d\, X_t \, , \qquad \text{(3-5)}$$

where (d) is the maximum number of differencing necessary to make the time series stationary in the mean (Box and Jenkins, 1976).

## 3.3 Normality

The time series observation of a given phenomenon required a certain type of transformation (Hipel et. al., 1977). Before transformating data to normal distribution, the coefficients of skewness (Cs) and Coefficients of kurtosis ($C_k$) must be determined from the following equations:

$$(Cs)_m = \frac{\frac{1}{N}\sum_{r=1}^{N}(X_{r,m}-\bar{X}_m)^3}{(Sd_m)^3} \quad ,\ldots\ldots(3\text{-}6)$$

$$(C_k)_m = \frac{\frac{1}{N}\sum_{r=1}^{N}(X_{r,m}-\bar{X}_m)^4}{(Sd_m)^4} \quad ,\ldots\ldots(3\text{-}7)$$

Where $(Cs)_m$ and $(C_k)_m$ are coefficients of skewness and Coefficients of kurtosis respectively, $X_{r,m}$ is origin time series for r year and m month, $\bar{X}_m$ seasonal mean, $Sd_m$ seasonal standard deviation and N number of years.

The purpose of transformation is to remove non normality in the residuals of the time series. (Hipel and McLeod, 1994)

It is often better to transform the data to the normal distribution to utilize its simple properties, its familiarity to most engineers, and to obtain satisfactory fit to data (Hipel and McLeod, 1994).

The other reason for transforming the date includes stabilizing the variance and improving the normality assumption of the white noise series.

Several transformation may be used to normalize the data but the most common and useful class of transforms for stabilizing the variance is known as the Box - Cox transformation ( Chandra et. al, 1978 ). The transformation as follows:

$$\begin{bmatrix} Y_{i,j} = (X_{i,j}^{\lambda} - 1)/\lambda & \lambda \neq 0 \\ Y_{i,j} = \log X_{i,j} & \lambda = 0 \end{bmatrix} \quad ,\ldots\ldots(3\text{-}8)$$

Where $Y_{i,j}$ is the transformed series. $X_{i,j}$ is the origin time series .

$\lambda$ is the constant of transformation.

The relationships between $\lambda$ and $C_S$ is being some form of second degree polynomial as

$$\lambda = B_0 + B_1 C_S + B_2 C_S^2 \quad ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3\text{-}9)$$

The value of ($\lambda$) is found by choosing random ten values of ($\lambda$) between (0) and (1) and computing the corresponding ($C_s$) values for the series after transforming it by equation (3-8). Then by fitting equation (3-9) to these ten points, the value of ($\lambda$) is found as equal to ($B_o$) corresponding to $C_S = 0$. For normally distributed data the ($C_S \approx 0$) and ($C_k \approx 3$).

A square root transformation is also applied by using the following equation.

$$Y_{i,j} = \sqrt{X_{i,j}} \quad ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3\text{-}10)$$

where $Y_{i,j}$ & $X_{i,j}$ are defined above.

The best procedure which displays the value of skewness coefficient ($C_s \approx 0$) and kurtosis coefficient ($C_k \approx 3$). The normality was tested by plotting observed cumulative probability against expected cumulative probability where

$$p(x) = \frac{R}{N+1} \quad , \quad ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3\text{-}11)$$

where p(x) is the observed cumulative probability of the value (x) of transformed parameter, R is the rank of (x) in ascending order and N is the number of tested data.

## 3.4 Autocorrelation and Periodic Autocorrelation

The sample autocorrelation coefficients measure the statistical correlation between observation (series and itself) at different time lags. The autocorrelation coefficient between observations at lag k for any random variable $X_t$ is given by the following equation (Chatfield, 1982):

$$r_k = \frac{\sum_{t=1}^{n-k}(X_t-\overline{X})(X_{t+k}-\overline{X})}{\sum_{t=1}^{n}(X_t-\overline{X})^2} \quad ,\ldots\ldots(3\text{-}12)$$

Where $r_k$ is the autocorrelation coefficient at lag k, k is the number of lag, and $\overline{X}$ is the overall mean of the observations which is computed as follows:

$$\overline{X} = \frac{1}{n}\sum_{t=1}^{n} X_t \quad \ldots\ldots(3\text{-}13)$$

The autocorrelation coefficients are usually calculated by computing the series of autocovariance coefficients,$c_k$, which is given as follows:

$$c_k = \frac{1}{n}\sum_{t=1}^{n-k}(X_t - \overline{X})(X_{t+k} - \overline{X}) \quad ,\ldots\ldots(3\text{-}14)$$

where $c_k$ is the autocovariance coefficient at lag k.

Then, the autocorrelation coefficients are computed as follows:

$$r_k = \frac{c_k}{c_0} \quad ,\ldots\ldots(3\text{-}15)$$

where $c_0$ is the autocovariance coefficient at lag zero.

The periodic autocorrelation coefficients may be computed as follows:

$$(r_{k,m}) = \frac{\sigma_{k,m}}{\sqrt{\sigma_{0,m}\sigma_{0,m-k}}} \quad ,\ldots\ldots(3\text{-}16)$$

$$\sigma_{k,m} = \frac{1}{N}\sum_{r=1}^{N}(Y_{r,m} - \overline{Y}_m)\,(Y_{r,m-k} - \overline{Y}_{m-k}) \quad ,\ldots\ldots(3\text{-}17)$$

when $m - k < 1$ the terms, $r = 1$, $Y_{r,m-k}$, $\overline{Y}_{m-k}$, $\sigma_{m-k}$ replaced by $r = 2$, $Y_{r-1,m-k+s}$, $\overline{Y}_{m-k+s}$, $\sigma_{m-k+s}$ (Sveinsson et. al, 2007).

## 3.5 Removal of Periodic or Seasonal Component

The time series has seasonal behavior or seasonal component when it has periodic fluctuations over time (Haghaparast, 2007). This time period is named seasonal cycle (S) which may be day, month, season, or year. When a time series has monthly data, seasonal pattern usually repeats itself every year, which means

S=12. The seasonal factor is found in series with a periodic behavior and it is very common in monthly series (Cortez et. al., 2004). Cyclic behavior may appear in more than one statistical parameter such as mean, standard deviation, skewness, kurtosis, and serial correlation coefficients.

It is easy to discover the seasonal periods from the graph of the stationary time series. If the time series is nonstationary, the discovery of the seasonal periods will not be easy.

Detection of periodicity or seasonality of any time series can be made by autocorrelation analysis. If the series is periodic, the plot of autocorrelation coefficient with different time lags will be periodic also otherwise it is not (Chatfield, 1982).

A time series with seasonal variation may be considered stationary if the theoretical autocorrelation function and theoretical partial autocorrelation function are zero after a lag k = 2S + 2. The sample autocorrelation and sample partial autocorrelation function already do not meet the above condition; therefore, the time series can be considered stationary when the ACF and PACF cut off at lags less than 2S + 2 (Trajkovic, 1999). The removal of the periodicity or seasonality of any time series can be made by two methods as follows.

### 3.5.1 Removal by Seasonal Differencing Method

To remove the seasonal component from homogeneous time series ( $Y_t$) when the Box-Jenkins seasonal models are used, one or more of the seasonal differences must be taken as follows (Box and Jenkins, 1976):

$W_t = \nabla_s^D Y_t$ , ……………………………………...…………………..(3-18)

Where $\nabla_s^D$ is the seasonal backward difference of order D and season S, D is the degree of seasonal differencing (for seasonality removal), and S is the seasonal or periodic cycle such as 6, 12, 24 months, etc.

Then, for D = 1 and S=12 the following equation is obtained:

$W_t = \nabla_{12} Y_t = Y_t - Y_{t-12}$

$= Y_t - \beta^{12} Y_t$

$= (1 - \beta^{12})Y_t$ ,……………………………………………………………(3-19)

Where $\beta^{12}$ is the seasonal backward shift operator of order 12 . Then,

$\beta^{12} Y_t = Y_{t-12}$, ………………………………………………………...(3-20)

In general, if D seasonal differences are taken from the original homogeneous time series, the stochastic component can be written as follows:

$W_t = \nabla_s^D Y_t = (1 - \beta^s)^D Y_t$ ,………………………………………………(3-21)

Where $\beta^s$ is the seasonal backward shift operator of order s.

### 3.5.2 Removal by Nonparametric Method

Before fitting PARMA model the nonparametric method is practical method may be used to remove the periodic component from a time series.

Removing periodicity in the original series is necessary and can be employed for monthly sequences to deseasonalize the series by subtracting estimated monthly mean and dividing by the estimated monthly standard deviation for the series as follows (Hipel and McLeod, 1994) :

$$W_{i,j} = \frac{Y_{i,j} - \hat{\mu}_i}{\hat{\sigma}_i}$$ ,………………………………………………………… (3-22)

Where $W_{i,j}$ is the stochastic component at year i, and month j. $Y_{i,j}$ is the normality series.

$\hat{\mu}_j, \hat{\sigma}_j$ are the mean and standard deviation for month (j) respectively computed by:-

$$\hat{\mu}_j = \frac{1}{n}\sum_{i=1}^{n} Y_{i,j} \quad , \quad \ldots\ldots(3\text{-}23)$$

$$\hat{\sigma}_j = \left[\frac{1}{n-1}\sum_{i=1}^{n}(Y_{i,j} - \hat{\mu}_j)^2\right]^{1/2} \quad \ldots\ldots(3\text{-}24)$$

Where n is the number of years.

## 3.6 Stochastic Component

Stochastic component is the remaining part after the separation of periodic component by seasonal differencing in seasonal models from homogenous (free from jump and trend components) time series. It is termed as the dependent stochastic component in the presses and denote as $W_{i,j}$. The stochastic component contains dependent and independent part. The dependent part may be represented by one of the time series models that is known as the Box-Jenkins seasonal SARIMA (p, d, q) models or PARMA (p, q) models, where p and q is the order of the autoregressive and moving average models respectively and d is the degree of differencing. These models may be for Singlesite, Multisite, Univariate, or Multivariate stochastic process. The independent part can only be described by some probability distribution functions.

## 3.7 PARMA$_S$ (p,q) Model

Many seasonal time series cannot be filtered, standardized or differenced to achieve second-order stationarity because the series exhibits a strong seasonal behavior such that the entire correlation structure of the series depends on the season; hence such homogeneity assumption sometimes fails (Tiao and Grupe, 1980). For instance, in a river where high runoffs occur in the spring and low flows occur in the summer, the streamflow correlations between spring months may be different from the correlations between summer months (Vecchia, 1985). A more realistic family of models characterizing those kinds of seasonal time series is the Periodic Autoregressive Moving Average (PARMA) models. A model of this type assume that season-to-season correlations are the same for a given lag. (Salas, 1993 quoted in Sveinsson, et.al., 2007)

This type of process is important when dealing with hydrologic data that are defined at time intervals smaller than a year such as seasons, months, weeks, and days. For instance, monthly stream-flow data are generally characterized by periodic statistical properties, such as periodic mean and variance, periodic skewness, and periodic covariance (Salas, et.al. 2005).The PARMA models consists of having separate ARMA model for each season of the year in which the model orders and parameters are allowed to vary among seasons.(Hipel and McLeod, 1994).

These models are applied to single time series in one point and are classified as flows:

### 3.7.1 Periodic Autoregressive Model (PAR(p) or PARMA(p, 0))

Let us consider the original periodic series $X_{r,m}$ , where r denotes the year (r =1,2,…..n) and n is number of years, m is season (m = 1,2….,s) and s is the number of time intervals in the year where (s=12 for monthly data,52 for weekly data and 365 for daily data). Assuming that the distribution of the series is skewed, an appropriated transformation can be used to transform $X_{r,m}$ to the normality series $Y_{r,m}$. Then $W_{r,m}$ is series transformation to standardization from $Y_{r,m}$ normality series as the flowing: (Hipel and McLeod, 1994).

$$W_{r,m} = \frac{Y_{r,m} - \mu_m}{\sigma_m} \quad ,\dots\dots\dots(3\text{-}25)$$

where: $Y_{r,m}$ is series transform to normality from origin series $X_{r,m}$, $W_{r,m}$ is series transform to standardization from normality series $Y_{r,m}$ and $\mu_m$ , $\sigma_m$ are the periodic mean and periodic standard deviation for series $Y_{r,m}$.

In this model the current season value of the dependent stochastic process ( $W_{r,m}$) is expressed as a finite linear aggregate of previous season values of the ( $W_{r,m}$) and a shock process ($a_{r,m}$) for same year. The PAR (p) model is represented in the flowing form.

$$W_{r,m} = \sum_{i=1}^{P} \phi_{i,m} \cdot W_{r,m-i} + a_{r,m} \quad ,\dots\dots\dots(3\text{-}26)$$

where: $\phi_{1,m}, \dots\dots\dots, \phi_{P,m}$ are periodic autoregressive model parameters for each season, p is the order of the model for each season, r is year, m is season and $W_{r,m}$ is the value of stochastic series at time (r,m). (Valenca et. Al., 2005).

By utilizing the backward shift operator β, where $\beta^k W_{r,m} = W_{r,m-k}$ the model in (3-26) can be written as

$$\phi_m(\beta)\ W_{r,m} = a_{r,m} \quad , m=1,2,\dots\dots,s \quad ,\dots\dots\dots(3\text{-}27)$$

where:

$$\phi_m(\beta) = 1 - \phi_{1,m}\,\beta - \phi_{2,m}\,\beta^2 - \dots\dots - \phi_{P,m}\,\beta^p \quad ,\dots\dots\dots(3\text{-}28)$$

This model contains (p + 2) unknown parameters, $\mu_m$ , $\phi_{1,m}$ , $\phi_{2,m}, \dots, \phi_{P,m}$, $(\sigma_a^{\ 2})_m$ which in practice have to be estimated from the data. Where $\mu_m$ is the periodic mean of the stochastic series ($W_{r,m}$) and the additional parameter ($\sigma_a^{\ 2}$) is the periodic variance of the white noise process ($a_{r,m}$).

### 3.7.2 Periodic Moving Average Model (PMA(q) or PARMA(0, q))

In this model each value of $W_{r,m}$ is assumed to arise from the effect of the previous season values of the independent stochastic component ($a_{r,m}$) rather than the dependent stochastic component ($W_{r,m}$) for same year. The following equation is written to this model.

$W_{r,m} = a_{r,m} - \sum_{j=1}^{q} \theta_{j,m} \cdot a_{r,m-j}$ ,……………………………………………(3-29)

where: q is the order of the model for each season m, and($\theta_{1,m}$ ,………, $\theta_{q,m}$) are periodic moving average model parameters for each season.

By utilizing the backward shift operator β, the model in (3-29) can be written as

$W_{r,m} = \theta_m(\beta)\, a_{r,m}$ , m=1,2,………,s ,……………………(3-30)

Where:

$\theta_m(\beta) = 1 - \theta_{1,m}\beta - \theta_{2,m}\beta^2 - \ldots - \theta_{q,m}\beta^q$ ,…………………..(3-31)

This model contains (q + 2) unknown parameters, $\mu_m$ , $\theta_{1,m}$ , $\theta_{2,m}$,…, $\theta_{q,m}$, $(\sigma_a^2)_m$ which in practice have to be estimated from the data. Where $\mu_m$ is the periodic mean of the stochastic series ($W_{r,m}$) and the additional parameter ($\sigma_a^2$) is the periodic variance of the white noise process ($a_{r,m}$).

### 3.7.3 Mixed Model (PARMA$_s$ (p, q)) Periodic Autoregressive Moving Average Model

A useful class of model for time series is formed from a linear combination of PAR(p) and PMA(q) model. A mixed periodic autoregressive moving average model contains PAR (p) term and PMA (q) term that is abbreviated to an PARMA$_s$

(p, q) model of order (p, q ) (Hipel and McLeod, 1994).It is given by the following equation:

$W_{r,m} = \sum_{i=1}^{p} \phi_{i,m} \, . \, W_{r,m-i} - \sum_{j=1}^{q} \theta_{j,m} \, . \, a_{r,m-j} + a_{r,m}$ ,………………(3-32)

Where $\phi_{i,m}$ and $\theta_{j,m}$ are time varying autoregressive and moving average coefficients, respectively, and $a_{r,m}$, is an independent and identically distributed normal random variable. R represents year, m is season and s number of season equal 12 for monthly data in this study.

If the backward shift operator (β) is used, Equation (3-32) may be written in the

Following form:

$\phi_m(\beta) \; W_{r,m} = \theta_m(\beta) \, a_{r,m}$ ,……………………………………………(3-33)

This model contains (P+q+2) unknown parameters,{$\mu_m$, $\phi_{1,m}$ , $\phi_{2,m}$,…, $\phi_{P,m}$, $\theta_{1,m}$ , $\theta_{2,m}$,…, $\theta_{q,m}$, $(\sigma_a^{\,2})_m$}which in practice have to be estimated from the data.

## 3.8 Building of PARMA Model

The general method of forecasting does not assume any particular pattern for the historical data of the series. It uses an iterative approach of identifying a possible useful model from a general class of models. Then, the chosen model is checked against the historical data to see if it accurately describes the series. The model is appropriate if the residuals between the forecasting and the historical data points are small (close to zero), randomly distributed, and independent. If the specified model is not satisfactory, the process is repeated until a satisfactory model is found (Chatfield, 1982).

The three stages that are used in building of statistical models are: (1) Model Identification, (2) Model Estimation (estimation of model parameters), and (3) Model Diagnostic Checking (model verification).( Box and Jenkins, 1976, Chatfield, 2000). Before forecasting future time series by using PARMA models, these three stages of model building are completed as follows:

### 3.8.1 Model Identification

Model identification is the identification of a possible model based on an available realization, i.e., determining the type of the model with appropriate orders. Parameter estimation is the estimation of the model parameters. At this stage, the orders of the model may be further reduced by significance tests on parameters. Diagnostic checks are directed to the residuals of the fitted model to verify the assumptions on the white noise terms such as independence and normality. If verifications fail, model identification stage is to be repeated leading to a new possible model. (Akgun, 2003).

For PARMA models, the periodic autocorrelation function (PeACF) and the periodic partial autocorrelation function (PePACF) serve as useful indicators of the correlation or of the dependence between the values of the series so that they play an important role in model identification (Hipel and McLeod, 1994, Shao, and lund, 2004).see appendix (A-2 and A-3) details about (PeACF & PePACF). The choice of orders of the PMA and PAR parts requires a detailed analysis of these functions, respectively, whose shape and value determine the order of the model.

### 3.8.2 Estimation of Model Parameters

When an appropriate model is proposed for the periodic process using identification procedures, next stage is to estimate the model parameters. Estimation and likelihood evaluation methods for PARMA processes remain complicated, when the number of seasons, s, is large, estimation becomes harder, since this large number of seasons results increase in the number of parameters to be estimated. (Akgun, 2003).

The method of moments, least squares estimation method and maximum likelihood estimation method are widely used parameter estimation methods in hydrological time series analysis as well. The method of moments is one of the most common in time series context but it has serious disadvantages for some situations. Although method of moments can produce good estimators in the case of pure AR processes, they lead to unsatisfactory or even infeasible estimates when the model involves an MA component that estimation becomes difficult because a number of non-linear equations arise which are to be solved simultaneously. Similarly, for PAR processes, method of moments is straightforward and satisfactory, but same problems arise for PARMA and PMA processes. In this study used the Least Squares (LS) method is generally a more efficient parameter estimation method. In this method, the parameters $\phi_{i,m}$ and $\theta_{j,m}$ are estimated by minimizing conditional sum of squares of the residuals defined by (Sveinsson, et.al, 2007).

$SS = \sum_{r=1}^{n} (S_m)$ ,……………………………………………………….(3-34)

$(S_m) = \sum_{m=1}^{s} (a_{r,m}^2)$ ,…………………………………………………….(3-35)

Where s=12 is the number of seasons and n is the number of years of data for r year and m season. For the PARMA (p,q) model, the residuals are defined as

$a_{r,m} = W_{r,m} - \sum_{i=1}^{p} \phi_{i,m} . W_{r,m-i} - \sum_{j=1}^{q} \theta_{j,m} . a_{r,m-j}$ ,…………………...(3-36)

Once the $\phi_{i,m}$ and $\theta_{j,m}$ are determined the seasonal residuals variance $(\sigma_a{}^2)$ can be estimated by

$(\sigma_a{}^2)_m = \frac{1}{n} \sum_{r=1}^{n} (a_{r,m}^2)$ ……………………………………………….(3-37)

It can be seen from (3.35) that, each season m has its own sum of squares of errors $S_m$ and those sum up to the conditional sum of squares SS. Thus, it is apparent that, for any season m, minimizing SS with respect to $\phi$, $\theta$ coefficients in order to obtain least squares estimates (LSE) is equivalent to minimizing $S_m$ for each season separately.

### 3.8.3 Model Diagnostic Checking

After identification of model and estimation of parameters, the fitted model must be tested to determine whether the model complies with the model assumptions and whether the model is capable of reproducing the historical statistical properties of the data at hand.

The PeACF and PePACF are often used to get an idea of the order of the PARMA (p,q) model to fit. An alternative is to use information criteria for selecting the best-fit model. The two information criteria available in this study are the corrected Aikaike information criterion (AIC) and the Schwarz information criterion (SIC) often referred to as the Bayesian information criterion.

The AIC is given by (Sveinsson, et.al, 2007).

$$AIC_m = n \ln \left(\sigma_a^{\ 2}\right)_m + n + \frac{2(k+1)n}{n-k-2} \quad ,\ldots\ldots(3\text{-}38)$$

Where n is the size of the sample used for fitting, m is season, k is the number of parameters excluding constant terms (k = p + q for the PARMA (p,q) model), and $(\sigma_a^{\ 2})_m$ is the maximum likelihood estimate of the seasonal residual variance (biased). The AIC statistic is efficient but not consistent and is good for small samples but tends to overfit for large samples and large k.

The SIC is given by (Sveinsson, et.al, 2007).

$$SIC_m = n \ln \left(\sigma_a^{\ 2}\right)_m + n + k \ln n \quad ,\ldots\ldots(3\text{-}39)$$

Where n, k and $(\sigma_a^{\ 2})_m$ are defined in the same way as for the AIC statistic. In general the SIC is good for large samples, but tends to underfit for small samples.

Diagnostic checking is concerned with the goodness of- fit of a model, and, if the fit is poor, suggests a necessary modification, which means that the whole process of model identification, estimation and diagnostic checking must be repeated. The analysis on residuals resulting from the fitted model is one approach for diagnostic checking. The flowing statistical tests may be used in this study to checks residuals of fitted model; if residuals are random (independent) or not, if residuals are white noise or not and if residuals are normally distributed or not.

**(1)Port Manteau Lack of Fit Test**

It is a test of the residual independency and uses the lag k residual autocorrelation for the m-th season is given by

$$r_k^m\left(a_{r,m}\right) = \frac{\sum_{r=1}^{n} a_{r,m}\, a_{r,m-k}}{\sqrt{\sum_{r=1}^{n}(a_{r,m})^2 \sum_{r=1}^{n}(a_{r,m-k})^2}} \quad ,\ldots\ldots(3\text{-}40)$$

The Port Manteau Lack (Q-statistic) test defined by flowing equation (Hipel and McLeod, 1994).

$$Q_M^m = n\sum_{k=1}^{M}(r_{k,m})^2\,(a_{r,m}) \quad ,\ldots\ldots(3\text{-}41)$$

The Port Manteau Lack (Q-statistic) test modified by Li and Hui 1988 as the flowing equation (Li, 2004).

$$Q_M^m = \frac{(r_{k,m})^2}{\sqrt{\operatorname{var}\left(r_{k,m}(a_{r,m})\right)}} \quad ,\ldots\ldots(3\text{-}42)$$

$$\operatorname{var}\left(r_{k,m}(a_{r,m})\right) = (n - ((k - m + s) / s)) / n^2 \quad ,\ldots\ldots(3\text{-}43)$$

The $a_{r,m}$ is independent If $Q < \chi^2_{\alpha,(M - p_m)}$. Where k=1, …., $\frac{n}{4}$ ,n is size of sample, α is the level of significant, $p_m$ , $q_m$ is the number of model parameters for season m, and the expression (M-$q_m$- $p_m$) represents the degree of freedom, and S=12 number of season. And M is the maximum lag number that is considered about n / 4 (Box and Jenkins, 1976).

**(2)Residual Autocorrelation Function (RACF) Test**

The test for whiteness of the residuals is carried out to determine whether the residuals $a_{r,m}$ are white noise, i.e., PARMA (0,0), or not. An appropriate procedure is to check the residual, to be sure they are random (white noise series) by checking the autocorrelations of residuals to be sure they are not significantly different from zero or within 95% confidence limits. That a series is said to be white noise if its PeACF and PePACF values all fall inside $\pm 1.96/\sqrt{n}$ limits .

**(3)The Normality of Residuals Test**

The normality of residuals, which provides stronger conclusions on the model, is tested through plot the histogram of the residuals and normal probability plot. All these tests for residual autocorrelation function and the normality of residuals are performed through using the Software SPSS (Statistical Package for Social Science).

## 3.9 Forecasting by PARMA$_S$ (p,q) Model

An observation $Z_{r,m}$ for r year and m season my expressed directly in terms of difference equation by

$$\phi_m(\beta)\ Z_{r,m} = \theta_m(\beta)\ a_{r,m} \qquad ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3\text{-}44)$$

The approach for calculating minimum mean square error forecasting by PARMA model at origin r,m for lead time $\ell$, is the conditional expectation $Z_{r,m+\ell}$ of at time r,m. When $\hat{Z}_{r,m}(\ell)$ is as a function of $\ell$ for fixed r, m, it will be called the forecast function for origin r, m. Taking the conditional expectation for equation (3-58) to obtain when $\ell$ is lead time for forecast (Hipel and McLeod, 1994).

$[Z_{r,m+\ell}] = \hat{Z}_{r,m}(\ell) = \phi_{1,m+\ell}[Z_{r,m+\ell-1}] + \phi_{2,m+\ell}[Z_{r,m+\ell-2}] + \ldots + \phi_{p,m+\ell}$ $[Z_{r,m+\ell-p}] + [a_{r,m+\ell}] - \theta_{1,m+\ell}[a_{r,m+\ell-1}] - \theta_{2,m+\ell}[a_{r,m+\ell-2}] - \ldots - \theta_{q,m+\ell}$ $[a_{r,m+\ell-q}]$ ,……………………………………………………………………...(3-45)

By following the four roles listed below, equation (3-59) can be employed for calculating the forecasted for $Z_{r,m}$ and lead time $\ell = 1,2,\ldots$..

1- $E_t[Z_{r,m-\ell}] = Z_{r,m-\ell}$ , $\ell = 0,1,2,\ldots$..
2- $E_t[Z_{r,m+\ell}] = \hat{Z}_{r,m}(\ell)$ , $\ell = 1,2,\ldots$..
3- $E_t[a_{r,m-\ell}] = a_{r,m-\ell}$ , $\ell = 0,1,2,\ldots$..
4- $E_t[a_{r,m+\ell}] = 0$ , $\ell = 1,2,\ldots$..

After calculating the forecasting, value must transform the result to inverse standardization and then inverse normalization.

To obtain probability limits for these forecasts at lead time $\ell$ and also to allow new forecasts to be calculated by a process of updating the old for each season, an approximate $1-\varepsilon$ probability limits $Z_{r,m+\ell}(+)$ and $Z_{r,m+\ell}(-)$ for $Z_{r,m+\ell}$ will be given by

$$Z_{r,m+\ell}(\pm) = \hat{Z}_{r,m}(\ell) \pm u_{\varepsilon/2}\{1+\sum_{j=1}^{\ell-1}\Psi_j^2\}^{\frac{1}{2}}(\sigma_a)_m \quad ,\ldots\ldots(3\text{-}46)$$

Where r,m is the origin season time , $\ell$ is lead time , $Z_{r,m+\ell}(\pm)$ is the upper and lower probability limits for each season, $\hat{Z}_{r,m}(\ell)$ is forecast series , $(\sigma_a)_m$ is residual standard deviation , $u_{\varepsilon/2}$ is the deviate exceeded by a proportion $u_{\varepsilon/2}$=0.68 , or 1.96 where the probability limits of the future value lies in interval 50 % or 95% respectively.

The ψ weights may be computed from the following equation

$$\left.\begin{aligned} &\Psi_1 = \phi_1 - \theta_1 \\ &\Psi_2 = \phi_1 \Psi_1 + \phi_2 - \theta_2 \\ &. \quad . \quad . \quad . \\ &. \quad . \quad . \quad . \\ &\Psi_j = \phi_1 \Psi_{j-1} + \cdots + \phi_p \Psi_{j-p} - \theta_j \end{aligned}\right\} \quad ,\ldots\ldots\ldots\ldots(3\text{-}47)$$

Where $\Psi_0=1$, $\Psi_j=0$ if $j < 0$, and $\theta_j=0$ if $j>q$

In order to evaluate the accuracy of the forecasting and to compare the forecasting ability of the two modeling procedures by following equation

1- The minimum mean absolute relative error (MARE) is represented by (Kurunc ,et. al., 2005)

$$MARE\% = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{X_t - F_t}{X_t}\right| * 100 \quad ,\ldots\ldots\ldots\ldots(3\text{-}48)$$

2- The minimum mean absolute error (MAE) is represented by

$$MAE = \frac{1}{n}\sum_{t=1}^{n}| X_t - F_t| \quad ,\ldots\ldots\ldots\ldots(3\text{-}49)$$

3-The minimum mean square error (MSE) is represented by

$$MSE = \frac{1}{n}\sum_{t=1}^{n}| X_t - F_t|^2 \quad ,\ldots\ldots\ldots\ldots(3\text{-}50)$$

Where $X_t$ is the observed data at time t, $F_t$ is the forecasted data at time t and n is the number of observation. The best model is the one that gives minimum value from equations above.

## 3.10 SARIMA Models

Box-Jenkins approach was developed for handling most time series of data analysis. This approach is about to match one Autoregressive Integrated Moving Average (ARIMA) model based on the historical data that currently available in making forecast. For stationary time series, ARIMA models can be the Autoregressive (AR), Moving Average (MA), or the combination of these two models known as Autoregressive Moving Average (ARMA). If the data is nonstationary, a simple modification of the ARMA model known as integrated (I) processes will be performed to produce an ARI, IMA, or ARIMA model. If there is seasonality in the data, the models become Seasonal Autoregressive (SAR), Seasonal Moving Average (SMA), or Seasonal Autoregressive Moving Average (SARMA) for stationary time series while for nonstationary time series, the models become SARI, SIMA, or SARIMA (Mahpol, 2005).

Box and Jenkins (1970) have generalized the autoregressive integrated moving average, ARIMA (p, d, q), model to deal with seasonality and define a general multiplicative seasonal model. This model is multiplicative of an ARIMA (p, d, q) ×ARIMA (P, D, Q)$_s$. Thus, in addition to the nonseasonal parameters, seasonal parameters for a specified lag need to be estimated (Sabry et. al., 2007). See appendix A for derived these models. The general equation of this model is as follows :( Brockwell and Davis, 2002).

$$\phi_p(\beta)\,\Phi_P(\beta^s)\,W_t = \theta_q(\beta)\,\Theta_Q(\beta^s)\,a_t\ ,\qquad\text{(3-51)}$$

Where $\phi_p$, $\Phi_P$, $\theta_q$ and $\Theta_Q$ are polynomials of order p, P, q, and Q respectively and $a_t$ is purely random process with mean zero and variance $\sigma_a^2$ . According to Box and Jenkins (1970) the parameters value for seasonal ARIMA models ($\phi_p$, $\Phi_P$, $\theta_q$ and $\Theta_Q$ ) are restricted to lie between –1 and +1 (Mahpol, 2005). The values of $W_t$ are derived from the normalized series (the more suitable transformation is natural log transformation) by differencing to remove both trend and seasonal component by the following equation for monthly series:

$W_t = \nabla^d \nabla_{12}^D Z_t$ ,……………………………………………………...(3-52)

Where $Z_t$ is the normalized time series (=ln $X_t$), d is the degree of simple differencing (for trend removal), and D is the degree of seasonal differencing (for seasonality removal). Then, If d=D=1, Equation 3-24 is written as follows:

$$W_t = \nabla \nabla_{12} Z_t = \nabla_{12} Z_t - \nabla_{12} Z_{t-1}$$

$$= (Z_t - Z_{t-12}) - (Z_{t-1} - Z_{t-13})$$

$$= Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13} \text{ ,…………………………………………(3-53)}$$

## 3.11 Building of SARIMA Models

Three stages that are used in building of statistical models are: (1) Model Identification, (2) Model Estimation (estimation of model parameters), and (3) Model Diagnostic Checking (Box and Jenkins, 1976). Before forecasting future time series by using the Box-Jenkins seasonal models, these three stages of model building are completed as follows:

### 3.11.1 Model Identification

Identification means using of the data and any information on how the series was generated to suggest a subclass of parsimonious models. Therefore, the concept of model parsimony is followed in the model identification, i.e. a model with the smallest possible number of parameters is preferable (katsamaki et. al., 1998, quoted in Al-Tikriti, 2001). Identification of model includes the use of the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the dependent stochastic series (Wt). The plot of ACF and PACF of the Wt with time lags (k) is known the correlogram. In plotting correlogram, we can plot approximate 95% confidence limits at -1/N ± 2/√N, which are often further approximated to ±2/√N. The observed values of autocorrelation coefficients which fall outside these limits are significantly different from zero at the 5% level. The same thing is done for the observed partial autocorrelation coefficients, i.e. all values fall outside the above limits are significantly different from zero at the 5% level (Chatfield, 1982).

For stationary series, the correlogram can be used to indicate if a SIAR (p, d)×(P, D)s, SIMA(d, q)×(D, Q)s, or SARIMA (p, d, q)×(P, D, Q)s are appropriate and also to indicate the order of such a process. The ACF of seasonal autoregressive model tails off at lag p and P, where P takes periodic cycles values (for monthly data equal to 12, 24, 36,…), while its PACF has a cut off at lag p and P. Conversely, the ACF of seasonal moving average model has a cut off at lag q and Q, where P takes periodic cycles values (for monthly data equal to 12, 24, 36,…), while its PACF tails off at lag q and Q. If both ACF and PACF tail off, a mixed seasonal autoregressive integrated moving average model is suggested.

### 3.11.2 Model Estimation

The identification process leads to a tentative formulation for the model and needs to obtain an efficient estimate of the parameters (Box and Jenkins, 1976). The conditional and unconditional sum of squares of independent stochastic components ($a_t$) are used in iterative procedures in the exact Estimation of model parameters. The minimum sum of squares determines the maximum likelihood estimate of model parameters. The calculation of the unconditional sum of squares of ARIMA model was explained by Box and Jenkins (1976) who showed that the difference between the conditional and unconditional sum of squares is usually very small. Therefore, the unconditional sum of squares method is used to estimate the parameters of SARIMA models.

Best estimates of model parameters in the least squares sense are those which minimize the sum of squared errors ($\Sigma a_t^2$). If the distribution of the $a_t$'s is normal, the least squares estimates are the close approximation to determine the maximum likelihood estimates of model parameters. The likelihood function actually involves a factor other than $\Sigma a_t^2$ which is a function of the parameters but its influence is small (Naylor et. al., 1972, quated in Al-Ta'ee, 2009).

### 3.11.3 Model Diagnostic Checking (Model validation)

After identifying and estimation of model parameters, Akaike information criteria (AIC) test and the Schwarz information criterion (SIC) also often referred to as the Bayesian information criterion. It has been used to select the best Box & Jenkins seasonal model from the various models. AIC ( p,q) is used to test the parsimonious model ( the best fitted model ) The most parsimonious model is the one that gives minimum AIC or SIC value, defined as

$$AIC = n \ln \sigma_a^2 + 2(p+q) \quad ,\ldots\ldots(3\text{-}54)$$

$$SIC = \ln \{ \sigma_a^2/n \} + (p+d+q) \ln (n)/n \quad ,\ldots\ldots(3\text{-}55)$$

Where $\sigma_a$ is the maximum likelihood estimate of the residual variance, n is sample size and (p,d,q) the number of parameters. The Portmanteau Lack of Fit Test may be used to select the best fitted model for Box-Jenkins seasonal model.

The Box-Pierce (1970) method is based on the calculation of ACF residuals. If the model is adequate at describing the behavior of the time series, the residuals are not correlated. The Portmanteau lack of fit test investigates the first maximum lag (M) for ACF values of the residuals using Box-Pierce chi square statistics which is given in the following expression (Trajkovic, 1999):

$$Q= (n-d-DS) \sum_{k=1}^{M} r_k^2(a_t) \quad ,\ldots\ldots(3\text{-}56)$$

Where n is number of years ,d is normal differencing ,D seasonal differencing and S equal 12 ,the expression (n−d−DS) represents number of terms in the differenced series, $r_k(a_t)$ is the autocorrelation coefficient of the residual component ($a_t$) at lag k, and M is the maximum lag number that is considered about $2\sqrt{n}$ . (Li, 2004) SARIMA model is considered adequate if $Q < \chi^2_{\alpha,(M-np)}$ . Where α is the level of significant, np is the number of model parameters, and the expression (M-np) represents the degree of freedom.

Ljung and Box (1978) suggested the use of the modified Q statistic as flowing (Anderson,et.al., 1982).

$$Q = n(n+2)\sum_{k=1}^{M}\{r_k^2(a_t)/(n-k)\} \quad ,\ldots\ldots(3\text{-}57)$$

Li and Mcloed 1981 recommended the modification Q statistic as flowing (Li, 2004)

$$Q^* = Q + \frac{M(M+1)}{2n} \quad ,\ldots\ldots(3\text{-}58)$$

Where $Q = n\sum_{k=1}^{M} r_k^2(a_t)$ ,……(3-59)

The test of normal distribution for residual as explained in the previous section.

## 3.12 Forecasting by SARIMA Models

We shall be concerned with the forecasting a value $Z_{t+\ell}$, $\ell \geq 1$ when we are currently standing at time t. This forecasting is said to be made at origin t and lead time ℓ (Box and Jenkins, 1976). To obtain this forecast which denoted by $\hat{Z}_t(\ell)$, the following form is written down for the case of seasonal ARIMA (p, d, q) × (P, D, Q)s model as follows:

$$\nabla \nabla_s Z_{t+\ell}\, \phi_p(\beta)\, \Phi_P(\beta^s) = \theta_q(\beta)\, \Theta_Q(\beta^s)\, a_{t+\ell} \quad ,\ldots\ldots(3\text{-}60)$$

Where t is the origin time and $\ell$ is the lead time forecasts.

To obtain probability limits for these forecasts at lead time $\ell$ and also to allow new forecasts to be calculated by a process of updating the old, the following equation gives the upper and the lower probability limits for forecasts at lead time $\ell$ (Box and Jenkins, 1976):

$$Z_{t+\ell}\,(\pm) = \hat{Z}_t\,(\ell) \pm u_{\varepsilon/2}\{1+\sum_{j=1}^{\ell-1}\Psi_j^2\}^{1/2}\sigma_a \quad \text{.........(3-61)}$$

Where $Z_{t+\ell}$ ± is the upper and lower probability limits, $\hat{Z}_t\,(\ell)$ is the forecasted Series at origin time t and lead time $\ell$, $\sigma_a$ is the residual standard deviation (an estimated $S_a^2$ (sampled variance) may be obtained from the time series and may be substituted for ($\sigma_a{}^2$), and $u_{\varepsilon/2}$ is the deviate exceeded by a proportion $u_{\varepsilon/2}$= 0.68 or 1.96 where the probability limits of the future value lies in interval 50 % or 95% respectively.

For seasonal models, suppose there is moving average operator of order one in the following form:

$$(1-\theta\beta)(1 - \Theta\beta^{12}) = (\nabla + \lambda\beta)(\nabla_{12} + \Lambda\beta^{12}) \quad ,\text{.........(3-62)}$$

where $\lambda$=1-$\theta$, $\Lambda$=1-$\Theta$, and $\nabla_{12}$=1- $\beta^{12}$. Hence, the model of order (0, 1, 1)×( 0, 1, 1)$_{12}$ may be written in the following form:

$$\nabla\,\nabla_{12}\,Z_t = (\nabla + \lambda\,\beta)(\nabla_{12} + \Lambda\beta^{12})\,a_t \quad ,\text{.........(3-63)}$$

Writing $\Psi_j$. as, $\Psi_{r,m}$ where r =1, 2, 3, ... and m=1, 2, 3, …, 12 refers respectively to years and months. then,

$$\Psi_{r,m} = \lambda\,(1+r\Lambda) + \delta\,\Lambda \quad ,\text{.........(3-64)}$$

$$\text{where } \delta = \begin{cases} 1 \text{ when } m = 12n \text{ where n is the integer number } (1,2,3, \dots) \\ 0 \text{ when } m \neq 12 \end{cases}$$

Thus, the $\Psi$ weights for this process are as follows:

$\Psi_0 = 1$ ,…………………………………………………………………….(3-65)

$\Psi_1 = \Psi_2 = \ldots = \Psi_{11} = \lambda$ $\Psi_{12} = \lambda + \Lambda$ ,……………………...(3-66)

$\Psi_{13} = \Psi_{14} = \ldots = \Psi_{23} = \lambda\,(1+\Lambda)$ $\Psi_{24} = \lambda\,(1+\Lambda) + \Lambda$ ,…………….(3-67)

$\Psi_{25} = \Psi_{26} = \ldots = \Psi_{35} = \lambda\,(1+2\Lambda)$ $\Psi_{36} = \lambda\,(1+2\Lambda) + \Lambda$ ,………....(3-68)

and so on.

# *Chapter Four*

# *APPLICATION OF PARMA MODEL AND SARIMA MODEL*

## 4.1 Objectives of time-series analysis

Suppose we have data on one or more time series. How do we go about analyzing them? The special feature of time-series data is that successive observations are usually not independent and so the analysis must take into account the order in which the observations are collected. Effectively each observation on the measured variable is a bivariate observation with time as the second variable.

The main objectives of time-series analysis are (Chatfield, 2000):

**(a) Description**. To describe the data using summary statistics and/or graphical methods. A time plot of the data is particularly valuable.

**(b) Modeling**. To find a suitable statistical model to describe the data generating process. A univariate model for a given variable is based only on past values of that variable, while a multivariate model for a given variable may be based, not only on past values of that variable, but also on present and past values of other (predictor) variables. In the latter case, the variation in one series may help to explain the variation in another series. Of course, all models are approximations and model building is an art as much as a science.

**(c) Forecasting**. To estimate the future values of the series. Most authors use the terms 'forecasting' and 'prediction' interchangeably and we follow this convention. There is a clear distinction between steady-state forecasting, where we expect the future to be much like the past, and What-if forecasting where a multivariate model is used to explore the effect of changing policy variables.

**(d) Control**. Good forecasts enable the analyst to take action so as to control a given process, whether it is an industrial process, or an economy or whatever. This is linked to What-if forecasting.

## 4.2 Characteristics of Hydrological Time Series

The structure of hydrological time series consists of mainly one or more of these three basic structural properties and components (Salas, 1993, Tesfaye, 2005):

1. Over year trends and other deterministic changes (such as shifts in the parameters).

   In general, natural and human induced factors may produce gradual and instantaneous trends and shift in hydrological time series Intermittency in the processes, mainly consisting of the hydrology of intermittent sequences of zero and non-zero values.

2. Seasonal or periodic changes of days, weeks or months within the annual cycle.

   Periodicity means that the statistical characteristic changes periodically within the year. For example, in hydrologic data concerning river flows, we expect high runoff periods in the spring and low flow periods in the summer. Thus, the river flow correlations between spring months may be different from the correlations between summer months.

3. Stochasticity or random variations.

The special feature of time series analysis is the fact that successive observations are usually not independent and that the analysis must take into account the time order of the observation. When successive observations are dependent future value may be predicted from past observations. If a time series can be predicted exactly, it is said to be deterministic. But most time series are stochastic in that the future is only partly determined by past values. For stochastic series exact predictions are impossible and must be replaced by the idea that future values have a probability distribution which is conditioned by knowledge of past values, (Chatfield, 1982).

## 4.3 Description of the Study Area and Data

The Greater Zab River is one of the tributaries of Tigris River. It rises in the mountains of Kurdistan and flows south and southwest through southeastern Turkey and northern Iraq see Table 4.1, joining the Tigris south of Mosul city near vestiges Nimrud and flow through the region mountain chain and undulant. Greater Zab River is considered an important tributary of Tigris river so that provides Tigris river with about 33% from the water. See Figure 4.1. (Rosovsky, et. al., 1999).

**Table 4.1: Length and basin area within riparian states for greater Zab River.**

| Riparian states | Basin area | | % | Length of river | | % |
|---|---|---|---|---|---|---|
| | Miles squares | Kilometers squares | | Miles | Kilometers | |
| **Turkey** | **2737** | **7089** | **35** | **137** | **221** | **27.5** |
| **Iraq** | **1274** | **3300** | **65** | **362** | **583** | **72.5** |

We use monthly river flow data for the greater Zab River in this study, The monthly discharge for Zab River from (October 1933 to September 2002) about 70 year are given in table B.1 in appendix B. which collected from Eski-Kelek gauging station on mainstream greater Zab River (planning report on Bekhme dam project, 1986. quoted from Hussein, 2006).The maximum monthly mean discharge for 70 years was in month April about (1002 $m^3/s$ ) and the minimum monthly mean discharge for 70 years was in month September about (126 $m^3/s$ ). The maximum annual mean discharge was in year 1969 about (749 $m^3/s$) and the minimum annual mean discharge was in year 1989 about (117$m^3/s$).

This flow data river from year (1933-1992) is used for basic analysis and ten years (1993-2002) are used for comparison with the series of generation by using the time series models.

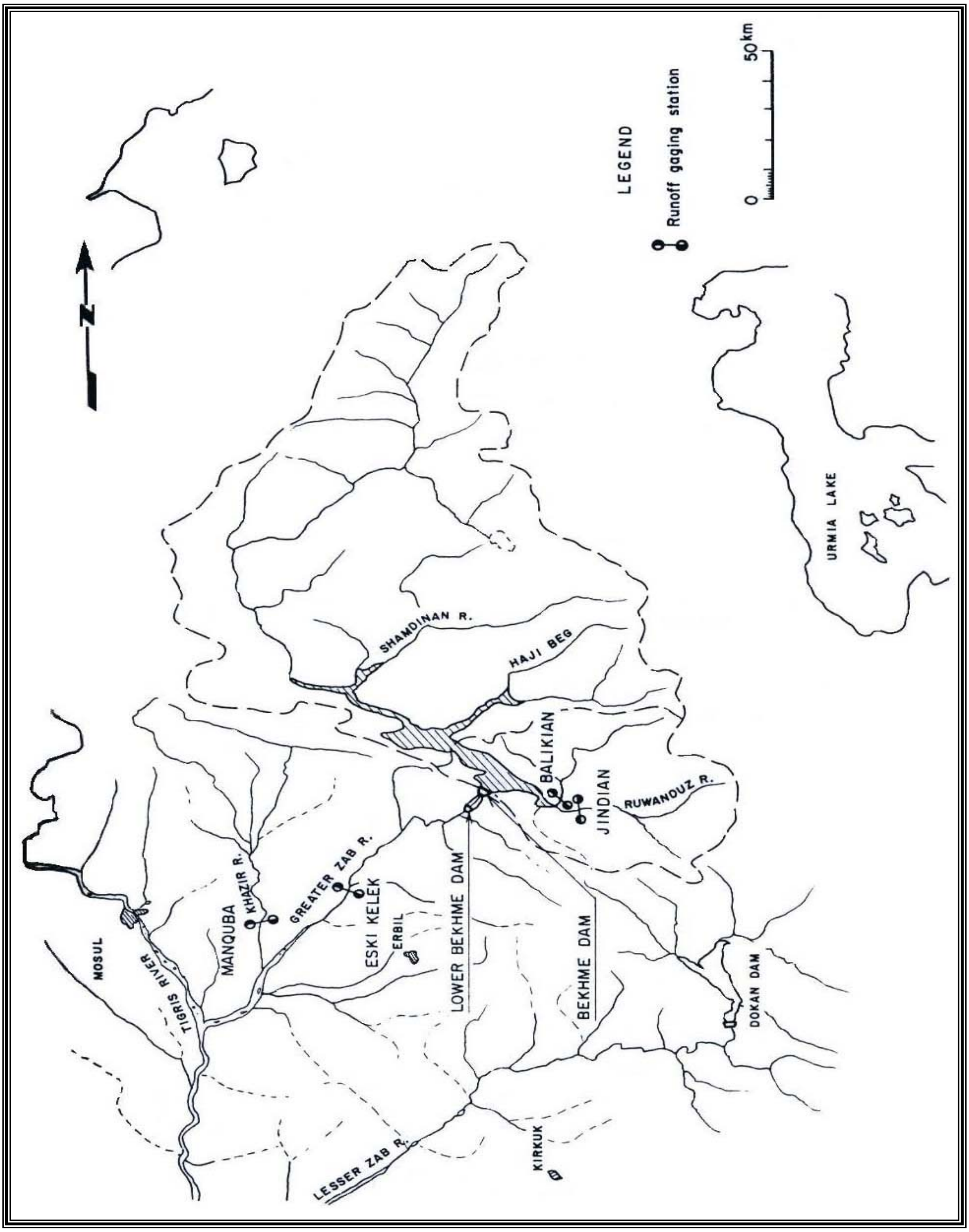



**Figure 4.1: Map of studying area**

## 4.4 Application PARMA$_{12}$(p,q) Model

The series which is considered in this study consists of average monthly flow in (m$^3$/ sec.) values for the Greater Zab River. The series contains 70 years of data from October 1933 to September 2002 and the numbers of observations are 840. We use in this study 60 years of data for building of PARMA model from October 1933 to September 1992 and the number of observations are 720. The rest of the series which corresponds to 10 years from 1993 to 2002 are used for verification of the PARMA model. In this series season 1 corresponds to October and season 12 corresponds to September. The observations are recorded monthly and the period (s) is equal to 12. The original time series is plotted as shown in Figure 4.2. Hence, the periodic behavior is apparent of monthly flow. The sample mean, standard deviation and autocorrelations at lag 1 and lag 2 are given in Table 4.2 (see also Figure 4.3). The nonstationarity of the series is apparent since the mean, standard deviation and correlation functions vary significantly from month to month. For example, the confidence intervals for the means are non-overlapping, indicating statically significant difference. Removing the periodicity in mean and variance will not yield a stationary series. Therefore, a periodically stationary time series model is appropriate Tesfaye (2005).

Before the building of PARMA model, historical season series are transformed to the normal distribution. The periodic component can be detected by plotting the autocorrelation coefficients of the normalized data against the lags. After converting the data to the normal distribution, standardization is applied to the series, and the remaining series is used for building the stochastic models.

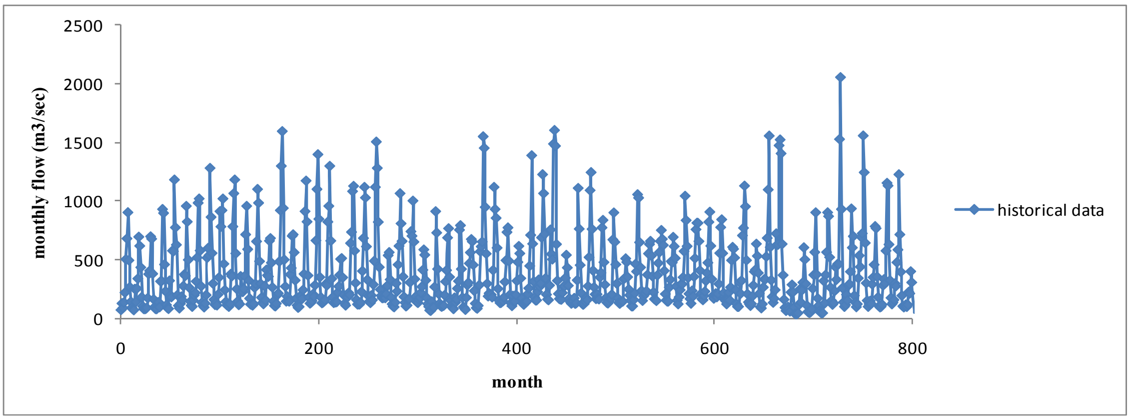



**Figure 4.2 : Historical Monthly Flow for Greater Zab River**

**Table 4.2: Sample mean, standard deviation and autocorrelation at lag 1 and 2 of historical monthly flow series for the Greater Zab River from 1933-2002.**

| season | Month | Parameter | | | |
|---|---|---|---|---|---|
| | | $\hat{\mu}$ | $\hat{\sigma}$ | $\hat{\rho}(1)$ | $\hat{\rho}(2)$ |
| 1 | OCT. | 128.085 | 42.53 | 0.779 | 0.76 |
| 2 | NOV. | 173.314 | 75.245 | 0.436 | 0.329 |
| 3 | DEC. | 232.514 | 135.463 | 0.46 | 0.221 |
| 4 | JAN. | 277.228 | 130.743 | 0.613 | 0.338 |
| 5 | FEB. | 405.214 | 169.058 | 0.649 | 0.449 |
| 6 | MAR. | 608.157 | 278.297 | 0.563 | 0.576 |
| 7 | APR. | 1001.714 | 340.004 | 0.645 | 0.546 |
| 8 | MAY | 964.385 | 389.334 | 0.782 | 0.55 |
| 9 | JUN. | 574.285 | 203.849 | 0.839 | 0.713 |
| 10 | JUL. | 288.4 | 108.673 | 0.858 | 0.688 |
| 11 | AUG. | 163.785 | 56.173 | 0.944 | 0.787 |
| 12 | SEP. | 125.414 | 37.623 | 0.931 | 0.82 |

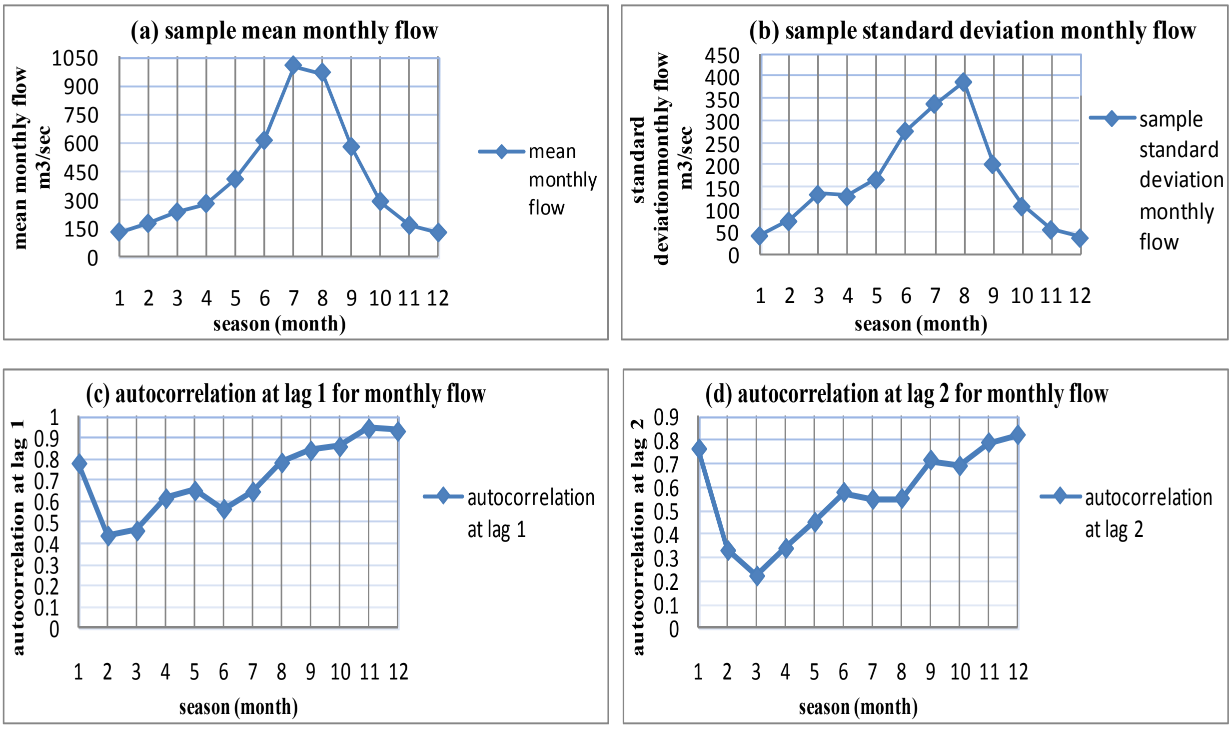



**Figure 4.3 : Sample mean, standard deviation and autocorrelation at lag 1 and 2 of historical monthly flow series for the Greater Zab River from 1933-2002.**

The procedure which is used for data analysis is shown schematically in Figure 4.4 and can be summarized by the following steps:

1. Use a suitable transformation like (Box-Cox, square root, or log transformation) to normalize the data.
2. Plot the correlogram of monthly means and standard deviations of normalized data to detect the periodicity.
3. Standardize the remaining series by subtracting mean and dividing by standard deviation.
4. Identify models by plotting the periodic autocorrelation function (PeACF) and periodic partial autocorrelation function (PePACF) of the series.
5. Estimation of model parameter by using the general least squares algorithm for the conditional model.

6. Apply diagnostic checks test independency and normality of the residuals.
7. Forecast and verify the models.

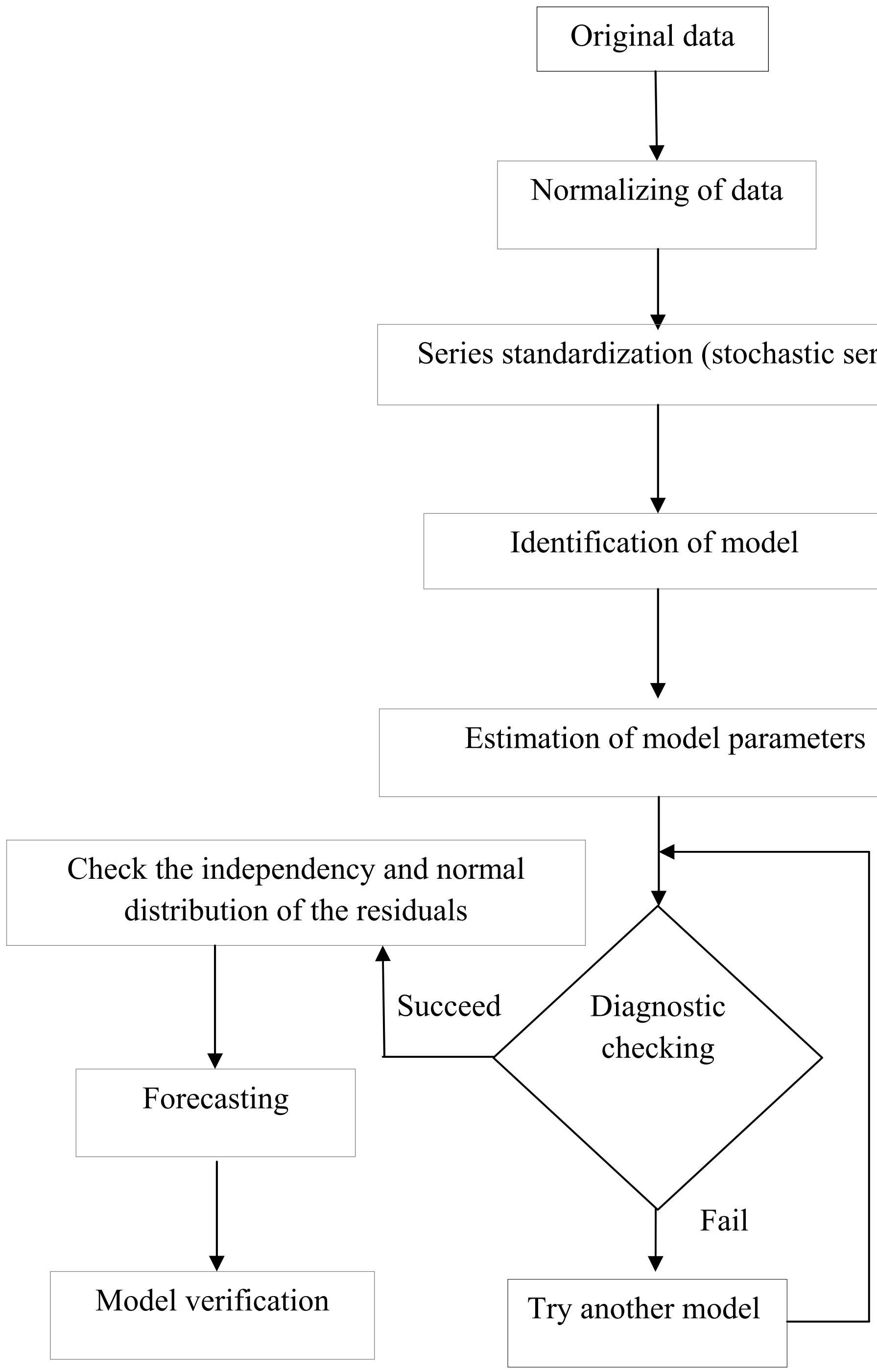



**Figure 4.4: Steps of PARMA Model for each season.**

## 4.5 Transformating Data to Normal Distribution

Before transformation data to normal distribution, the coefficients of skewness (Cs) and Coefficients of kurtosis ($C_k$) must be determined by equation (3-6 and 3-7). Table 4.3 shows the values of the four moments for each season of the original data series of monthly flow for Greater Zab river for period from 1933-1992, and this table indicates that the series is not normal because Cs ≠ 0 and $C_k \neq 3$. Several transformations are used to normalize the data but the most common and useful class of transforms for stabilizing the variance is known as the Box - Cox transformation ( Chandra et. al, 1978 ), as mentioned in section (3.3) and applied the equation (3-8 and 3-9), we choose variant values of $\lambda$ between 0 and 1 which give normally distributed data when ($C_s \approx 0$) and $(C_k \approx 3)$.

**Table 4.3: The values of the statistical characteristics of historical monthly flow for Greater Zab River of period 1933-1992.**

| season | month | parameter | | | |
|---|---|---|---|---|---|
| | | mean | standard deviation | Cs | $C_k$ |
| 1 | Oct. | 130.833 | 43.835 | 0.132 | 2.513 |
| 2 | Nov. | 174.417 | 71.91 | 0.862 | 3.7 |
| 3 | Dec. | 226.783 | 129.96 | 3.1 | 14.46 |
| 4 | Jan. | 272.433 | 130.61 | 1.299 | 4.51 |
| 5 | Feb. | 409.283 | 168.26 | 0.895 | 4.498 |
| 6 | Mar. | 624.733 | 287.61 | 1.637 | 6.318 |
| 7 | Apr. | 988.517 | 311.21 | 0.602 | 3.116 |
| 8 | May | 969.667 | 345.37 | 0.469 | 2.64 |
| 9 | Jun. | 583.817 | 186.57 | 0.142 | 3.411 |
| 10 | Jul. | 300.033 | 106.92 | 0.327 | 3.299 |
| 11 | Aug. | 169.983 | 55.953 | 0.301 | 3.41 |
| 12 | Sep. | 129.967 | 37.65 | 0.029 | 3.136 |

Table 4.4 shows values of mean, Sd, $C_s$ and $C_k$ of each season corresponding to different values of λ. For normally distributed data the ($C_s \approx 0$) and $(C_k \approx 3)$. Table 4.5 shows values of mean, Sd, $C_s \approx 0$ and $(C_k \approx 3)$ of each season corresponding to best values of λ. Figure 4.5 show skewnees test of normality.

**Table 4.4: Values of mean, Sd, $C_s$ and $C_k$ of each season corresponding to different values of λ**

| season | λ | mean | Sd | Cs | $C_k$ |
|---|---|---|---|---|---|
| 1 | 0.1 | 6.1878 | 0.6 | -0.674 | 3.221 |
| | 0.2 | 8.1202 | 0.9594 | -0.575 | 3.055 |
| | 0.3 | 10.864 | 1.5371 | -0.479 | 2.914 |
| | 0.4 | 14.806 | 2.4674 | -0.385 | 2.796 |
| | 0.5 | 20.53 | 3.9683 | -0.293 | 2.7 |
| | 0.6 | 28.923 | 6.3937 | -0.203 | 2.626 |
| | 0.7 | 41.341 | 10.32 | -0.116 | 2.571 |
| | 0.8 | 59.867 | 16.685 | -0.031 | 2.535 |
| | 0.9 | 87.708 | 27.023 | 0.051 | 2.516 |
| | 1 | 129.83 | 43.835 | 0.132 | 2.513 |
| 2 | 0.1 | 6.626 | 0.7072 | -0.323 | 3.447 |
| | 0.2 | 8.8462 | 1.1683 | -0.174 | 3.303 |
| | 0.3 | 12.069 | 1.9361 | -0.029 | 3.21 |
| | 0.4 | 16.808 | 3.2181 | 0.111 | 3.164 |
| | 0.5 | 23.863 | 5.3648 | 0.247 | 3.162 |
| | 0.6 | 34.48 | 8.9686 | 0.378 | 3.201 |
| | 0.7 | 50.62 | 15.034 | 0.505 | 3.277 |
| | 0.8 | 75.383 | 25.268 | 0.628 | 3.387 |
| | 0.9 | 113.69 | 42.575 | 0.746 | 3.529 |
| | 1 | 173.42 | 71.91 | 0.862 | 3.7 |
| 3 | 0.1 | 7.0375 | 0.741 | 1.056 | 5.577 |
| | 0.2 | 9.5413 | 1.2926 | 1.279 | 6.228 |
| | 0.3 | 13.247 | 2.2646 | 1.507 | 6.986 |
| | 0.4 | 18.812 | 3.9853 | 1.738 | 7.847 |
| | 0.5 | 27.283 | 7.0454 | 1.972 | 8.802 |
| | 0.6 | 40.339 | 12.511 | 2.205 | 9.838 |
| | 0.7 | 60.694 | 22.317 | 2.437 | 10.94 |
| | 0.8 | 92.768 | 39.981 | 2.665 | 12.09 |
| | 0.9 | 143.81 | 71.935 | 2.887 | 13.27 |
| | 1 | 225.78 | 129.96 | 3.1 | 14.46 |

**Table 4.4:Cont.**

| season | λ | mean | Sd | Cs | $C_k$ |
|---|---|---|---|---|---|
| 4 | 0.1 | 7.3581 | 0.7839 | 0.243 | 2.937 |
| | 0.2 | 10.096 | 1.3688 | 0.369 | 2.984 |
| | 0.3 | 14.207 | 2.397 | 0.492 | 3.067 |
| | 0.4 | 20.476 | 4.2095 | 0.614 | 3.186 |
| | 0.5 | 30.168 | 7.4129 | 0.733 | 3.338 |
| | 0.6 | 45.345 | 13.09 | 0.85 | 3.52 |
| | 0.7 | 69.387 | 23.175 | 0.966 | 3.731 |
| | 0.8 | 107.87 | 41.139 | 1.079 | 3.967 |
| | 0.9 | 170.06 | 73.211 | 1.19 | 4.228 |
| | 1 | 271.43 | 130.61 | 1.299 | 4.51 |
| 5 | 0.1 | 8.098 | 0.8149 | -0.893 | 5.53 |
| | 0.2 | 11.41 | 1.4466 | -0.64 | 4.899 |
| | 0.3 | 16.545 | 2.5817 | -0.403 | 4.432 |
| | 0.4 | 24.639 | 4.6305 | -0.182 | 4.114 |
| | 0.5 | 37.592 | 8.344 | 0.025 | 3.927 |
| | 0.6 | 58.603 | 15.101 | 0.218 | 3.857 |
| | 0.7 | 93.098 | 27.442 | 0.401 | 3.89 |
| | 0.8 | 150.34 | 50.055 | 0.573 | 4.014 |
| | 0.9 | 246.24 | 91.623 | 0.737 | 4.219 |
| | 1 | 408.28 | 168.26 | 0.895 | 4.498 |
| 6 | 0.1 | 8.8828 | 0.7854 | 0.466 | 3.126 |
| | 0.2 | 12.859 | 1.4981 | 0.585 | 3.325 |
| | 0.3 | 19.227 | 2.8645 | 0.707 | 3.562 |
| | 0.4 | 29.618 | 5.4911 | 0.833 | 3.838 |
| | 0.5 | 46.863 | 10.553 | 0.961 | 4.154 |
| | 0.6 | 75.908 | 20.334 | 1.092 | 4.51 |
| | 0.7 | 125.48 | 39.282 | 1.226 | 4.906 |
| | 0.8 | 211.1 | 76.082 | 1.362 | 5.341 |
| | 0.9 | 360.52 | 147.74 | 1.499 | 5.812 |
| | 1 | 623.73 | 287.61 | 1.637 | 6.318 |
| 7 | 0.1 | 9.8397 | 0.6383 | -0.267 | 3.512 |
| | 0.2 | 14.701 | 1.2613 | -0.151 | 3.309 |
| | 0.3 | 22.778 | 2.4969 | -0.041 | 3.158 |
| | 0.4 | 36.472 | 4.9515 | 0.063 | 3.054 |
| | 0.5 | 60.107 | 9.8357 | 0.162 | 2.99 |
| | 0.6 | 101.54 | 19.569 | 0.257 | 2.961 |
| | 0.7 | 175.14 | 38.994 | 0.348 | 2.962 |
| | 0.8 | 307.42 | 77.816 | 0.435 | 2.991 |
| | 0.9 | 547.59 | 155.51 | 0.52 | 3.043 |
| | 1 | 987.52 | 311.21 | 0.602 | 3.116 |
| 8 | 0.1 | 9.7713 | 0.7451 | -0.459 | 3.569 |
| | 0.2 | 14.573 | 1.4611 | -0.327 | 3.284 |
| | 0.3 | 22.538 | 2.8721 | -0.204 | 3.063 |
| | 0.4 | 36.023 | 5.6587 | -0.089 | 2.896 |
| | 0.5 | 59.268 | 11.174 | 0.018 | 2.774 |
| | 0.6 | 99.969 | 22.109 | 0.119 | 2.692 |
| | 0.7 | 172.21 | 43.834 | 0.214 | 2.642 |
| | 0.8 | 301.96 | 87.07 | 0.303 | 2.619 |
| | 0.9 | 537.44 | 173.26 | 0.388 | 2.62 |
| | 1 | 968.67 | 345.37 | 0.469 | 2.64 |

**Table 4.4:Cont.**

| season | λ | mean | Sd | Cs | Ck |
|---|---|---|---|---|---|
| 9 | 0.1 | 8.8015 | 0.7119 | -1.47 | 7.317 |
| | 0.2 | 12.7 | 1.3023 | -1.232 | 6.349 |
| | 0.3 | 18.914 | 2.3926 | -1.012 | 5.559 |
| | 0.4 | 28.999 | 4.4136 | -0.808 | 4.927 |
| | 0.5 | 45.629 | 8.1725 | -0.619 | 4.433 |
| | 0.6 | 73.437 | 15.186 | -0.445 | 4.057 |
| | 0.7 | 120.52 | 28.31 | -0.283 | 3.781 |
| | 0.8 | 201.09 | 52.934 | -0.132 | 3.591 |
| | 0.9 | 340.29 | 99.251 | 0.009 | 3.471 |
| | 1 | 582.82 | 186.57 | 0.142 | 3.411 |
| 10 | 0.1 | 7.5688 | 0.737 | -1.388 | 7.375 |
| | 0.2 | 10.46 | 1.2584 | -1.109 | 6.167 |
| | 0.3 | 14.836 | 2.1606 | -0.859 | 5.237 |
| | 0.4 | 21.56 | 3.7286 | -0.635 | 4.538 |
| | 0.5 | 32.035 | 6.4641 | -0.435 | 4.031 |
| | 0.6 | 48.553 | 11.253 | -0.255 | 3.679 |
| | 0.7 | 74.893 | 19.662 | -0.091 | 3.452 |
| | 0.8 | 117.31 | 34.472 | 0.059 | 3.325 |
| | 0.9 | 186.21 | 60.624 | 0.198 | 3.279 |
| | 1 | 299.03 | 106.92 | 0.327 | 3.299 |
| 11 | 0.1 | 6.6182 | 0.6228 | -1.127 | 5.763 |
| | 0.2 | 8.8276 | 1.0137 | -0.93 | 5.151 |
| | 0.3 | 12.028 | 1.656 | -0.743 | 4.651 |
| | 0.4 | 16.723 | 2.7144 | -0.567 | 4.251 |
| | 0.5 | 23.691 | 4.4634 | -0.402 | 3.941 |
| | 0.6 | 34.141 | 7.3615 | -0.245 | 3.711 |
| | 0.7 | 49.963 | 12.176 | -0.097 | 3.551 |
| | 0.8 | 74.129 | 20.193 | 0.042 | 3.453 |
| | 0.9 | 111.32 | 33.574 | 0.175 | 3.408 |
| | 1 | 168.98 | 55.953 | 0.301 | 3.41 |
| 12 | 0.1 | 6.1983 | 0.5326 | -1.115 | 5.274 |
| | 0.2 | 8.1334 | 0.8473 | -0.964 | 4.83 |
| | 0.3 | 10.879 | 1.3512 | -0.82 | 4.447 |
| | 0.4 | 14.82 | 2.1598 | -0.682 | 4.121 |
| | 0.5 | 20.536 | 3.4602 | -0.549 | 3.847 |
| | 0.6 | 28.907 | 5.5554 | -0.423 | 3.623 |
| | 0.7 | 41.272 | 8.9379 | -0.302 | 3.443 |
| | 0.8 | 59.682 | 14.408 | -0.187 | 3.305 |
| | 0.9 | 87.291 | 23.27 | -0.077 | 3.204 |
| | 1 | 128.97 | 37.65 | 0.029 | 3.136 |

**Table 4.5: Values of mean, Sd, $C_s \approx 0$ and $(C_k \approx 3)$ of each season corresponding to best values of λ for converting data to normally distribution.**

| Season | Month | λ | mean | Sd | Cs | Ck |
|---|---|---|---|---|---|---|
| 1 | Oct | 0.837 | 68.864 | 19.94 | -0.0005 | 2.526 |
| 2 | Nov | 0.321 | 12.917 | 2.1536 | 0.0006 | 3.1967 |
| 3 | Dec | -0.405 | 2.1786 | 0.0476 | 0.0363 | 3.8937 |
| 4 | Jan | -0.088 | 4.3574 | 0.2771 | -0.0003 | 2.9587 |
| 5 | Feb | 0.488 | 35.691 | 7.7729 | 0.0006 | 3.9431 |
| 6 | Mar | -0.317 | 2.7293 | 0.0546 | 0.00424 | 2.6859 |
| 7 | Apr | 0.339 | 27.267 | 3.2604 | 0.00032 | 3.1125 |
| 8 | May | 0.483 | 54.357 | 9.9517 | 0.00023 | 2.792 |
| 9 | Jun | 0.893 | 327.85 | 94.97 | -0.0005 | 3.4771 |
| 10 | Jul | 0.759 | 97.427 | 27.373 | -0.0011 | 3.3661 |
| 11 | Aug | 0.769 | 65.505 | 17.258 | -0.0002 | 3.4772 |
| 12 | Sep | 0.972 | 115.51 | 32.899 | -0.0004 | 3.1518 |

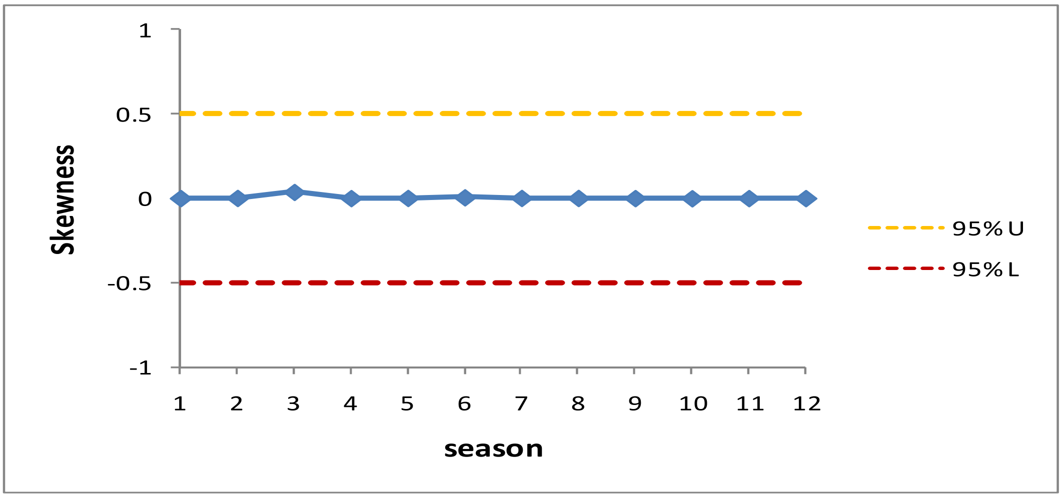



**Figure 4.5: Skewnees test of normality series of each season**

Table 4.6 shows a comparison between transformation procedures. The best procedure which displays the value of skewness coefficient ($C_s \approx 0$) and kurtosis coefficient $(Ck \approx 3)$ is the Box-Cox transformation for each season. The normality was tested by plotting observed cumulative probability against expected cumulative probability. The resulting plots are shown in Figure 4.6 .All these figures show good agreement.

**Table 4.6: Comparison Cs & Ck of among the three Transformation procedures for monthly flow Greater Zab river.**

| | **Cs** | | | **Ck** | | |
|---|---|---|---|---|---|---|
| **Transform / Season** | **Box-Cox** | **Log** | **Square root** | **Box-Cox** | **Log** | **Square root** |
| 1 | -0.0005 | -0.7752 | -0.2929 | 2.526 | 3.4123 | 2.7002 |
| 2 | 0.0006 | -0.4768 | 0.24672 | 3.1967 | 3.6434 | 3.1622 |
| 3 | 0.0363 | 0.83977 | 1.97159 | 3.8937 | 5.0353 | 8.8021 |
| 4 | -0.0003 | 0.11455 | 0.73308 | 2.9587 | 2.9302 | 3.3382 |
| 5 | 0.0006 | -1.1613 | 0.02468 | 3.9431 | 6.3424 | 3.9272 |
| 6 | 0.00424 | 0.35026 | 0.96121 | 2.6859 | 2.9651 | 4.1542 |
| 7 | 0.00032 | -0.3905 | 0.16234 | 3.1125 | 3.7743 | 2.9897 |
| 8 | 0.00023 | -0.6001 | 0.018 | 2.792 | 3.9276 | 2.7743 |
| 9 | -0.0005 | -1.7253 | -0.6194 | 3.4771 | 8.4831 | 4.4329 |
| 10 | -0.0011 | -1.6987 | -0.4349 | 3.3661 | 8.9007 | 4.0313 |
| 11 | -0.0002 | -1.3357 | -0.4016 | 3.4772 | 6.4977 | 3.9413 |
| 12 | -0.0004 | -1.2714 | -0.5493 | 3.1518 | 5.7811 | 3.8472 |

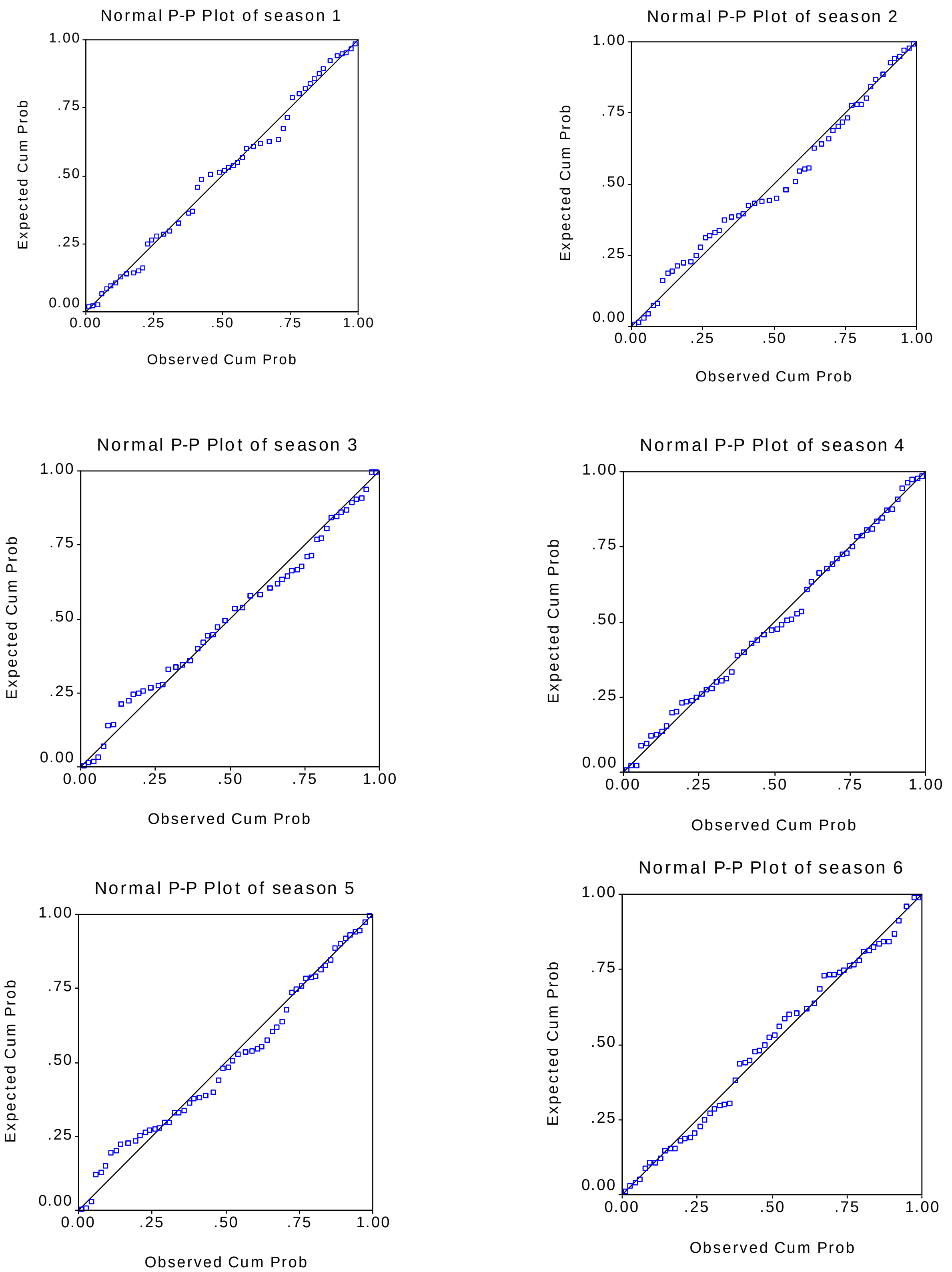
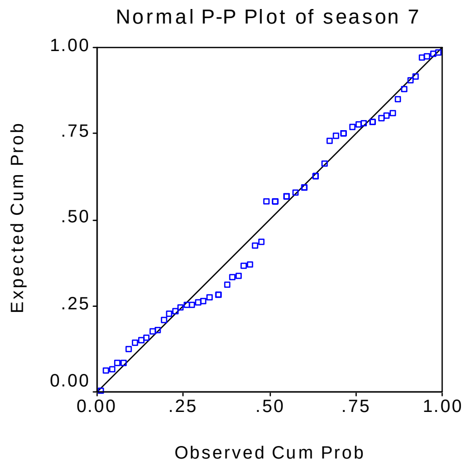
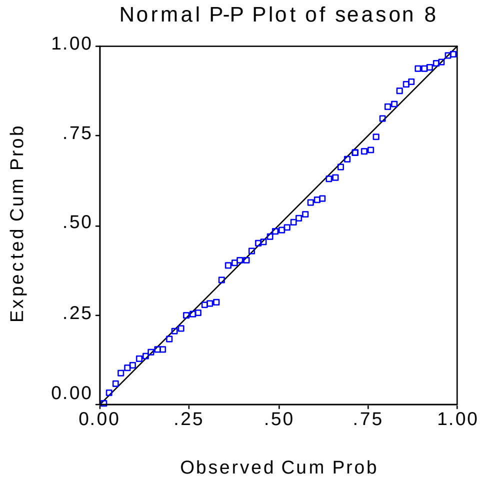
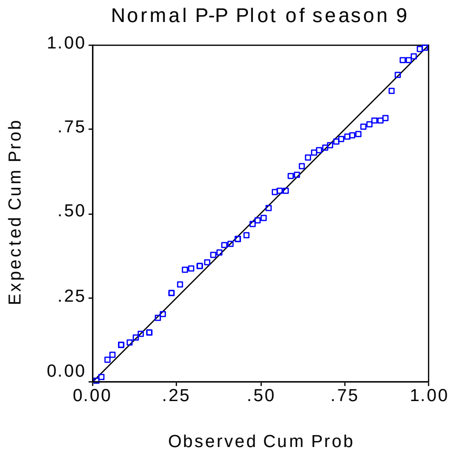
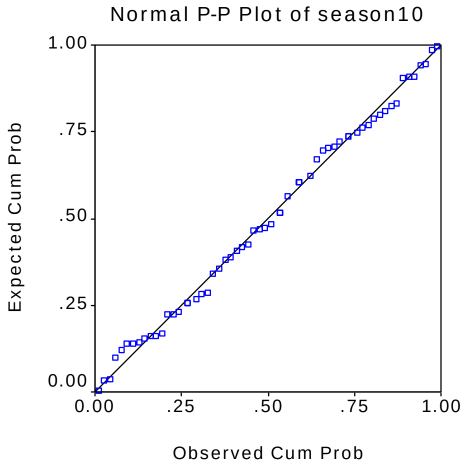


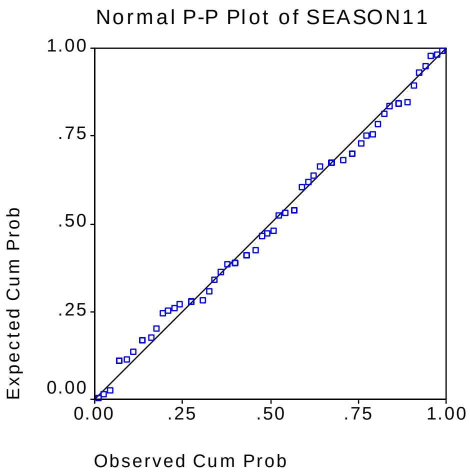

**Figure 4.6: Normal distribution test for data convert by Box-Cox transform**



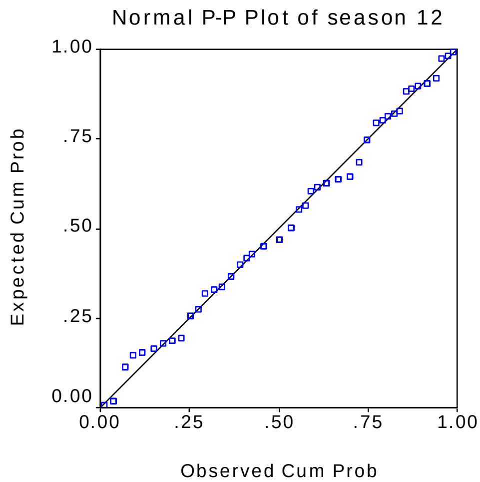




**Figure 4.6: Cont.**

## 4.6 Transformating Data to standardization

Generally, plotting normalized monthly flow data shows seasonal patterns which may be due to the influence of the annual cyclic pattern of the hydrological inputs to the rive ( Kurunc et. al., 2005). If a time series contains a seasonal fluctuation then the correlogram which is a plot of autocorrelation coefficient ($r_k$ ) against the lag ( k) will also exhibit an oscillation at the same frequency (Chatfield, 1982), then the periodicity can be detected by the correlogram. The correlogram of normalized data for monthly flow Greater Zab River are shown in Figure 4.7. This figure indicates that the periodicity is found in the series because the correlogram is periodic too.

Removal of periodicity from data is done by equation (3-22). This method is referred to as the non-parametric method of cyclic standardization (Srikanthan and McMahon, 1982) and the result series is called the standardized series. Figure 4.8 shows correlogram series before and after standardization data.

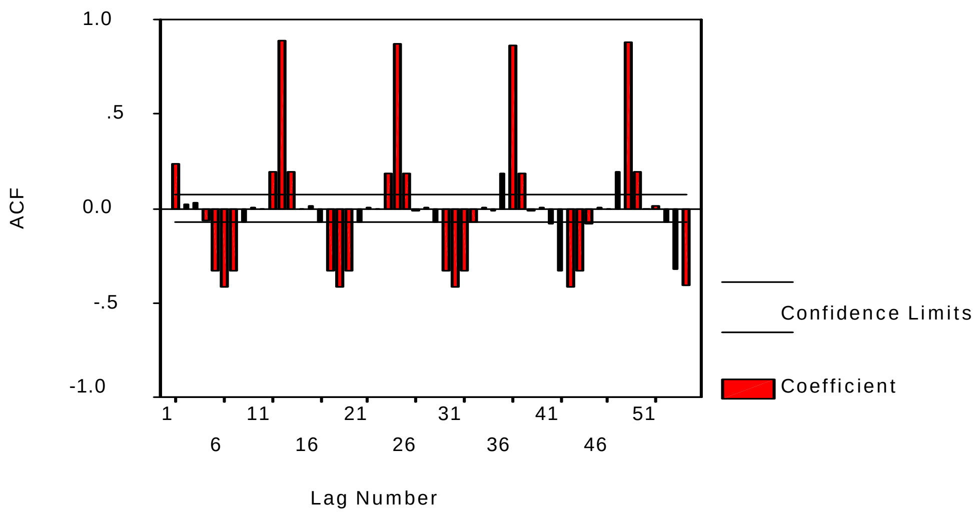



**Figure 4.7:Correlogram of normalized series explain the periodicity in series of monthly flow Greater Zab river**

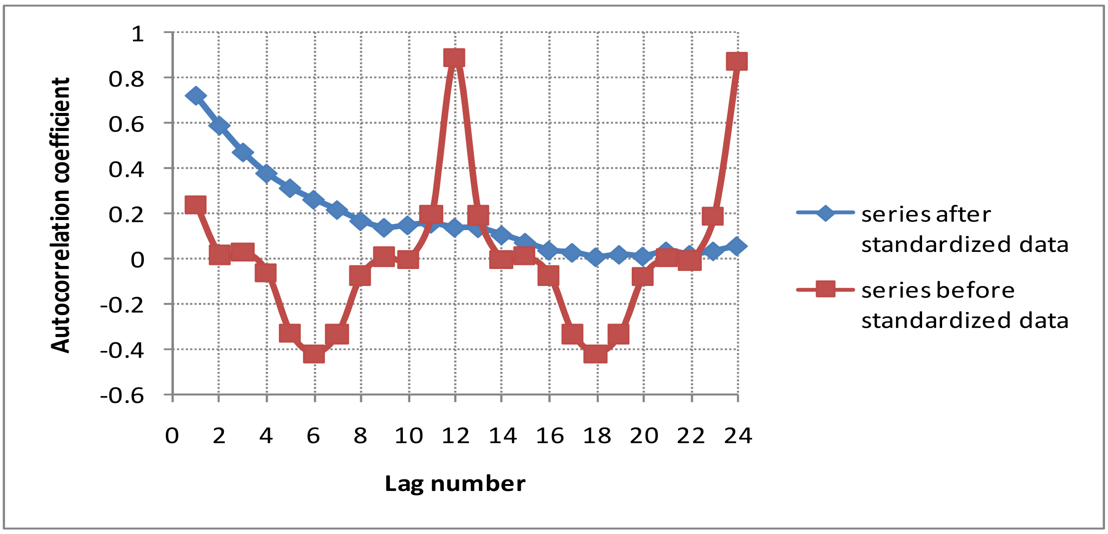



**Figure 4.8: Correlogram series before and after standardization**

## 4.7 Stochastic Model

This section contains three steps of model building, identification, estimation of model parameters and diagnostic checks. These stages are made after normalization and standardization.

### 4.7.1 Model Identification

As explained in section (3.8) periodic autocorrelation function (PeACF) and periodic partial autocorrelation function (PePACF) are an important guide to the properties of a time series because they often provide insight into the probability model which generated the data.

The sample PeACF and sample PePACF graphs for each 12 seasons are shown in Figures 4.9, which in turn show the behavior of the periodic autocorrelation function (PeACF) and periodic partial autocorrelation function (PePACF) of monthly flow for Greater Zab river with 95% confidence limits. The values are drawn versus time lag. In practice, maximum lag is taken as one fourth of the number of years, however, this value here is, $N / 4 = 60 /4 = 15$, For example if its suggested model is

PAR(1) because the (PeACF) ) tails off while its (PePACF) has a cut off after lag (1). In this study eight suggested models are PAR(1), PAR(2), PMA(1), PMA(2), PARMA(1,1), PARMA(2,2), PARMA(1,2), PARMA(2,1) and further estimation will be presented in next section.

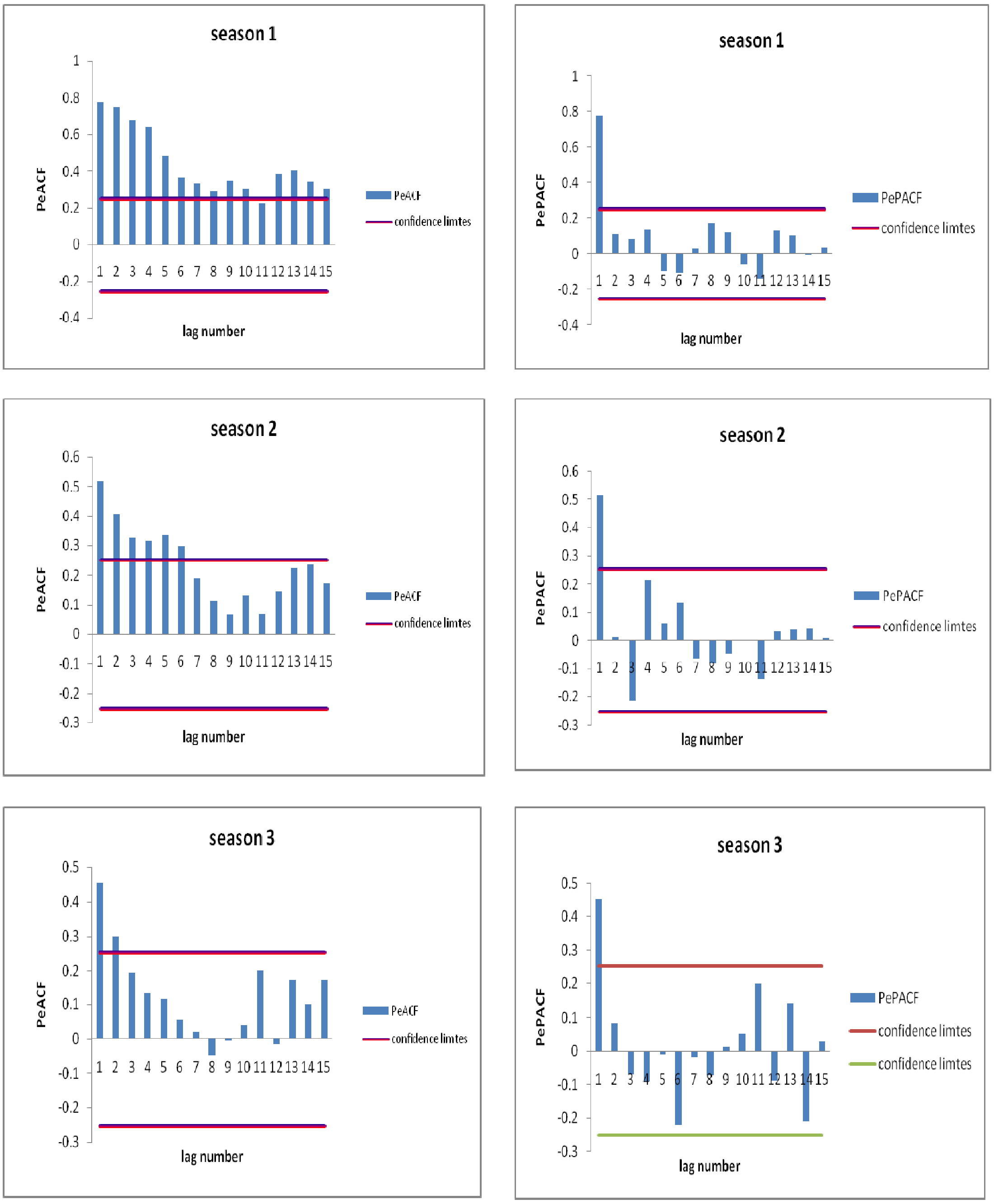



**Figure 4.9: Periodic autocorrelation function and periodic partial autocorrelation function for each season of monthly flow Zab River**

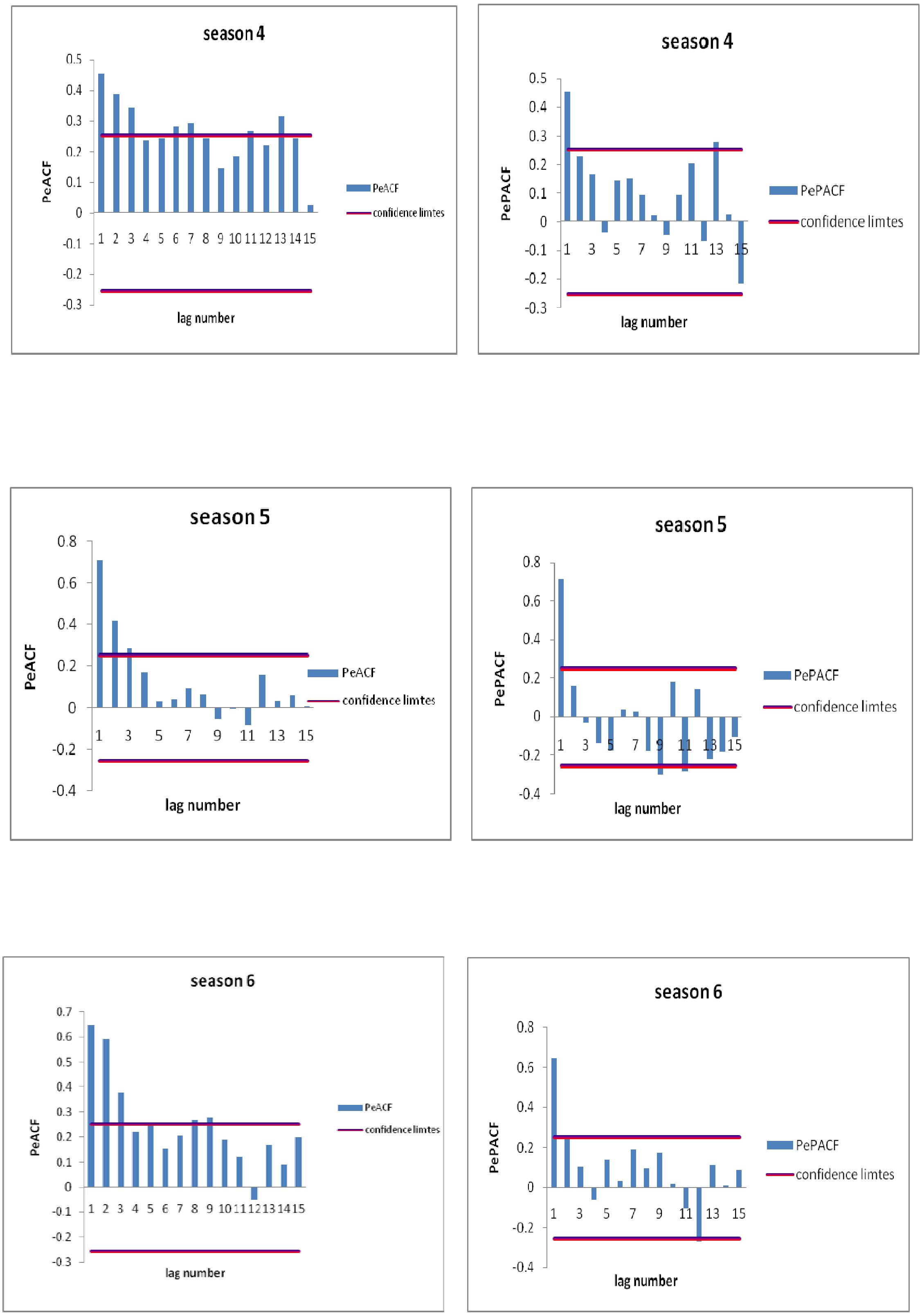



**Figure 4.9: Cont.**

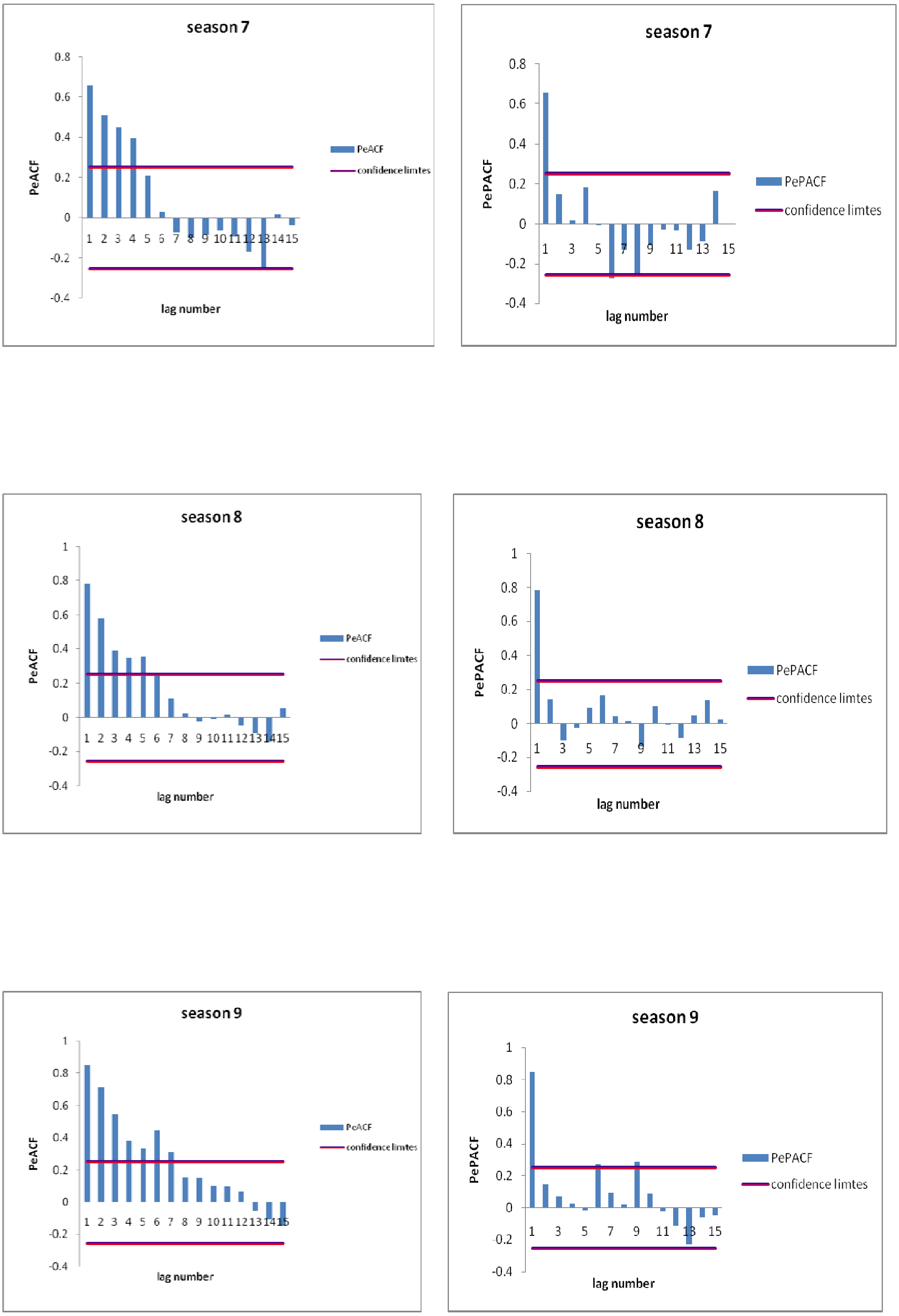



**Figure 4.9: Cont.**

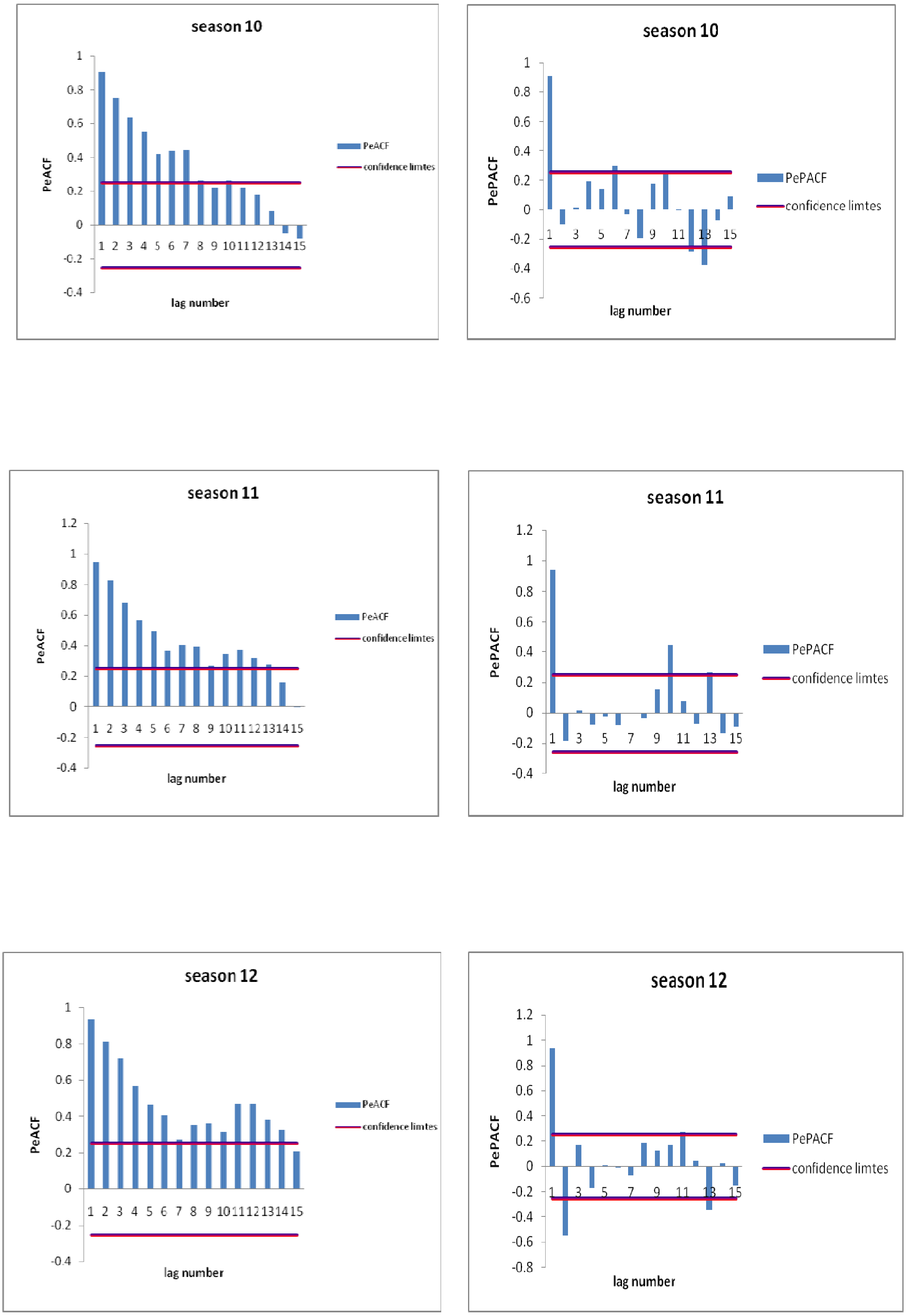



**Figure 4.9: Cont.**

## 4.7.2 Estimation of Model Parameters

The identification process has led to a tentative formulation for the model, then, it is needed to obtain efficient estimates of the model parameters. Conditional sum of squares method explained in chapter 3 is used in estimation of model parameter by using computer program. The process is started by using equation (3-36), initial values of $(\phi, \theta)$ between (1 , -1) are used. The value (0.01) is added to the values $(\phi \,\&\, \theta)$ of the last iteration, the process is repeated with the next iteration until getting the minimum sum of squares $S(\phi, \theta)$.

As an example, the calculation of min. sum of squared for season one of monthly flow of greater Zab River is shown in Table 4.7 for PAR(1) or PARMA(1,0) model and other tables for seasons are given in appendix (B)(tables from (B.2) to (B.12)). Table 4.8 shows the results of estimation of parameters for suggested models of each season.

## Table 4.7:Calculated min sum of square for season one with PARMA(1,0) model and $\phi_{1,1} = 0.77$

| year | $W_{r,1}$ | $W_{r,12}$ | $\phi_{1,1}W_{r,12}$ | $a_{r,1}$ | $a^2_{r,1}$ | year | $W_{r,1}$ | $W_{r,12}$ | $\phi_{1,1}W_{r,12}$ | $a_{r,1}$ | $a^2_{r,1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | 0 | | | | 31 | -0.98789 | 1.93828 | -0.75359 | -0.2343 | 0.0549 |
| 1 | -1.30412 | -0.86624 | 0 | -1.30412 | 1.70073 | 32 | 2.1889 | 0.294091 | 1.502167 | 0.686722 | 0.47159 |
| 2 | -1.13281 | -1.21165 | -0.67133 | -0.46147 | 0.21296 | 33 | 0.2996 | -0.46928 | 0.227921 | 0.071683 | 0.00514 |
| 3 | -1.08431 | -1.21165 | -0.93903 | -0.14528 | 0.02111 | 34 | 1.67254 | -0.12647 | -0.3637 | 2.036233 | 4.14624 |
| 4 | -1.08431 | -0.97237 | -0.93903 | -0.14528 | 0.02111 | 35 | 0.34385 | 0.922473 | -0.09802 | 0.441863 | 0.19524 |
| 5 | -1.03601 | -1.05205 | -0.75359 | -0.28242 | 0.07976 | 36 | 0.80268 | 1.183521 | 0.714917 | 0.087756 | 0.0077 |
| 6 | 0.07684 | -0.33731 | -0.81534 | 0.892189 | 0.796 | 37 | 0.84586 | 1.313887 | 0.917229 | -0.07137 | 0.00509 |
| 7 | -0.56238 | -0.44288 | -0.26141 | -0.30097 | 0.09058 | 38 | 0.99636 | 0.136545 | 1.018263 | -0.0219 | 0.00048 |
| 8 | -0.67929 | -0.17914 | -0.34323 | -0.33606 | 0.11294 | 39 | 0.27745 | -0.12647 | 0.105823 | 0.171622 | 0.02945 |
| 9 | 0.32174 | -0.2582 | -0.13884 | 0.460575 | 0.21213 | 40 | 0.05442 | 2.43112 | -0.09802 | 0.152436 | 0.02324 |
| 10 | -0.33123 | -0.20549 | -0.2001 | -0.13112 | 0.01719 | 41 | 1.06056 | 0.346564 | 1.884118 | -0.82356 | 0.67826 |
| 11 | 0.00949 | -0.07382 | -0.15925 | 0.168746 | 0.02848 | 42 | 0.16626 | 0.346564 | 0.268587 | -0.10232 | 0.01047 |
| 12 | -0.10333 | 0.32033 | -0.05721 | -0.04612 | 0.00213 | 43 | 0.09924 | 0.896344 | 0.268587 | -0.16935 | 0.02868 |
| 13 | 0.03197 | -0.41647 | 0.248256 | -0.21629 | 0.04678 | 44 | -0.44638 | 0.844074 | 0.694667 | -1.14105 | 1.30199 |
| 14 | -0.44638 | 0.660982 | -0.32277 | -0.12361 | 0.01528 | 45 | 1.6308 | 1.23568 | 0.654157 | 0.976643 | 0.95383 |
| 15 | 1.56808 | -0.89276 | 0.512261 | 1.055814 | 1.11474 | 46 | 0.91048 | 0.948598 | 0.957652 | -0.04717 | 0.00223 |
| 16 | -0.58569 | 0.162816 | -0.69189 | 0.106199 | 0.01128 | 47 | 0.5636 | 0.005108 | 0.735163 | -0.17157 | 0.02944 |
| 17 | 0.32174 | 0.267847 | 0.126183 | 0.195555 | 0.03824 | 48 | 1.42112 | 1.313887 | 0.003958 | 1.417162 | 2.00835 |
| 18 | 0.32174 | 0.32033 | 0.207581 | 0.114156 | 0.01303 | 49 | 0.12161 | 1.261753 | 1.018263 | -0.89666 | 0.804 |
| 19 | 0.45402 | -0.33731 | 0.248256 | 0.205764 | 0.04234 | 50 | 0.84586 | 1.392058 | 0.977859 | -0.132 | 0.01742 |
| 20 | 1.42112 | -0.07382 | -0.26141 | 1.682535 | 2.83092 | 51 | 1.85954 | 0.817931 | 1.078845 | 0.780689 | 0.60947 |
| 21 | -0.03555 | 0.660982 | -0.05721 | 0.02166 | 0.00047 | 52 | 0.00949 | -0.65433 | 0.633897 | -0.62441 | 0.38989 |
| 22 | 0.25526 | 2.094047 | 0.512261 | -0.257 | 0.06605 | 53 | -0.56238 | 0.005108 | -0.50711 | -0.05528 | 0.00306 |
| 23 | 1.25211 | -0.60143 | 1.622887 | -0.37078 | 0.13748 | 54 | -0.35419 | -1.02549 | 0.003958 | -0.35815 | 0.12827 |
| 24 | -0.63242 | 0.372793 | -0.4661 | -0.16631 | 0.02766 | 55 | 0.00949 | 0.372793 | -0.79475 | 0.804245 | 0.64681 |
| 25 | -0.44638 | 0.477658 | 0.288915 | -0.73529 | 0.54066 | 56 | 1.14589 | -0.89276 | 0.288915 | 0.856969 | 0.7344 |
| 26 | 0.27745 | -0.65433 | 0.370185 | -0.09274 | 0.0086 | 57 | -1.50319 | -2.53007 | -0.69189 | -0.8113 | 0.65821 |
| 27 | -1.37834 | -0.44288 | -0.50711 | -0.87123 | 0.75905 | 58 | -2.04646 | -2.12294 | -1.9608 | -0.08565 | 0.00734 |
| 28 | -0.53911 | -0.91929 | -0.34323 | -0.19588 | 0.03837 | 59 | -1.96667 | -2.12294 | -1.64528 | -0.32139 | 0.10329 |
| 29 | -1.06014 | -1.02549 | -0.71245 | -0.34769 | 0.12089 | 60 | -2.07325 | -0.12647 | -1.64528 | -0.42797 | 0.18315 |
| 30 | -1.25492 | -0.97237 | -0.79475 | -0.46016 | 0.21175 | | | | | SS= | 23.55631 |

**Table 4.8:Final estimation of model parameter for each season of suggested models**

| season | Suggested model | PAR parameter | | PMA parameter | | SS | $\sigma_a{}^2$ |
|---|---|---|---|---|---|---|---|
| | | $\phi_1$ | $\phi_2$ | $\theta_1$ | $\theta_2$ | | |
| 1 | PAR(1) | 0.775 | | | | 23.556 | 0.393 |
| | PAR(2) | 0.592 | 0.195 | | | 23.667 | 0.394 |
| | PMA(1) | | | -0.728 | | 44.567 | 0.742 |
| | PMA(2) | | | -0.759 | -0.894 | 32.796 | 0.546 |
| | PARMA(1,1) | 0.787 | | 0.146 | | 23.86 | 0.397 |
| | PARMA(2,2) | 1.726 | -0.776 | 1.101 | 0.815 | 23.663 | 0.394 |
| | PARMA(1,2) | 0.830 | | 0.203 | 0.228 | 23.788 | 0.396 |
| | PARMA(2,1) | 0.533 | 0.25 | -0.088 | | 23.654 | 0.394 |
| 2 | PAR(1) | 0.516 | | | | 43.274 | 0.721 |
| | PAR(2) | 0.502 | 0.017 | | | 44.038 | 0.734 |
| | PMA(1) | | | -0.466 | | 50.295 | 0.838 |
| | PMA(2) | | | -0.485 | -0.437 | 47.588 | 0.793 |
| | PARMA(1,1) | 0.507 | | -0.019 | | 44.04 | 0.734 |
| | PARMA(2,2) | 1.892 | -1.155 | 1.381 | -1.086 | 44.965 | 0.749 |
| | PARMA(1,2) | 0.434007 | | -0.092 | -0.831 | 45.631 | 0.761 |
| | PARMA(2,1) | -1.529 | 1.592 | -2.058 | | 45.75 | 0.763 |
| 3 | PAR(1) | 0.452 | | | | 46.933 | 0.782 |
| | PAR(2) | 0.395 | 0.067 | | | 48.697 | 0.811 |
| | PMA(1) | | | -0.429 | | 50.708 | 0.845 |
| | PMA(2) | | | -0.398 | -0.352 | 48.346 | 0.805 |
| | PARMA(1,1) | 0.528 | | 0.134 | | 48.685 | 0.811 |
| | PARMA(2,2) | 0.62 | -0.115 | 0.248 | -0.171 | 48.019 | 0.800 |
| | PARMA(1,2) | 0.494 | | 0.128 | -0.111 | 48.18 | 0.803 |
| | PARMA(2,1) | 0.716 | -0.098 | 0.33 | | 48.559 | 0.809 |

**Table 4.8: Cont.**

| season | Suggested model | $\phi_1$ | $\phi_2$ | $\theta_1$ | $\theta_2$ | SS | ${\sigma_a}^2$ |
|---|---|---|---|---|---|---|---|
| 4 | PAR(1) | 0.455 | | | | 46.813 | 0.78 |
| | PAR(2) | 0.399 | 0.208 | | | 47.54 | 0.792 |
| | PMA(1) | | | -0.443 | | 50.041 | 0.834 |
| | PMA(2) | | | -0.399 | -0.315 | 47.568 | 0.792 |
| | PARMA(1,1) | 0.918 | | 0.529 | | 48.082 | 0.801 |
| | PARMA(2,2) | 0.211 | 0.48 | -0.176 | 0.287 | 47.739 | 0.796 |
| | PARMA(1,2) | 1.019 | | 0.633 | 0.086 | 47.885 | 0.798 |
| | PARMA(2,1) | 0.604 | 0.12 | 0.206 | | 48.526 | 0.809 |
| 5 | PAR(1) | 0.711 | | | | 29.158 | 0.486 |
| | PAR(2) | 0.658 | 0.102 | | | 29.442 | 0.490 |
| | PMA(1) | | | -0.665 | | 37.836 | 0.630 |
| | PMA(2) | | | -0.677 | -0.345 | 32.387 | 0.539 |
| | PARMA(1,1) | 0.82 | | 0.156 | | 29.615 | 0.493 |
| | PARMA(2,2) | -0.889 | 1.492 | -1.568 | 0.792 | 27.759 | 0.462 |
| | PARMA(1,2) | 0.731 | | 0.066 | -0.077 | 29.466 | 0.491 |
| | PARMA(2,1) | 0.586 | 0.136 | -0.074 | | 29.43 | 0.490 |
| 6 | PAR(1) | 0.646 | | | | 34.398 | 0.573 |
| | PAR(2) | 0.428 | 0.283 | | | 34.886 | 0.581 |
| | PMA(1) | | | -0.564 | | 47.946 | 0.799 |
| | PMA(2) | | | -0.435 | -0.528 | 40.548 | 0.675 |
| | PARMA(1,1) | 0.84 | | 0.428 | | 35.53 | 0.592 |
| | PARMA(2,2) | 0.032 | 0.797 | -0.407 | 0.329 | 34.605 | 0.577 |
| | PARMA(1,2) | 1.019 | | 0.617 | 0.184 | 34.763 | 0.579 |
| | PARMA(2,1) | 2.071 | -0.88 | 1.67 | | 34.56 | 0.576 |

**Table 4.8:Cont.**

| season | Suggested model | $\phi_1$ | $\phi_2$ | $\theta_1$ | $\theta_2$ | SS | $\sigma_a^2$ |
|---|---|---|---|---|---|---|---|
| 7 | PAR(1) | 0.66 | | | | 33.309 | 0.555 |
| | PAR(2) | 0.569 | 0.153 | | | 33.566 | 0.559 |
| | PMA(1) | | | -0.612 | | 42.023 | 0.700 |
| | PMA(2) | | | -0.634 | -0.422 | 37.884 | 0.631 |
| | PARMA(1,1) | 0.8 | | 0.24 | | 33.554 | 0.559 |
| | PARMA(2,2) | -0.122 | 0.79 | -0.701 | 0.433 | 34.194 | 0.569 |
| | PARMA(1,2) | 0.804 | | 0.263 | -0.023 | 34.283 | 0.571 |
| | PARMA(2,1) | 0.891 | -0.049 | 0.358 | | 34.136 | 0.569 |
| 8 | PAR(1) | 0.784 | | | | 22.758 | 0.379 |
| | PAR(2) | 0.709 | 0.113 | | | 22.923 | 0.382 |
| | PMA(1) | | | -0.725 | | 37.9 | 0.631 |
| | PMA(2) | | | -0.709 | -0.603 | 26.135 | 0.435 |
| | PARMA(1,1) | 0.858 | | 0.137 | | 22.866 | 0.381 |
| | PARMA(2,2) | 0.187 | 0.48 | -0.555 | 0.037 | 23.259 | 0.387 |
| | PARMA(1,2) | 0.788 | | 0.068 | -0.108 | 23.611 | 0.394 |
| | PARMA(2,1) | 0.365 | 0.341 | -0.356 | | 23.567 | 0.393 |
| 9 | PAR(1) | 0.849 | | | | 16.438 | 0.274 |
| | PAR(2) | 0.752 | 0.124 | | | 16.962 | 0.283 |
| | PMA(1) | | | -0.802 | | 35.569 | 0.592 |
| | PMA(2) | | | -0.785 | -0.621 | 29.253 | 0.487 |
| | PARMA(1,1) | 0.911 | | 0.164 | | 17.337 | 0.289 |
| | PARMA(2,2) | 1.489 | -0.497 | 0.765 | -0.084 | 17.86 | 0.298 |
| | PARMA(1,2) | 0.924 | | 0.171 | 0.027 | 17.369 | 0.289 |
| | PARMA(2,1) | 0.668 | 0.189 | -0.085 | | 17.358 | 0.289 |

**Table 4.8: Cont.**

| season | model | $\phi_1$ | $\phi_2$ | $\theta_1$ | $\theta_2$ | SS | $\sigma_a^2$ |
|---|---|---|---|---|---|---|---|
| 10 | PAR(1) | 0.908 | | | | 10.384 | 0.173 |
| | PAR(2) | 0.976 | -0.079 | | | 10.455 | 0.174 |
| | PMA(1) | | | -0.899 | | 31.196 | 0.519 |
| | PMA(2) | | | -0.934 | -0.658 | 23.116 | 0.385 |
| | PARMA(1,1) | 0.884 | | -0.085 | | 10.476 | 0.174 |
| | PARMA(2,2) | 0.624 | 0.254 | -0.353 | 0.097 | 10.411 | 0.173 |
| | PARMA(1,2) | 0.898 | | -0.069 | 0.034 | 10.46 | 0.174 |
| | PARMA(2,1) | 1.05 | -0.143 | 0.077 | | 10.453 | 0.174 |
| 11 | PAR(1) | 0.941 | | | | 6.728 | 0.112 |
| | PAR(2) | 1.076 | -0.148 | | | 7.6106 | 0.127 |
| | PMA(1) | | | -0.987 | | 29.588 | 0.493 |
| | PMA(2) | | | -0.987 | -0.848 | 16.401 | 0.273 |
| | PARMA(1,1) | 0.912 | | -0.162 | | 7.6137 | 0.127 |
| | PARMA(2,2) | -1.897 | 2.491 | -2.993 | -0.223 | 7.1219 | 0.119 |
| | PARMA(1,2) | 0.907 | | -0.171 | -0.016 | 7.6016 | 0.127 |
| | PARMA(2,1) | 0.894 | 0.016 | -0.183 | | 7.6069 | 0.127 |
| 12 | PAR(1) | 0.933 | | | | 7.603 | 0.127 |
| | PAR(2) | 1.483 | -0.584 | | | 7.999 | 0.133 |
| | PMA(1) | | | -1.02 | | 29.159 | 0.486 |
| | PMA(2) | | | -1.081 | -0.842 | 24.413 | 0.406 |
| | PARMA(1,1) | 0.861 | | -0.652 | | 8.2293 | 0.137 |
| | PARMA(2,2) | 1.666 | -0.726 | 0.159 | 0.172 | 8.2182 | 0.137 |
| | PARMA(1,2) | 0.871 | | -0.641 | 0.043 | 8.2344 | 0.137 |
| | PARMA(2,1) | 0.591 | 0.255 | -0.923 | | 8.2043 | 0.137 |

## 4.7.3 Diagnostic Checking

After exact estimation, Akaik information criterion (AIC) and the Schwarz information criterion (SIC) as explained in sec. (3.8) are used to select the best model from a group of suggested models for each season. The best model is the one that gives minimum (AIC) and (SIC). (AIC) and (SIC) values are summarized in Table 4.9. However, the adequacy of model fitting is also checked by statistical tests are used in this study to check residuals of fitted model for independent, white noise ($a_{r,m}$) and normally distributed of residuals.

**Table 4.9: AIC and SIC values of suggested models for each season**

| season \ model | test | PAR(1) | PAR(2) | PMA(1) | PMA(2) | PARMA(1,1) | PARMA(2,2) | PARMA(1,2) | PARMA(2,1) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | AIC | 8.173* | 10.5 | 46.306 | 30.12 | 10.999 | 15.226 | 13.146 | 12.843 |
| | SIC | 8.057* | 12.304 | 46.189 | 31.88 | 12.759 | 20.493 | 16.702 | 16.398 |
| 2 | AIC | 44.583* | 47.873 | 53.606 | 52.512 | 47.873 | 53.77 | 52.339 | 52.497 |
| | SIC | 44.467* | 49.633 | 53.49 | 54.272 | 49.633 | 59.036 | 55.895 | 56.053 |
| 3 | AIC | 49.456* | 53.859 | 54.105 | 53.413 | 53.859 | 57.722 | 55.563 | 56.009 |
| | SIC | 49.34* | 55.619 | 53.989 | 55.173 | 55.619 | 62.988 | 59.118 | 59.565 |
| 4 | AIC | 49.302* | 52.436 | 53.319 | 52.436 | 53.114 | 57.421 | 55.188 | 56.009 |
| | SIC | 49.186* | 54.197 | 53.203 | 54.197 | 54.875 | 62.688 | 58.744 | 59.565 |
| 5 | AIC | 20.917* | 23.627 | 36.488 | 29.346 | 23.993 | 24.779 | 26.048 | 25.926 |
| | SIC | 20.801* | 25.387 | 36.372 | 31.106 | 25.753 | 30.045 | 29.604 | 29.482 |
| 6 | AIC | 30.798* | 33.848 | 50.746 | 42.846 | 34.973 | 38.116 | 35.94 | 35.628 |
| | SIC | 30.682* | 35.608 | 50.63 | 44.606 | 36.733 | 43.382 | 39.495 | 39.184 |
| 7 | AIC | 28.883* | 31.532 | 42.81 | 38.801 | 31.532 | 37.278 | 35.105 | 34.894 |
| | SIC | 28.767* | 33.292 | 42.693 | 40.561 | 33.292 | 42.544 | 38.661 | 38.45 |
| 8 | AIC | 5.997* | 8.688 | 36.583 | 16.484 | 8.531 | 14.151 | 12.843 | 12.69 |
| | SIC | 5.881* | 10.448 | 36.467 | 18.244 | 10.291 | 19.417 | 16.398 | 16.246 |
| 9 | AIC | -13.467* | -9.309 | 32.755 | 23.259 | -8.051 | -1.528 | -5.752 | -5.752 |
| | SIC | -13.583* | -7.549 | 32.639 | 25.019 | -6.291 | 3.737 | -2.196 | -2.196 |
| 10 | AIC | -41.057* | -38.493 | 24.859 | 9.157 | -38.493 | -34.156 | -36.194 | -36.194 |
| | SIC | -41.173* | -36.733 | 24.743 | 10.917 | -36.733 | -28.89 | -32.638 | -32.638 |
| 11 | AIC | -67.144* | -57.385 | 21.775 | -11.468 | -57.385 | -56.606 | -55.086 | -55.086 |
| | SIC | -67.261* | -55.625 | 21.659 | -9.708 | -55.625 | -51.34 | -51.531 | -51.531 |
| 12 | AIC | -59.603* | -54.615 | 20.917 | 12.344 | -52.837 | -48.155 | -50.539 | -50.539 |
| | SIC | -59.719* | -52.855 | 20.801 | 14.104 | -51.077 | -42.889 | -46.983 | -46.983 |

*Min. AIC & SIC

From the result in Table 4.9, it's found that the minimum (AIC and SIC) value is the $PAR_{12}$ (1) or $PARMA_{12}$ (1, 0) model from group of suggested models for each season, so that checking this model to diagnostic checking.

Table 4.10 shows the Porte Manteau lack of fit test. The table shows that all models are succeeded for each season because the calculated (Q) by using equations (3-41, 3-42) with M=15 is less than the $\chi^2$-table with degree of freedom (M-p-q).

**Table 4.10: Diagnostic check results with 95% confidence limits and (M-p-q) degree of freedom, where M=15.**

| season | $\chi^2$-table | Q1-calculated | Q2-calculated | Result test |
|---|---|---|---|---|
| 1 | 23.6848 | 0.17598 | 0.000404 | succeeded |
| 2 | 23.6848 | 0.44971 | 0.000659 | succeeded |
| 3 | 23.6848 | 1.89288 | 0.001836 | succeeded |
| 4 | 23.6848 | 1.77854 | 0.001616 | succeeded |
| 5 | 23.6848 | 2.35188 | 0.002026 | succeeded |
| 6 | 23.6848 | 1.61311 | 0.001349 | succeeded |
| 7 | 23.6848 | 4.35021 | 0.017104 | succeeded |
| 8 | 23.6848 | 5.66478 | 0.005076 | succeeded |
| 9 | 23.6848 | 7.15965 | 0.011645 | succeeded |
| 10 | 23.6848 | 11.0418 | 0.028054 | succeeded |
| 11 | 23.6848 | 7.76376 | 0.052976 | succeeded |
| 12 | 23.6848 | 6.84814 | 0.068697 | succeeded |

The second test is the independency of the resulting ($a_{r,m}$) series for each season. The correlogram of these series are computed for lag (M=15) as shown in Figure 4.10. The result shows that the most of computed lags lie inside the tolerance interval ($\pm 1.96/\sqrt{n}$, at 95% confidence limits). Hence, the selected models can be considered as appropriate models because their capability of removing the dependency from data.

The third test for the normality of residuals, which provides stronger conclusions about the model. Figure 4.11 shows plot of the histogram and normal probability of the residuals which indicate that transform (Box-Cox) is good fit for the residual. The final $PAR_{12}$ (1) or $PARMA_{12}$ (1, 0) models for each season and model equation which have minimum AIC and succeeded in the above tests are shown in Table 4.11.

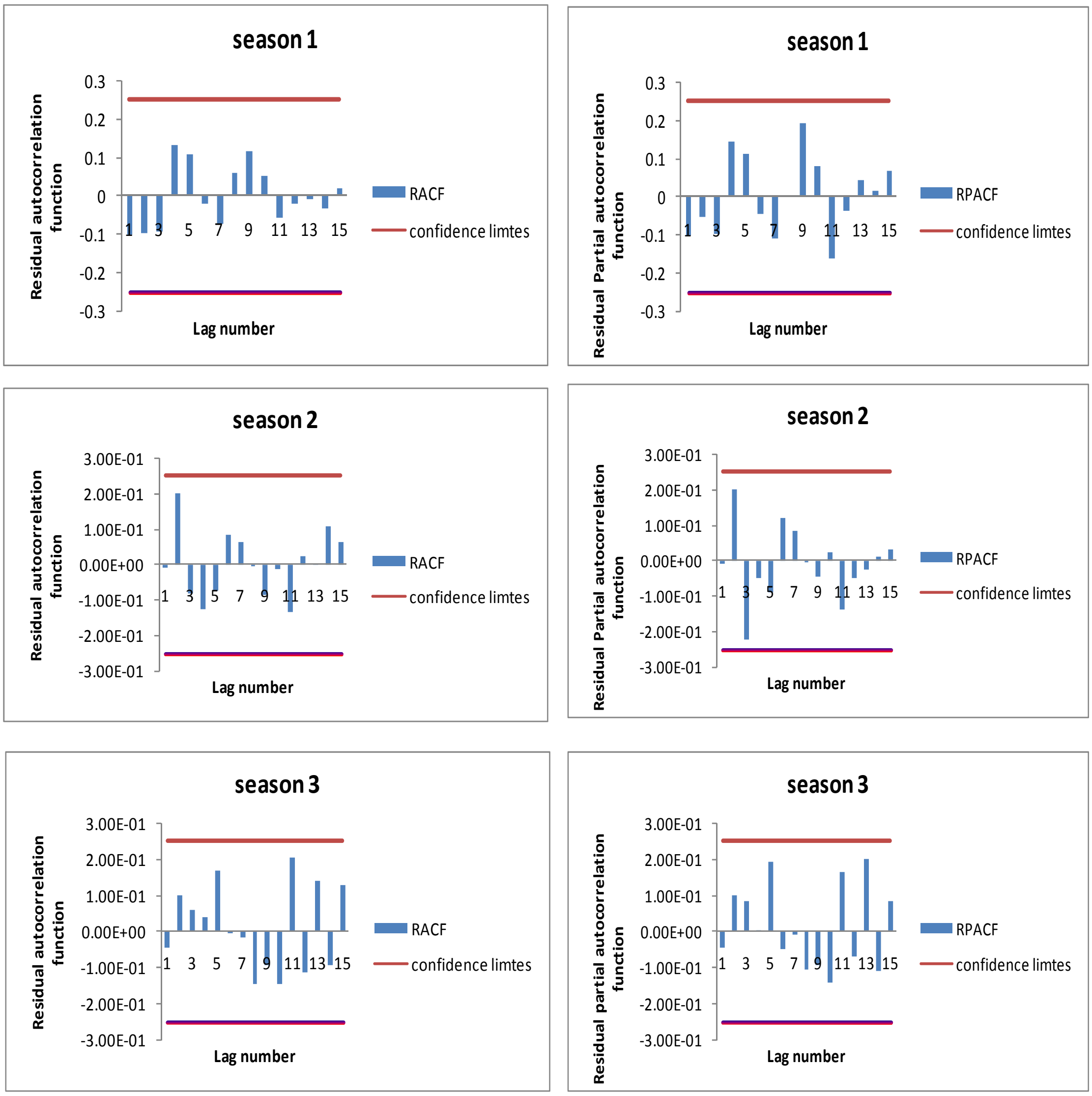



**Figure 4.10:ACF & PACF for $PARMA_{12}$(1,0) model residual ,showing the 95% confidence limits $\pm 1.96/\sqrt{n}$**

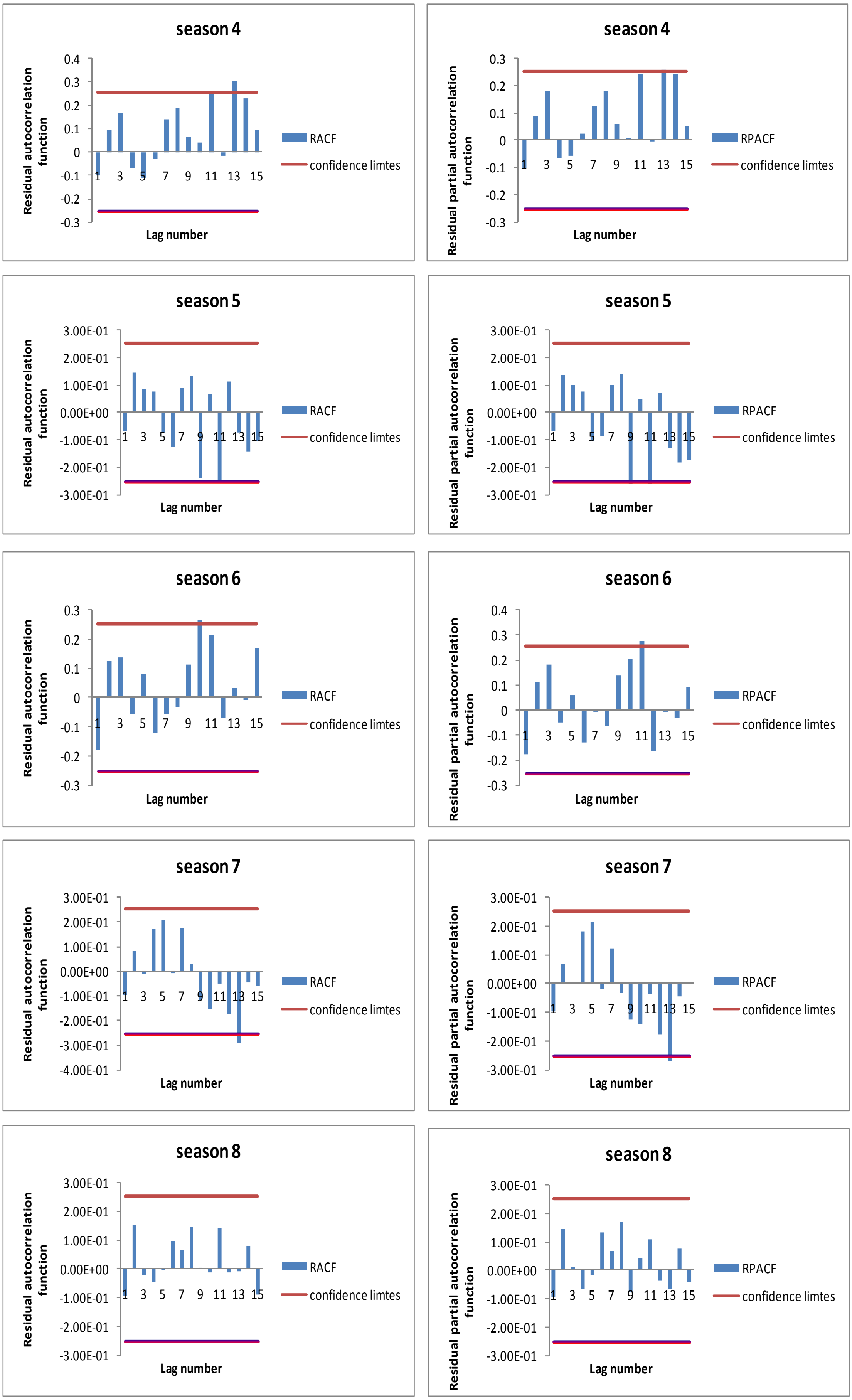



**Figure 4.10:Cont.**

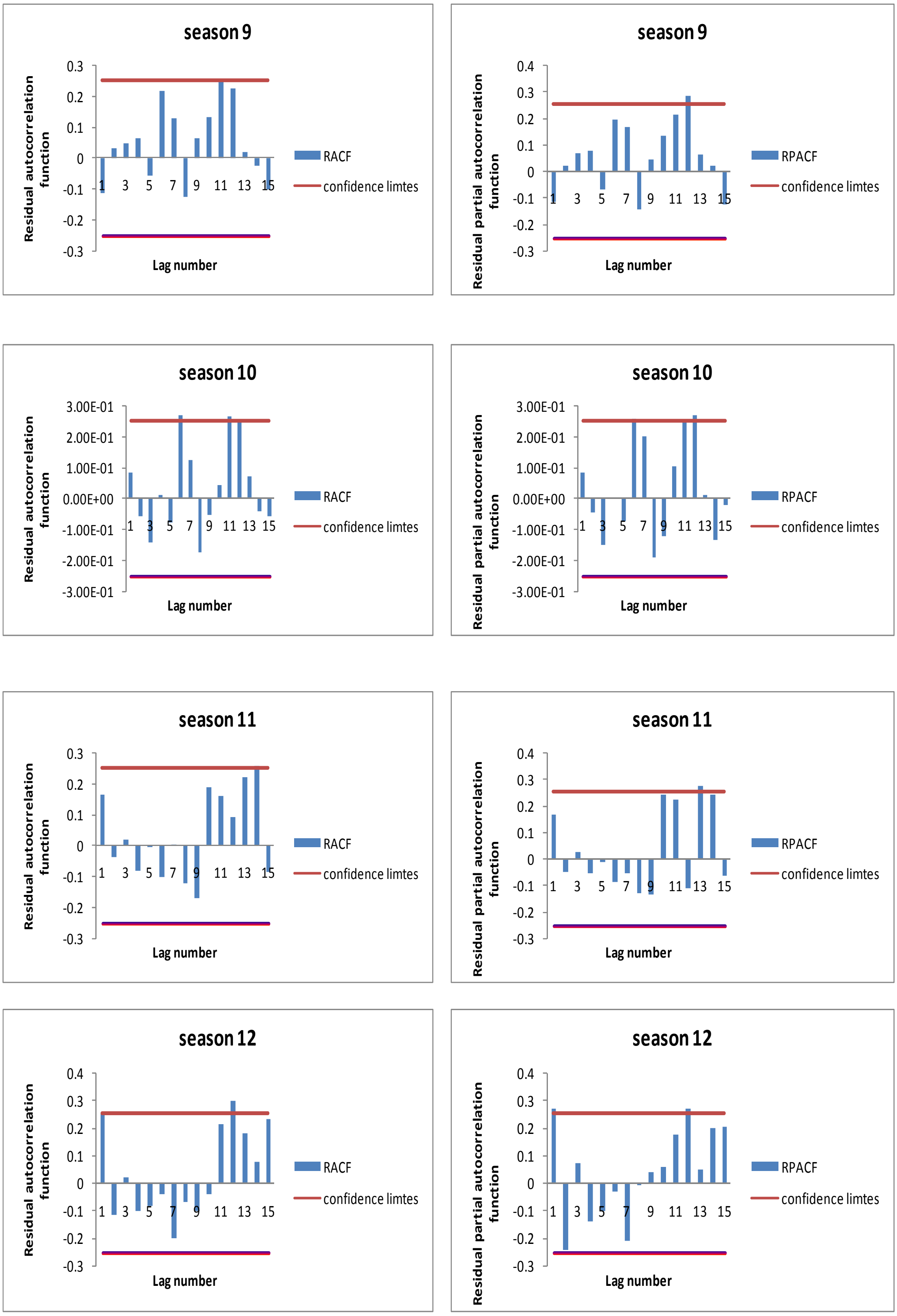



**Figure 4.10: Cont.**

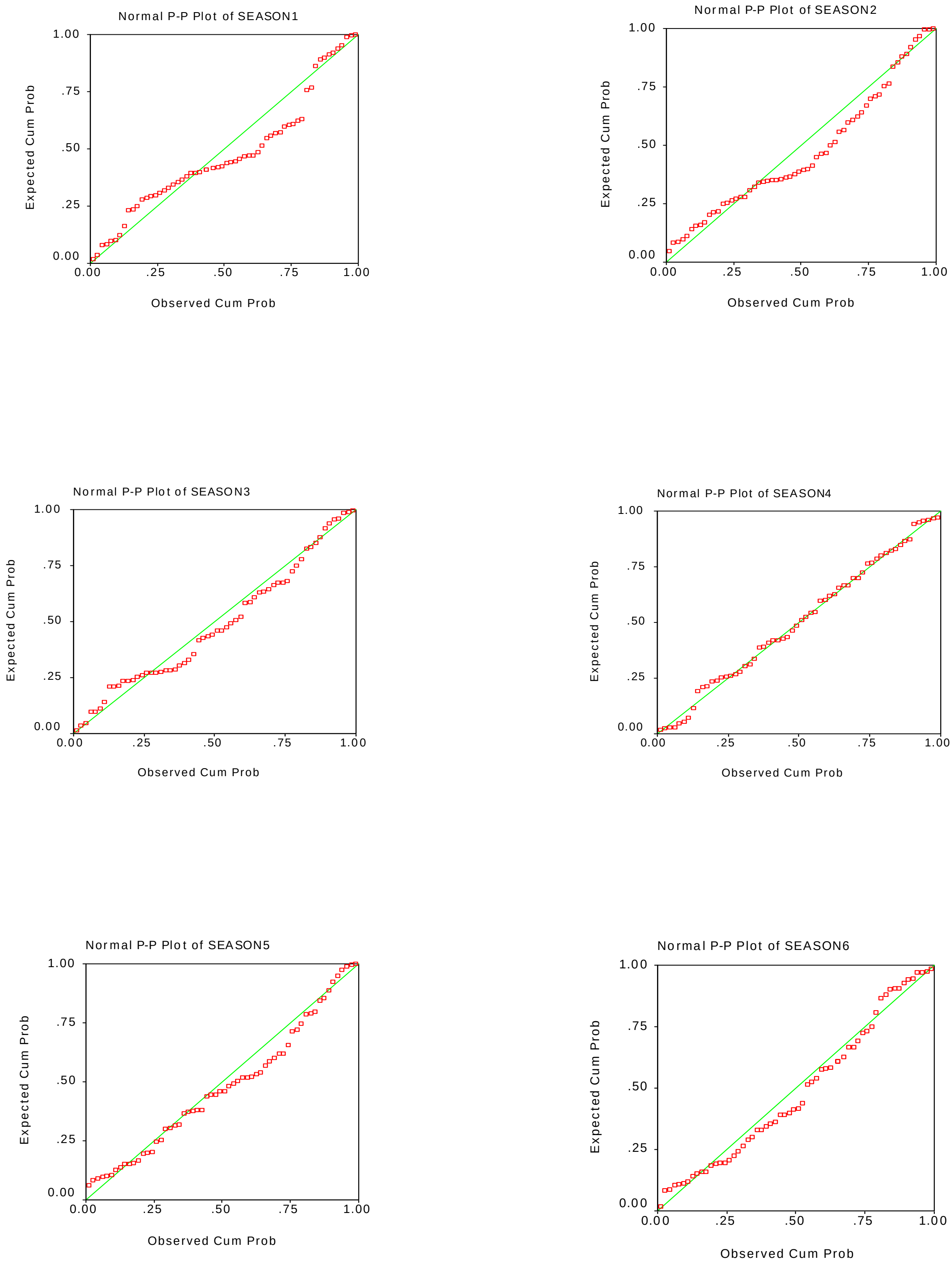



**Figure 4.11: Probability plot for PARMA(1,0) model residuals for 12 season**

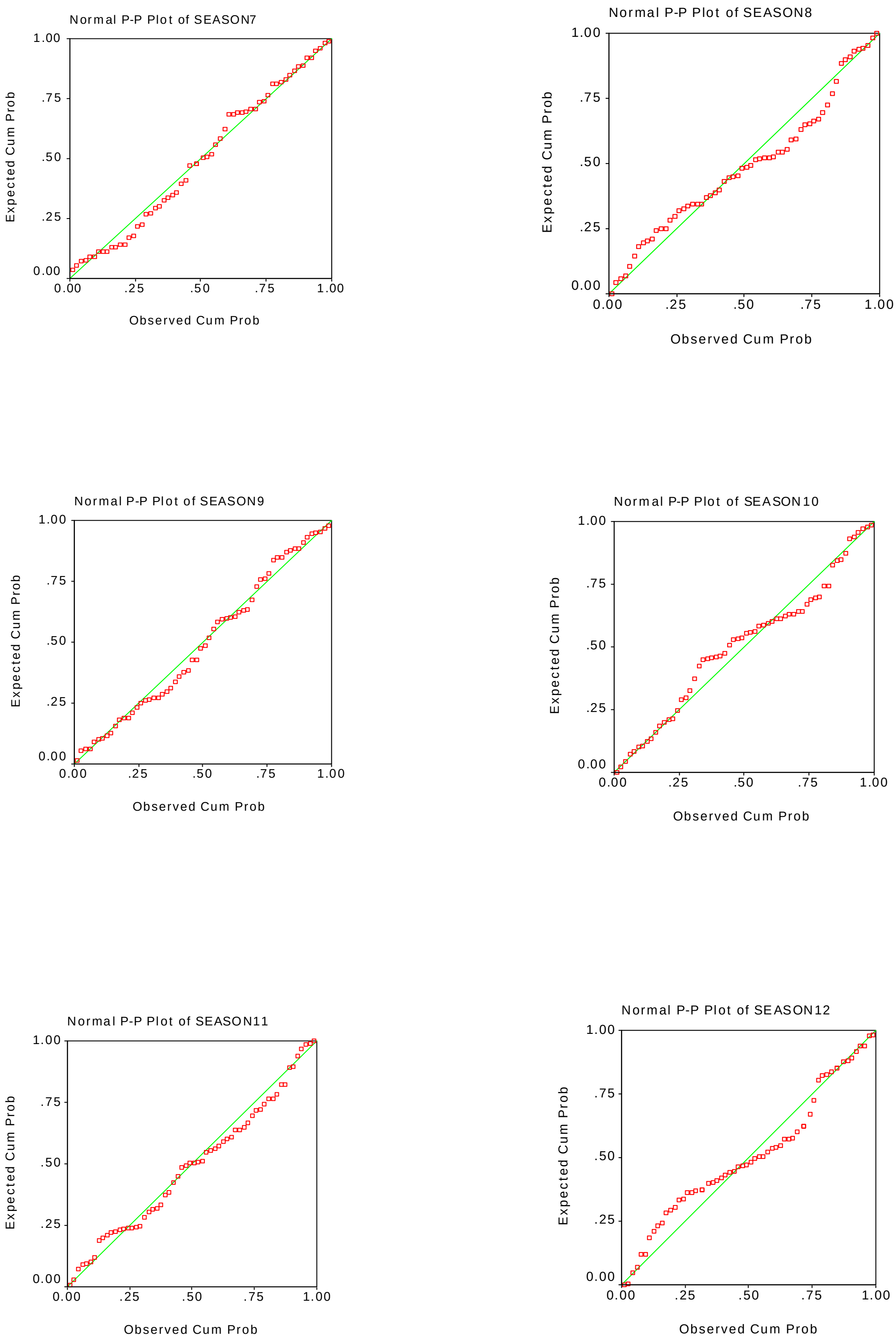



**Figure 4.11: Cont.**

**Table 4.11: Final model types of each season for Greater Zab River.**

| season | Month | model | model equation |
|---|---|---|---|
| 1 | Oct | PARMA(1,0) | $W_{r,m}$= 0.775 $W_{r,m-1}$ + $a_{r,m}$ |
| 2 | Nov | PARMA(1,0) | $W_{r,m}$= o.516 $W_{r,m-1}$ + $a_{r,m}$ |
| 3 | Dec | PARMA(1,0) | $W_{r,m}$= 0.452 $W_{r,m-1}$ + $a_{r,m}$ |
| 4 | Jan | PARMA(1,0) | $W_{r,m}$= 0.455 $W_{r,m-1}$ + $a_{r,m}$ |
| 5 | Feb | PARMA(1,0) | $W_{r,m}$= 0.711 $W_{r,m-1}$ + $a_{r,m}$ |
| 6 | Mar | PARMA(1,0) | $W_{r,m}$= 0.646 $W_{r,m-1}$ + $a_{r,m}$ |
| 7 | Apr | PARMA(1,0) | $W_{r,m}$= 0.66 $W_{r,m-1}$ + $a_{r,m}$ |
| 8 | May | PARMA(1,0) | $W_{r,m}$= 0.784 $W_{r,m-1}$ + $a_{r,m}$ |
| 9 | Jun | PARMA(1,0) | $W_{r,m}$= 0.849 $W_{r,m-1}$ + $a_{r,m}$ |
| 10 | Jul | PARMA(1,0) | $W_{r,m}$= 0.908 $W_{r,m-1}$ + $a_{r,m}$ |
| 11 | Aug | PARMA(1,0) | $W_{r,m}$= 0.941 $W_{r,m-1}$ + $a_{r,m}$ |
| 12 | Sep | PARMA(1,0) | $W_{r,m}$= 0.933 $W_{r,m-1}$ + $a_{r,m}$ |

## 4.8 Fitting SARIMA (p,d,q)(P,D,Q)$_{12}$ model

### 4.8.1 Identification of Models

For fitting SARIMA (p, d, q)× (P, D, Q)s model which is a multiplicative nonstationary seasonal autoregressive integrated moving average model to a time series of monthly flow data, three stages procedure are adopted. These stages are model identification, estimation of model parameters, and diagnostic checking of the estimated parameters as explained in chapter three (Section 3.11).

The period of observation of monthly flow of Greater Zab river is from (1933-2002). The record is from (1933-1992) used in building SARIMA models. Data from (1993-2002) are used to choose best fit model depending on minimum forecasted error.

First of all, the observation is plotted against time. This plot will show up important features such as trend, seasonality, discontinuities, and outlier. Figure 4.12 show that there is little trend, high fluctuation, and seasonal variation for monthly flow for Greater Zab river.

The procedure of fitting SARIMA model is summarized by the following steps (Box and Jenkins, 1976):

1. Transform the data by using natural log transformation, which was found the most appropriate transformation (Box and Jenkins, 1976; McLeod et. al., 1987). As shown in Figure 4.13, the series are converted to stationary in variance by log transform method.

2. Remove trend component by trying the first order differencing. Figure 4.14 shows that the series is transformed to a stationary in mean with approximately no trend.
3. Remove the seasonal variation by trying the first order seasonal differencing. Figure 4.15 shows ACF & PACF after taking log transform and first differencing for flow data, indicating seasonal variance that must be removed by first order seasonal differencing as shown in Figure 4.16.
4. Identify appropriate model by plotting ACF and PACF after logarithmic transformation, first simple differencing and first seasonal differencing of monthly observations.

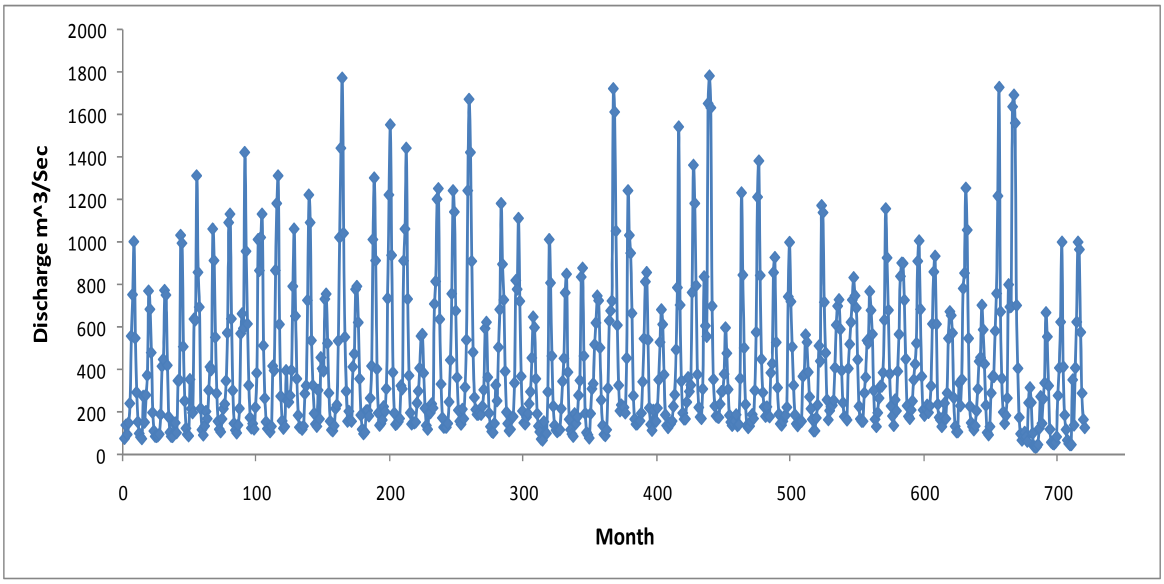



**Figure 4.12: Monthly discharge for Greater Zab River from 1933 to 1992**

Figure 4.17 shows the (ACF) and (PACF) for monthly flow of greater Zab river which plotted by using the Software SPSS (Statistical Package for Social Science). Chatfield (1982) explained that the (ACF) of moving average (MA(q)) cuts off after lag q whereas the (PACF) is a mixture of damped exponentials sinusoids and dies out slowly (or attenuates), then, these figures indicate that the model is seasonal multiplicative model. i.e. these models may be seasonal integrated moving average model (SIMA(q)) or seasonal autoregressive integrated model (SARI(p)) or seasonal autoregressive integrated moving average model (SARIMA(p,q)) which are given in Table 4.12 (see Appendix A.4 for derivation these models). The selected model is the one which gives minimum sum of squares (SS).

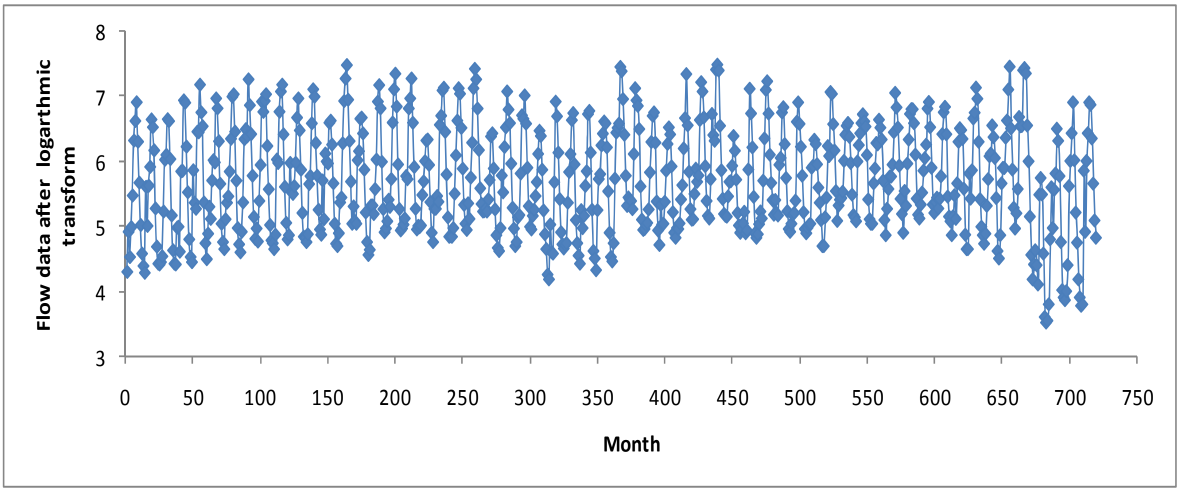



**Figure 4.13:Monthly flow after log transform for stationary in variance**

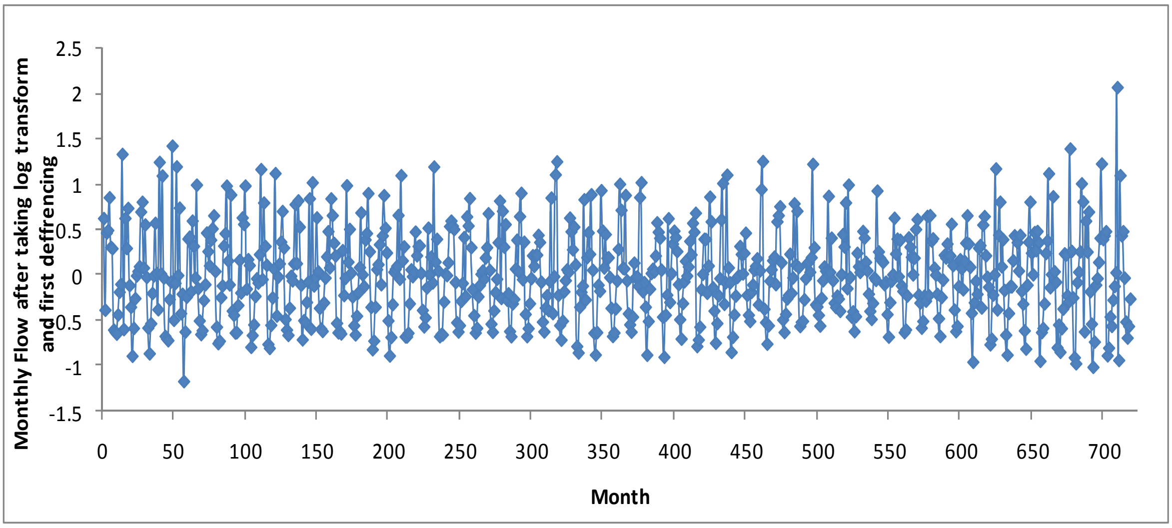



**Figure 4.14: Monthly Flow after taking log transform and first differencing for stationary in mean.**

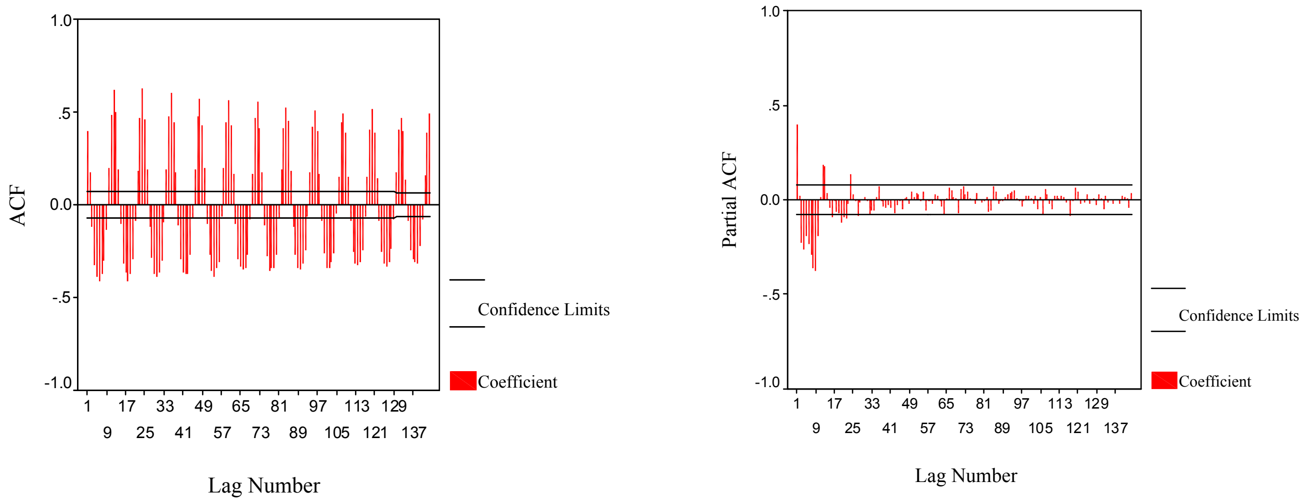



**Figure 4.15: ACF & PACF after taking Log transform and first differencing for flow data.**

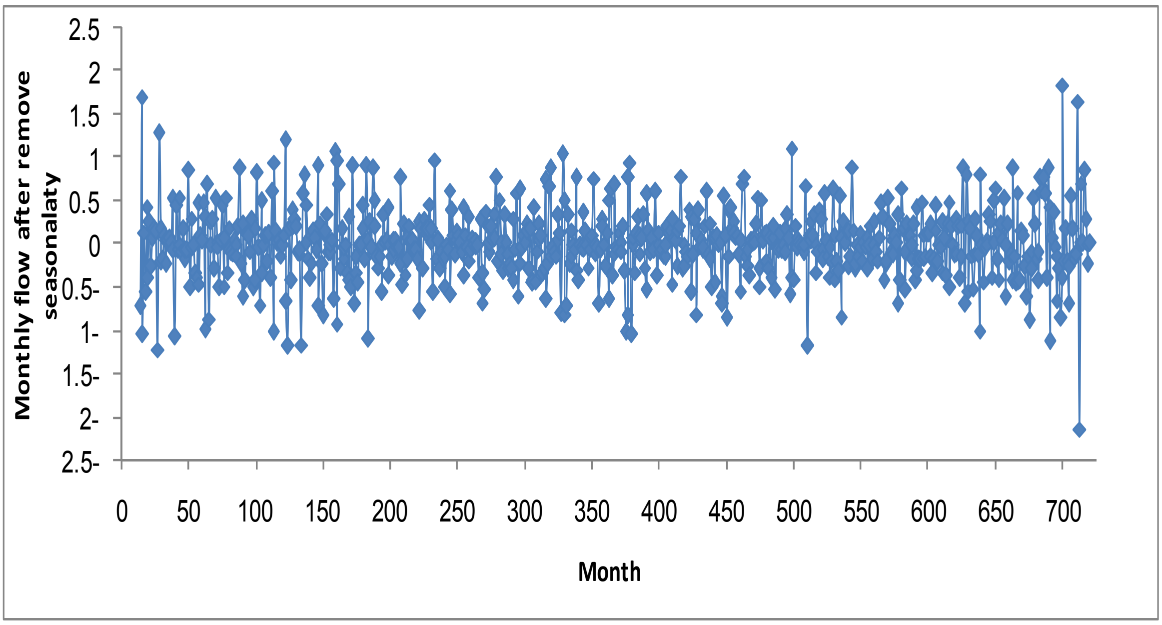



**Figure 4.16: Monthly flow after taking Log transform,first differencing and first seasonal differencing**

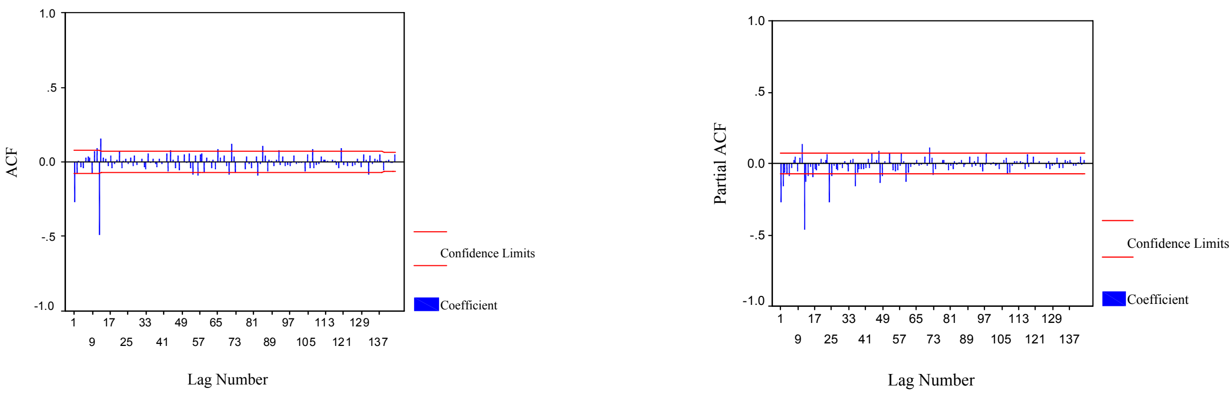



**Figure 4.17: ACF & PACF for Monthly Flow for Greater Zab River**

**Table 4.12: Best Fit SARIMA Models for Greater Zab River.**

| model | Seasonal part of model | Nonseasonal part of model | SARIMA models | Equation of models |
|---|---|---|---|---|
| **SARIMA(0,1,1)x(0,1,1)$_{12}$** | $W_t=(1-\Theta\beta^{12})a_t$ | $W_t=(1-\theta\beta)a_t$ | $W_t=(1-\theta\beta)(1-\Theta\beta^{12})\ a_t$ | $W_t= a_t - \theta a_{t-1} - \Theta a_{t-12} + \theta\Theta a_{t-13}$ |
| **SARIMA(0,1,2)x(0,1,1)$_{12}$** | $W_t=(1-\Theta\beta^{12})a_t$ | $W_t=(1-\theta_1\beta-\theta_2\beta^2)a_t$ | $W_t=(1-\theta_1\beta-\theta_2\beta^2)(1-\Theta\beta^{12})a_t$ | $W_t= a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \Theta a_{t-12}+\theta_1\Theta a_{t-13}+\theta_2\Theta a_{t-14}$ |
| **SARIMA(1,1,0)x(1,1,0)$_{12}$** | $a_t=(1-\Phi\beta^{12})W_t$ | $a_t=(1-\phi\beta)W_t$ | $a_t=(1-\phi\beta)(1-\Phi\beta^{12})W_t$ | $a_t=W_t-\phi W_{t-1} - \Phi W_{t-12}+\phi\Phi W_{t-13}$ |
| **SARIMA(2,1,0)x(1,1,0)$_{12}$** | $a_t=(1-\Phi\beta^{12})W_t$ | $a_t=(1-\phi_1\beta-\phi_2\beta^2)W_t$ | $a_t=(1-\phi_1\beta-\phi_2\beta^2)(1-\Phi\beta^{12})W_t$ | $a_t=W_t - \phi_1 W_{t-1} - \phi_2 W_{t-2}-\Phi W_{t-12}+\phi_1\Phi W_{t-13}+\phi_2\Phi W_{t-14}$ |

## 4.8.2 Estimation of Parameters

The unconditional sum of squares is used to estimate the model parameters by using computer program. To illustrate this procedure we select, for example , the SARIMA(0,1,1)x(0,1,1) model. This model of monthly flow is:

$$\nabla\nabla_{12} Z_t = W_t= (1-\theta\beta)\ (1-\Theta\beta^{12})\ a_t \quad ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4\text{-}1)$$

where $Z_t$ is the transformed series ( $Z_t$=ln $X_t$) and $X_t$ is the original series. The number of observations (N) is 720 (number of months from 1933 to 1992), n=N-d-SD=720−1−12=707 is the number of dependent stochastic component ($W_t$), d is the degree of simple difference (d=1), D is the degree of seasonal difference (D=1), and S is the length of periodic cycle (S=12). The values of both $\Theta$ and $\theta$ vary from (-1 to 1). We choose $\Theta = \theta = -1$ as first trial values and the sum of squares (SS) is computed. The procedure is repeated with different values of $\Theta$ and $\theta$ until we obtain minimum sum of squares. Table (B.13) are given in appendix (B) shows calculation with $\theta = 0.361$ and $\Theta = 0.95$. These values correspond to minimum sum of square.

The calculations which are shown in Table (B.13) are explained as follows:

The model may be written in either forward form as shown in the following equation

$[a_t]= [W_t]+ \theta[a_{t-1}] + \Theta[a_{t-12}] -\theta\Theta[a_{t-13}]$ ,……………………..(4-2)

or backward form as follows:

$[e_t]= [W_t]+ \theta[e_{t+1}] + \Theta[e_{t+12}] -\theta\Theta[e_{t+13}]$ ,……………………..(4-3)

Then, it is convenient to use a numbering system so that the first observation in the $X_t$ series (the second column in Table (B.13)) has a subscript -12, the last observation in the $X_t$ has a subscript 707, the first observation in the $W_t$ series (the eight column in Table (B.13)) has a subscript -12, and the last observation in the $W_t$ series has a subscript 707. The beginning of calculation of the unconditional sum of squares (SS) is done by backward form (Equation 4-3), i.e. when t=707, to compute the [ $e_t$],s (column 12 in Table (B.13)) for t=707, 706, …, 1 by setting [ $e_t$]=0 for t=708, 709, 710, and so on. Then, the [ $W_t$] series for t=0, - 1, -2, …., -12 is computed from Equation (4-3) by setting [ $e_t$]=0 for t=0, -1, -2, …., -12. After that the [ $a_t$],s (fourth column in Table (B.13)) are computed by forward form Equation (4-2) for t=-12, - 11, -10, …, 707 by setting [ $a_t$],s=0 for t=-13, -14, -15, and so on. Hence, the sum of squares (SS) is computed by the following equation:

$$SS=\sum_{t=-12}^{t=707}(a_t^2) \quad ,\ldots\ldots(4\text{-}4)$$

Table (B.13) shows that the minimum sum of squares of monthly flow for greater Zab river is 60.580 when $\theta$ =0.361 and $\Theta$ =0.95. Table 4.13 gives the estimates of model parameters for each the models given in Table 4.12.

**Table 5.13: Summary of estimation model parameters for monthly flow for Greater Zab River.**

| Suggested model | ARI parameter | | SARI parameter | IMA parameter | | SIMA parameter | Min sum of square (SS) |
|---|---|---|---|---|---|---|---|
| | $\Phi_1$ | $\Phi_2$ | $\Phi$ | $\theta_1$ | $\theta_2$ | $\Theta$ | |
| **SARIMA(0,1,1)x(0,1,1)$_{12}$** | | | | **0.361** | | **0.95** | **60.580** |
| **SARIMA(0,1,2)x(0,1,1)$_{12}$** | | | | **0.35** | **0.15** | **0.95** | **61.537** |
| **SARIMA(1,1,0)x(1,1,0)$_{12}$** | **-0.267** | | **-0.533** | | | | **83.234** |
| **SARIMA(2,1,0)x(1,1,0)$_{12}$** | **-0.31** | **-0.15** | **-0.53** | | | | **81.312** |

## 4.8.3 Diagnostic Checking

After estimating the model parameters, we must select one model which is best from four suggested models by using Akaike information criteria (AIC) test and the Schwarz information criterion (SIC) also often referred to as the Bayesian information criterion as mentioned in Chapter three section (3.11). The most adequate model is the one that gives minimum AIC and SIC value from equation (3.54 and 3.55). Table 4.14 summaries the estimation of (AIC) and (SIC) for monthly flow for Greater Zab River for all models in Table 4.12. From this table we found that the minimum (AIC and SIC) value was the SARIMA $(0,1,1)(0,1,1)_{12}$ model from group of suggested models.

**Table 4.14: The (AIC) and (SIC) for monthly flow of Greater Zab River for all Suggested models**

| Suggested model | SS | $\sigma_a^2$ | NO. of parameter | n | AIC | SIC |
|---|---|---|---|---|---|---|
| SARIMA$(0,1,1)x(0,1,1)_{12}$ | 60.580 | 0.0841 | 4 | 720 | -1774.539* | -2.439* |
| SARIMA$(0,1,2)x(0,1,1)_{12}$ | 61.537 | 0.0854 | 5 | 720 | -1760.921 | -2.415 |
| SARIMA$(1,1,0)x(1,1,0)_{12}$ | 83.234 | 0.1156 | 4 | 720 | -1545.486 | -2.121 |
| SARIMA$(2,1,0)x(1,1,0)_{12}$ | 81.312 | 0.1129 | 5 | 720 | -1560.502 | -2.136 |

* **Min. value of AIC & SIC**

The diagnostic checking is applied to see if the model is adequate or not. Portmanteau lack of fit test (Section 3.11) is used for this purpose. The Q1 test is called Box-Pierce test, Q2 test is called Ljung-Box test and Q3 test is called Li- Mcloed test. The results of these tests are given in Table 4.15, which indicates that SARIMA$(0,1,1)(0,1,1)_{12}$ model is adequate for monthly flow of Greater Zab river because the calculated Q value is less than the value of $\chi 2$-table with 52 degree of freedom (M-P-p-Q-q) and 95% confidence limits where M=54 for monthly flow .

**Table 4.15: Diagnostic check results with 95% confidence limits and (M-p-q-P-Q) degree of freedom, where M=54**

| Type of model | Q1-calculated | Q2-calculated | Q3-calculated | $\chi^2$-table |
|---|---|---|---|---|
| SARIMA(0,1,1)x(0,1,1)$_{12}$ | 65.762 | 69.524 | 69.033 | 69.820 |

The SARIMA(0,1,1)(0,1,1)$_{12}$ model may be written in the following form:

$W_t = (1-0.361\beta)\ (1-0.95\beta^{12})\ a_t$ ,……………………………..(4-5)

or

$W_t = a_t - 0.361\ a_{t-1} - 0.95\ a_{t-12} + 0.343\ a_{t-13}$ ,…………………(4-6)

## 4.9 Confidence Region

When several parameters have been considered simultaneously it needs some means of judging the precision of the estimate jointly. One of means of doing this is to determine the confidence region. The approximate 1-ξ confidence region bounded by contour on the sum of squares surface obtained by (Box and Jenkins, 1976):

$$S(\beta) = S(\hat{\beta})\left\{1 + \frac{\chi_{\xi}^{2}(k)}{n}\right\}$$ ,………………………………(4-7)

Where $S(\beta)$ is the quadratic surface. $S(\hat{\beta})$ is the minimum sum of squares. $\chi^2_{\xi}(k)$ is the significant point expected by a proportion ξ of the $\chi^2$ distribution having k degree of freedom. K is the no. of model parameters. n=707.

Figure 4.18 shows the 95% confidence region bounded by the contour line with SS=61.58 with SS (θ,Θ) grids for monthly flow Greater Zab river.

| Θ \ θ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 331.18 | 302.46 | 280.78 | 265.60 | 257.15 | 287.01 | 271.02 | 325.04 | 631.84 | 714310.5 |
| 0.9 | 64.45 | 61.45 | 61.37 | 61.18 | 62.00 | 64.11 | 68.08 | 75.25 | 88.76 | 41612.7 |
| 0.8 | 66.86 | 64.72 | 63.49 | 63.19 | 63.93 | 66.03 | 70.22 | 78.02 | 92.85 | 13020.8 |
| 0.7 | 69.99 | 67.71 | 66.39 | 66.04 | 66.79 | 69.03 | 73.56 | 82.11 | 98.26 | 6234.28 |
| 0.6 | 73.83 | 71.41 | 70.00 | 69.64 | 70.46 | 72.90 | 77.86 | 87.23 | 104.68 | 3669.54 |
| 0.5 | 78.41 | 75.82 | 74.33 | 73.96 | 74.87 | 77.56 | 83.02 | 93.29 | 112.02 | 2439.01 |
| 0.4 | 83.78 | 81.01 | 79.42 | 79.04 | 80.07 | 83.04 | 89.07 | 100.34 | 120.35 | 1758.73 |
| 0.3 | 90.08 | 87.07 | 85.39 | 85.00 | 86.17 | 89.47 | 96.16 | 108.55 | 129.89 | 1346.41 |
| 0.2 | 97.48 | 94.25 | 92.42 | 92.02 | 93.35 | 97.04 | 104.50 | 118.18 | 140.96 | 1080.51 |
| 0.1 | 106.24 | 102.73 | 100.75 | 100.36 | 101.88 | 106.05 | 114.43 | 129.61 | 154.00 | 901.97 |

61.5

**Figure 4.18: Confidence region for monthly flow Greater Zab river when θ=0.361 & Θ=0.95 for SARIMA model (0,1,1)(0,1,1)$_{12}$.**

# Chapter Five
# FORECASTING

## 5.1 Forecasting by Using PARMA$_{12}$(1, 0) Model

Forecasted monthly data are computed for the period from 1993 to 2002 by the following steps:

1- Generation of data by using the following equations and rules given in section (3.10) for 12 season of PARMA$_{12}$(1,0) model:

Forecast for one month ahead

$$[Z_{r,m+1}] = [\hat{Z}_{r,m}(1)] = \phi_{m+1}[Z_{r,m}] + a_{r,m+1} \quad ,\ldots\ldots\ldots\ldots\ldots\ldots \text{(5-1)}$$

Two months ahead

$$Z_{r,m+2} = \hat{Z}_{r,m}(2) = \phi_{m+2}\hat{Z}_{r,m}(1) + a_{r,m+2} \quad ,\ldots\ldots\ldots\ldots\ldots\ldots \text{(5-2)}$$

and so on .

2- Reversing the standardization process (reversing equation 3-14) as shown in the following equation

$$Y_{r,m} = Z_{r,m}\,Sd_m + \mu_m \quad ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5-3)}$$

Where: $(\mu_m, Sd_m)$ are the periodic means and standard deviations respectively.

3- Applying the reverse Box-Cox transformation with λ for each season as shown in the following equation.

$$F_{r,m} = \exp((\ln(Y_{r,m} * \lambda) + 1)/\lambda) \quad ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5-4)}$$

where: $(F_{r,m})$ is the generated series of year (r) and month (m).

In this study we used software program (SAMS) (Stochastic Analysis, Modeling and Simulation, Version 2007), for forecasting of monthly flow for Greater Zab river by using $PARMA_{12}(1,0)$ model for period 10 Years from (1993-2002) as shown in Table 5.1.

**Table 5.1: Forecasting values for monthly flow Greater Zab river by $PARMA_{12}$ (1,0) model for period from 1993 to 2002**

| Forecasting monthly flow ($m^3$/sec) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Month / Year | Oct | Nov | Dec | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep |
| **1993** | 94.852 | 172.32 | 153.70 | 246.70 | 534.28 | 735.92 | 822.59 | 1164.1 | 546.14 | 235.25 | 115.31 | 136.38 |
| **1994** | 101.41 | 204.21 | 188.46 | 428.66 | 668.61 | 1086.6 | 985.80 | 881.06 | 633.9 | 370.39 | 213.16 | 165.49 |
| **1995** | 193.47 | 172.3 | 113.44 | 187.88 | 475.66 | 515.82 | 691.88 | 361.01 | 365.64 | 200.78 | 115.08 | 123.36 |
| **1996** | 124.53 | 131.38 | 110.07 | 217.00 | 561.94 | 1012.2 | 1867.6 | 2113.4 | 1118.6 | 543.47 | 293.30 | 197.40 |
| **1997** | 160.83 | 144.65 | 241.31 | 467.28 | 523.70 | 817.34 | 1082.7 | 1123.6 | 727.83 | 389.92 | 218.28 | 174.60 |
| **1998** | 168.26 | 156.64 | 246.95 | 261.16 | 375.04 | 482.09 | 629.63 | 732.64 | 439.58 | 268.64 | 142.95 | 98.522 |
| **1999** | 87.980 | 162.99 | 344.99 | 398.74 | 429.97 | 578.81 | 1014 | 1089.7 | 592.15 | 328.84 | 199.30 | 165.60 |
| **2000** | 190.44 | 119.43 | 83.294 | 105.82 | 130.12 | 386.40 | 681.09 | 698.39 | 406.69 | 238.65 | 140.61 | 105.72 |
| **2001** | 148.37 | 101.68 | 101.22 | 139.76 | 356.98 | 968.52 | 1315.5 | 810.89 | 601.31 | 308.94 | 170.56 | 149.93 |
| **2002** | 123.45 | 137.57 | 215.95 | 226.10 | 312.23 | 526.68 | 1050.3 | 1294.8 | 747.13 | 328.24 | 184.04 | 122.79 |

The accuracy of forecast may be provided by determining the upper and lower probability limits by using equation (3-38). Table 5.2 & Figure 5.1 show the forecast series with upper and lower probability limits (at 95% confidence limits) for each season of flow Greater Zab river by using $PARMA_{12}(1,0)$ model. Also see Figure 5.2 which shows forecast series with upper and lower probability limits (at 95% confidence limits) for monthly flow Greater Zab river by using $PARMA_{12}(1,0)$ model for period from 1993-2002. The L.L and U.L values are very close to the forecasted monthly flow for Greater Zab River, therefore, Figures 5.1 & 5.2 indicates there is no obvious difference between them (L.L and U.L) with respect to forecasted series.

**Table 5.2: Lower and upper probability limits for forecasted flow data for each season for Greater Zab river from 1993 to 2002.**

| year | season1 | | | season 2 | | | season 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | L.L | $F_{r,m}$ | U.L | L.L | $F_{r,m}$ | U.L | L.L | $F_{r,m}$ | U.L |
| 1993 | 92.91 | 94.852 | 96.793 | 170.38 | 172.32 | 174.27 | 151.76 | 153.7 | 155.64 |
| 1994 | 99.47 | 101.41 | 103.36 | 202.27 | 204.21 | 206.15 | 186.52 | 188.47 | 190.41 |
| 1995 | 191.5 | 193.48 | 195.42 | 170.36 | 172.3 | 174.24 | 111.5 | 113.44 | 115.39 |
| 1996 | 122.6 | 124.54 | 126.48 | 129.44 | 131.39 | 133.33 | 108.13 | 110.07 | 112.02 |
| 1997 | 158.9 | 160.83 | 162.78 | 142.71 | 144.65 | 146.6 | 239.37 | 241.31 | 243.26 |
| 1998 | 166.3 | 168.27 | 170.21 | 154.71 | 156.65 | 158.59 | 245.01 | 246.96 | 248.9 |
| 1999 | 86.04 | 87.981 | 89.922 | 161.05 | 163 | 164.94 | 343.05 | 344.99 | 346.94 |
| 2000 | 188.5 | 190.44 | 192.38 | 117.49 | 119.43 | 121.37 | 81.352 | 83.295 | 85.238 |
| 2001 | 146.4 | 148.38 | 150.32 | 99.74 | 101.68 | 103.63 | 99.277 | 101.22 | 103.16 |
| 2002 | 121.5 | 123.46 | 125.4 | 135.63 | 137.58 | 139.52 | 214.01 | 215.96 | 217.9 |
| year | season 4 | | | season 5 | | | season 6 | | |
| | L.L | $F_{r,m}$ | U.L | L.L | $F_{r,m}$ | U.L | L.L | $F_{r,m}$ | U.L |
| 1993 | 244.8 | 246.7 | 248.65 | 532.34 | 534.28 | 536.22 | 733.98 | 735.93 | 737.87 |
| 1994 | 426.7 | 428.67 | 430.61 | 666.67 | 668.62 | 670.56 | 1084.7 | 1086.7 | 1088.6 |
| 1995 | 185.9 | 187.89 | 189.83 | 473.72 | 475.66 | 477.6 | 513.89 | 515.83 | 517.77 |
| 1996 | 215.1 | 217.01 | 218.95 | 560 | 561.95 | 563.89 | 1010.3 | 1012.2 | 1014.2 |
| 1997 | 465.3 | 467.29 | 469.23 | 521.76 | 523.71 | 525.65 | 815.4 | 817.34 | 819.29 |
| 1998 | 259.2 | 261.16 | 263.11 | 373.1 | 375.04 | 376.98 | 480.15 | 482.09 | 484.04 |
| 1999 | 396.8 | 398.74 | 400.69 | 428.03 | 429.97 | 431.91 | 576.87 | 578.81 | 580.76 |
| 2000 | 103.9 | 105.83 | 107.77 | 128.18 | 130.12 | 132.07 | 384.46 | 386.41 | 388.35 |
| 2001 | 137.8 | 139.77 | 141.71 | 355.04 | 356.99 | 358.93 | 966.58 | 968.52 | 970.47 |
| 2002 | 224.2 | 226.1 | 228.05 | 310.3 | 312.24 | 314.18 | 524.74 | 526.68 | 528.63 |

**$F_{r,m}$ = Forecasting data ($m^3$/sec)**

**Table 5.2:Cont.**

| year | season 7 | | | season 8 | | | season 9 | | |
|---|---|---|---|---|---|---|---|---|---|
| | L.L | $F_{r,m}$ | U.L | L.L | $F_{r,m}$ | U.L | L.L | $F_{r,m}$ | U.L |
| 1993 | 820.7 | 822.6 | 824.54 | 1162.2 | 1164.2 | 1166.1 | 544.23 | 546.15 | 548.06 |
| 1994 | 983.9 | 985.8 | 987.75 | 879.13 | 881.07 | 883.01 | 631.99 | 633.9 | 635.82 |
| 1995 | 689.9 | 691.88 | 693.83 | 359.07 | 361.01 | 362.95 | 363.73 | 365.64 | 367.56 |
| 1996 | 1866 | 1867.6 | 1869.6 | 2111.5 | 2113.4 | 2115.3 | 1116.7 | 1118.7 | 1120.6 |
| 1997 | 1081 | 1082.8 | 1084.7 | 1121.7 | 1123.7 | 1125.6 | 725.92 | 727.83 | 729.75 |
| 1998 | 627.7 | 629.64 | 631.58 | 730.71 | 732.65 | 734.58 | 437.67 | 439.58 | 441.5 |
| 1999 | 1012 | 1014 | 1015.9 | 1087.8 | 1089.8 | 1091.7 | 590.24 | 592.15 | 594.07 |
| 2000 | 679.2 | 681.1 | 683.04 | 696.46 | 698.4 | 700.34 | 404.78 | 406.7 | 408.61 |
| 2001 | 1314 | 1315.5 | 1317.5 | 808.95 | 810.89 | 812.83 | 599.4 | 601.32 | 603.23 |
| 2002 | 1048 | 1050.3 | 1052.3 | 1292.9 | 1294.8 | 1296.8 | 745.22 | 747.14 | 749.05 |

| year | season 10 | | | season 11 | | | season 12 | | |
|---|---|---|---|---|---|---|---|---|---|
| | L.L | $F_{r,m}$ | U.L | L.L | $F_{r,m}$ | U.L | L.L | $F_{r,m}$ | U.L |
| 1993 | 233.4 | 235.25 | 237.08 | 113.65 | 115.32 | 116.98 | 134.67 | 136.39 | 138.11 |
| 1994 | 368.6 | 370.4 | 372.22 | 211.5 | 213.17 | 214.83 | 163.78 | 165.5 | 167.22 |
| 1995 | 199 | 200.79 | 202.61 | 113.42 | 115.09 | 116.75 | 121.65 | 123.37 | 125.08 |
| 1996 | 541.7 | 543.48 | 545.3 | 291.64 | 293.31 | 294.97 | 195.69 | 197.4 | 199.12 |
| 1997 | 388.1 | 389.92 | 391.75 | 216.62 | 218.28 | 219.95 | 172.89 | 174.6 | 176.32 |
| 1998 | 266.8 | 268.65 | 270.47 | 141.29 | 142.96 | 144.62 | 96.806 | 98.523 | 100.24 |
| 1999 | 327 | 328.84 | 330.67 | 197.64 | 199.3 | 200.97 | 163.89 | 165.6 | 167.32 |
| 2000 | 236.8 | 238.65 | 240.48 | 138.95 | 140.62 | 142.28 | 104 | 105.72 | 107.44 |
| 2001 | 307.1 | 308.94 | 310.77 | 168.9 | 170.56 | 172.23 | 148.22 | 149.94 | 151.66 |
| 2002 | 326.4 | 328.24 | 330.07 | 182.38 | 184.05 | 185.71 | 121.07 | 122.79 | 124.51 |


**Figuer 5.1:Probability limits of forecasted values for each season obtained by PARMA$_{12}$(1,0) model from 1993 to 2002 for Greater Zab river.**

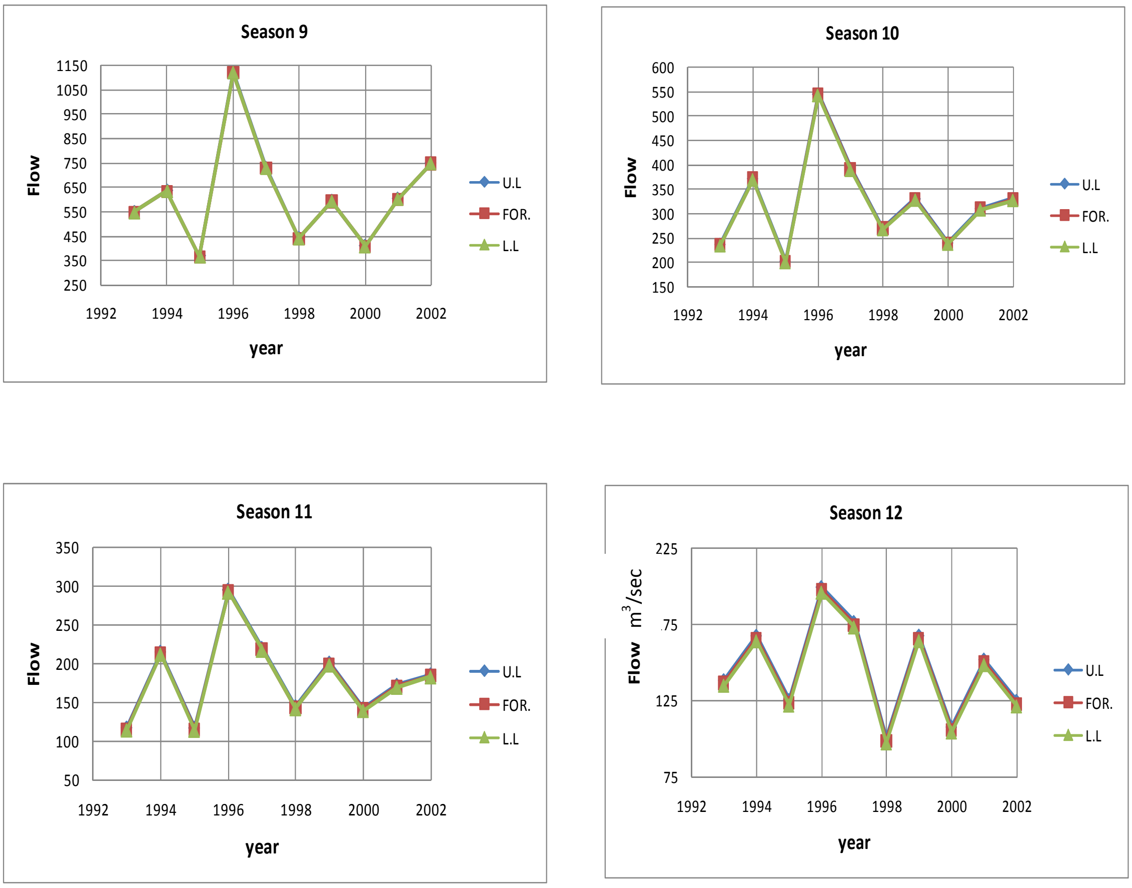



**Figure 5.1: Cont.**

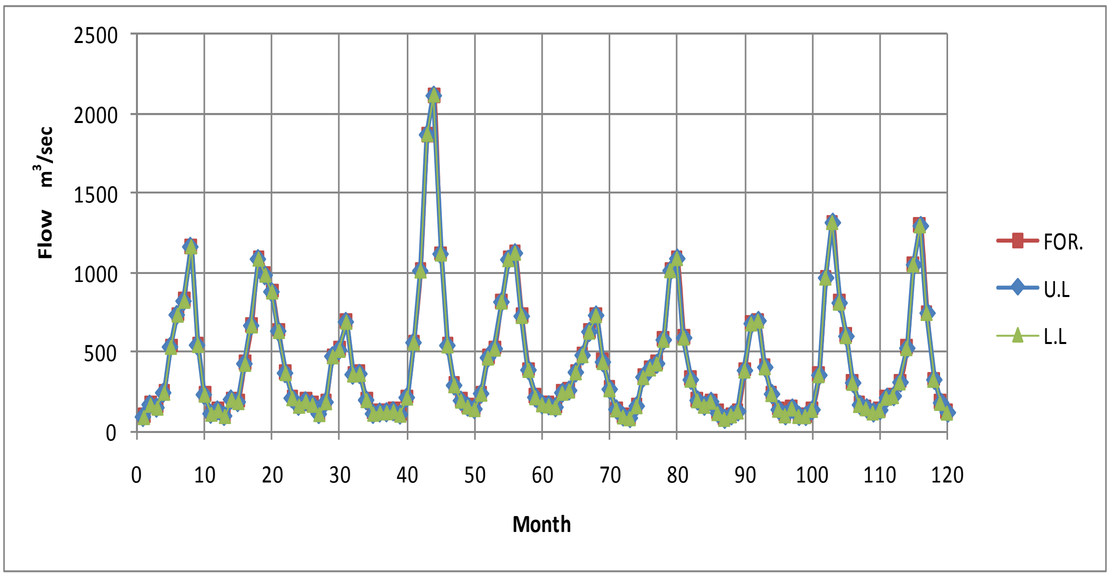



**Figure 5.2: Probability limits of forecasted values obtained by PARMA$_{12}$(1,0) model from 1993 to 2002 for monthly flow Greater Zab river.**

## 5.2 Forecasting by Using SARIMA $(0,1,1)x(0,1,1)_{12}$ model

Forecasted monthly data for Greater Zab River are computed for 10 years for the period from 1993 to 2002 by applying the forecasting equation is written in the following form:

$$\hat{Z}_t(\ell) = Z_{t+\ell-1} + Z_{t+\ell-12} - Z_{t+\ell-13} + a_{t+\ell} - \theta a_{t+\ell-1} - \Theta a_{t+\ell-12} + \theta\Theta a_{t+\ell-13} \quad ,\ldots\ldots(5\text{-}5)$$

where (t = 720) is the origin time and ℓ (=1, 2, 3, …, 120) is the lead time. After obtaining the forecasted series ( $Z_t$) for t=721, 722, 723, …., 840), the final series ( $X_t$) is determined by reversing (ln) transformation. Table 5.3 summarizes the forecasting of monthly flow for Greater Zab river by using seasonal ARIMA $(0,1,1)x(0,1,1)_{12}$ model. Equations (3-61 to 3-68) are used to calculate the upper and the lower probability limits (U.L. and L.L.) for forecasted series at 95% confidence limits. Table 5.4 gives the L.L and U.L for forecasted monthly flow for Greater Zab River. Figure 5.3 shows the L.L and U.L for forecasted monthly flow for Greater Zab River. The L.L and U.L values are very close to the forecasted monthly flow for Greater Zab River therefore, Figure 5.3 shows there is no obvious difference between them (L.L and U.L) with respect to forecasted series.

**Table 5.3: Forecasted monthly flow data for Greater Zab River obtained by seasonal ARIMA (0,1,1) (0,1,1)$_{12}$ model from 1993 to 2002.**

| i | Month | t | $Z_t$ | $X_t$ † | Year | Month | t | $Z_t$ | $X_t$ † |
|---|---|---|---|---|---|---|---|---|---|
| 1993 | Oct | 721 | 4.840 | 126.52 | 1996 | Oct | 757 | 4.865 | 129.63 |
| | Nov | 722 | 5.128 | 168.71 | | Nov | 758 | 5.153 | 172.86 |
| | Dec | 723 | 5.495 | 243.35 | | Dec | 759 | 5.519 | 249.34 |
| | Jan | 724 | 5.593 | 268.5 | | Jan | 760 | 5.617 | 275.11 |
| | Feb | 725 | 6.001 | 403.77 | | Feb | 761 | 6.025 | 413.7 |
| | Mar | 726 | 6.457 | 636.86 | | Mar | 762 | 6.481 | 652.52 |
| | Apr | 727 | 6.897 | 988.98 | | Apr | 763 | 6.921 | 1013.3 |
| | May | 728 | 6.814 | 910.82 | | May | 764 | 6.839 | 933.22 |
| | Jun | 729 | 6.305 | 547.31 | | Jun | 765 | 6.329 | 560.77 |
| | Jul | 730 | 5.633 | 279.63 | | Jul | 766 | 5.658 | 286.51 |
| | Aug | 731 | 5.085 | 161.65 | | Aug | 767 | 5.11 | 165.62 |
| | Sep | 732 | 4.850 | 127.78 | | Sep | 768 | 4.875 | 130.92 |
| 1994 | Oct | 733 | 4.849 | 127.55 | 1997 | Oct | 769 | 4.873 | 130.69 |
| | Nov | 734 | 5.136 | 170.09 | | Nov | 770 | 5.161 | 174.27 |
| | Dec | 735 | 5.503 | 245.33 | | Dec | 771 | 5.527 | 251.37 |
| | Jan | 736 | 5.601 | 270.69 | | Jan | 772 | 5.625 | 277.34 |
| | Feb | 737 | 6.009 | 407.05 | | Feb | 773 | 6.033 | 417.06 |
| | Mar | 738 | 6.465 | 642.04 | | Mar | 774 | 6.489 | 657.83 |
| | Apr | 739 | 6.905 | 997.02 | | Apr | 775 | 6.929 | 1021.5 |
| | May | 740 | 6.822 | 918.22 | | May | 776 | 6.847 | 940.81 |
| | Jun | 741 | 6.313 | 551.76 | | Jun | 777 | 6.337 | 565.33 |
| | Jul | 742 | 5.642 | 281.91 | | Jul | 778 | 5.666 | 288.84 |
| | Aug | 743 | 5.094 | 162.96 | | Aug | 779 | 5.118 | 166.97 |
| | Sep | 744 | 4.858 | 128.82 | | Sep | 780 | 4.883 | 131.98 |
| 1995 | Oct | 745 | 4.857 | 128.59 | 1998 | Oct | 781 | 4.881 | 131.75 |
| | Nov | 746 | 5.144 | 171.47 | | Nov | 782 | 5.169 | 175.69 |
| | Dec | 747 | 5.511 | 247.33 | | Dec | 783 | 5.535 | 253.41 |
| | Jan | 748 | 5.609 | 272.89 | | Jan | 784 | 5.633 | 279.6 |
| | Feb | 749 | 6.017 | 410.36 | | Feb | 785 | 6.041 | 420.45 |
| | Mar | 750 | 6.473 | 647.26 | | Mar | 786 | 6.497 | 663.18 |
| | Apr | 751 | 6.913 | 1005.1 | | Apr | 787 | 6.937 | 1029.9 |
| | May | 752 | 6.831 | 925.69 | | May | 788 | 6.855 | 948.46 |
| | Jun | 753 | 6.321 | 556.25 | | Jun | 789 | 6.346 | 569.93 |
| | Jul | 754 | 5.65 | 284.2 | | Jul | 790 | 5.674 | 291.19 |
| | Aug | 755 | 5.102 | 164.29 | | Aug | 791 | 5.126 | 168.33 |
| | Sep | 756 | 4.866 | 129.86 | | Sep | 792 | 4.891 | 133.06 |

† $X_t$ ($m^3/sec$) = exp ($Z_t$)

**Table 5.3: cont.**

| Year | Month | t | $Z_t$ | $X_t$ † | Year | Month | t | $Z_t$ | $X_t$ † |
|---|---|---|---|---|---|---|---|---|---|
| 1999 | Oct | 793 | 4.889 | 132.82 | 2001 | Oct | 817 | 4.905 | 134.99 |
| | Nov | 794 | 5.177 | 177.12 | | Nov | 818 | 5.193 | 180.01 |
| | Dec | 795 | 5.543 | 255.47 | | Dec | 819 | 5.559 | 259.64 |
| | Jan | 796 | 5.641 | 281.87 | | Jan | 820 | 5.658 | 286.48 |
| | Feb | 797 | 6.049 | 423.87 | | Feb | 821 | 6.066 | 430.80 |
| | Mar | 798 | 6.505 | 668.57 | | Mar | 822 | 6.521 | 679.49 |
| | Apr | 799 | 6.945 | 1038.2 | | Apr | 823 | 6.961 | 1055.2 |
| | May | 800 | 6.863 | 956.17 | | May | 824 | 6.879 | 971.79 |
| | Jun | 801 | 6.354 | 574.57 | | Jun | 825 | 6.370 | 583.95 |
| | Jul | 802 | 5.682 | 293.56 | | Jul | 826 | 5.698 | 298.35 |
| | Aug | 803 | 5.134 | 169.70 | | Aug | 827 | 5.150 | 172.47 |
| | Sep | 804 | 4.899 | 134.14 | | Sep | 828 | 4.915 | 136.33 |
| 2000 | Oct | 805 | 4.897 | 133.90 | 2002 | Oct | 829 | 4.913 | 136.09 |
| | Nov | 806 | 5.185 | 178.56 | | Nov | 830 | 5.201 | 181.47 |
| | Dec | 807 | 5.551 | 257.55 | | Dec | 831 | 5.567 | 261.75 |
| | Jan | 808 | 5.650 | 284.17 | | Jan | 832 | 5.666 | 288.81 |
| | Feb | 809 | 6.058 | 427.32 | | Feb | 833 | 6.074 | 434.30 |
| | Mar | 810 | 6.513 | 674.01 | | Mar | 834 | 6.529 | 685.02 |
| | Apr | 811 | 6.953 | 1046.7 | | Apr | 835 | 6.970 | 1063.8 |
| | May | 812 | 6.871 | 963.95 | | May | 836 | 6.887 | 979.69 |
| | Jun | 813 | 6.362 | 579.24 | | Jun | 837 | 6.378 | 588.70 |
| | Jul | 814 | 5.690 | 295.95 | | Jul | 838 | 5.706 | 300.78 |
| | Aug | 815 | 5.142 | 171.08 | | Aug | 839 | 5.158 | 173.87 |
| | Sep | 816 | 4.907 | 135.23 | | Sep | 840 | 4.923 | 137.44 |

† $X_t\ (\mathrm{m^3/sec}) = \exp(Z_t)$

**Table 5.4: Lower and upper probability limits for forecasted monthly flow data for Greater Zab river obtained by seasonal ARIMA (0,1,1) $(0,1,1)_{12}$ model from 1993 to 2002.**

| Year | Month | t | $X_t$ (-) | $X_t$ † | $X_t$ (+) | Year | Month | t | $X_t$ (-) | $X_t$ † | $X_t$ (+) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1993 | Oct | 721 | 121.55 | 126.52 | 131.49 | 1996 | Oct | 757 | 124.66 | 129.63 | 134.6 |
| | Nov | 722 | 163.74 | 168.71 | 173.69 | | Nov | 758 | 167.89 | 172.86 | 177.84 |
| | Dec | 723 | 238.38 | 243.35 | 248.32 | | Dec | 759 | 244.37 | 249.34 | 254.31 |
| | Jan | 724 | 263.53 | 268.5 | 273.47 | | Jan | 760 | 270.13 | 275.11 | 280.08 |
| | Feb | 725 | 398.79 | 403.77 | 408.74 | | Feb | 761 | 408.73 | 413.7 | 418.67 |
| | Mar | 726 | 631.89 | 636.86 | 641.83 | | Mar | 762 | 647.55 | 652.52 | 657.49 |
| | Apr | 727 | 984.01 | 988.98 | 993.95 | | Apr | 763 | 1008.3 | 1013.3 | 1018.3 |
| | May | 728 | 905.84 | 910.82 | 915.79 | | May | 764 | 928.25 | 933.22 | 938.19 |
| | Jun | 729 | 542.34 | 547.31 | 552.28 | | Jun | 765 | 555.8 | 560.77 | 565.74 |
| | Jul | 730 | 274.66 | 279.63 | 284.6 | | Jul | 766 | 281.54 | 286.51 | 291.48 |
| | Aug | 731 | 156.68 | 161.65 | 166.62 | | Aug | 767 | 160.65 | 165.62 | 170.6 |
| | Sep | 732 | 122.81 | 127.78 | 132.75 | | Sep | 768 | 125.95 | 130.92 | 135.89 |
| 1994 | Oct | 733 | 122.58 | 127.55 | 132.52 | 1997 | Oct | 769 | 125.72 | 130.69 | 135.66 |
| | Nov | 734 | 165.12 | 170.09 | 175.06 | | Nov | 770 | 169.3 | 174.27 | 179.24 |
| | Dec | 735 | 240.36 | 245.33 | 250.3 | | Dec | 771 | 246.39 | 251.37 | 256.34 |
| | Jan | 736 | 265.71 | 270.69 | 275.66 | | Jan | 772 | 272.37 | 277.34 | 282.31 |
| | Feb | 737 | 402.08 | 407.05 | 412.02 | | Feb | 773 | 412.09 | 417.06 | 422.03 |
| | Mar | 738 | 637.07 | 642.04 | 647.01 | | Mar | 774 | 652.86 | 657.83 | 662.8 |
| | Apr | 739 | 992.05 | 997.02 | 1002 | | Apr | 775 | 1016.6 | 1021.5 | 1026.5 |
| | May | 740 | 913.25 | 918.22 | 923.19 | | May | 776 | 935.84 | 940.81 | 945.78 |
| | Jun | 741 | 546.79 | 551.76 | 556.73 | | Jun | 777 | 560.36 | 565.33 | 570.3 |
| | Jul | 742 | 276.94 | 281.91 | 286.88 | | Jul | 778 | 283.87 | 288.84 | 293.81 |
| | Aug | 743 | 157.99 | 162.96 | 167.93 | | Aug | 779 | 162 | 166.97 | 171.94 |
| | Sep | 744 | 123.84 | 128.82 | 133.79 | | Sep | 780 | 127.01 | 131.98 | 136.95 |
| 1995 | Oct | 745 | 123.62 | 128.59 | 133.56 | 1998 | Oct | 781 | 126.78 | 131.75 | 136.72 |
| | Nov | 746 | 166.5 | 171.47 | 176.44 | | Nov | 782 | 170.72 | 175.69 | 180.66 |
| | Dec | 747 | 242.36 | 247.33 | 252.3 | | Dec | 783 | 248.44 | 253.41 | 258.38 |
| | Jan | 748 | 267.92 | 272.89 | 277.86 | | Jan | 784 | 274.63 | 279.6 | 284.57 |
| | Feb | 749 | 405.39 | 410.36 | 415.33 | | Feb | 785 | 415.48 | 420.45 | 425.42 |
| | Mar | 750 | 642.29 | 647.26 | 652.23 | | Mar | 786 | 658.21 | 663.18 | 668.15 |
| | Apr | 751 | 1000.2 | 1005.1 | 1010.1 | | Apr | 787 | 1024.9 | 1029.9 | 1034.8 |
| | May | 752 | 920.72 | 925.69 | 930.66 | | May | 788 | 943.49 | 948.46 | 953.43 |
| | Jun | 753 | 551.28 | 556.25 | 561.22 | | Jun | 789 | 564.96 | 569.93 | 574.9 |
| | Jul | 754 | 279.23 | 284.2 | 289.17 | | Jul | 790 | 286.22 | 291.19 | 296.16 |
| | Aug | 755 | 159.32 | 164.29 | 169.26 | | Aug | 791 | 163.36 | 168.33 | 173.3 |
| | Sep | 756 | 124.89 | 129.86 | 134.83 | | Sep | 792 | 128.09 | 133.06 | 138.03 |

**† Forecasted series ($m^3$/sec )**

**Table 5.4: cont.**

| Year | Month | t | $X_t$ (-) | $X_t$ † | $X_t$ (+) | Year | Month | t | $X_t$ (-) | $X_t$ † | $X_t$ (+) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1999 | Oct | 793 | 127.85 | 132.82 | 137.79 | 2001 | Oct | 817 | 130.02 | 134.99 | 139.96 |
| | Nov | 794 | 172.15 | 177.12 | 182.09 | | Nov | 818 | 175.04 | 180.01 | 184.98 |
| | Dec | 795 | 250.5 | 255.47 | 260.44 | | Dec | 819 | 254.67 | 259.64 | 264.61 |
| | Jan | 796 | 276.9 | 281.87 | 286.84 | | Jan | 820 | 281.51 | 286.48 | 291.45 |
| | Feb | 797 | 418.9 | 423.87 | 428.84 | | Feb | 821 | 425.82 | 430.8 | 435.77 |
| | Mar | 798 | 663.6 | 668.57 | 673.54 | | Mar | 822 | 674.52 | 679.49 | 684.46 |
| | Apr | 799 | 1033.3 | 1038.2 | 1043.2 | | Apr | 823 | 1050.2 | 1055.2 | 1060.2 |
| | May | 800 | 951.2 | 956.17 | 961.14 | | May | 824 | 966.82 | 971.79 | 976.76 |
| | Jun | 801 | 569.59 | 574.57 | 579.54 | | Jun | 825 | 578.98 | 583.95 | 588.92 |
| | Jul | 802 | 288.59 | 293.56 | 298.53 | | Jul | 826 | 293.38 | 298.35 | 303.32 |
| | Aug | 803 | 164.73 | 169.7 | 174.67 | | Aug | 827 | 167.5 | 172.47 | 177.44 |
| | Sep | 804 | 129.17 | 134.14 | 139.11 | | Sep | 828 | 131.36 | 136.33 | 141.3 |
| 2000 | Oct | 805 | 128.93 | 133.9 | 138.87 | 2002 | Oct | 829 | 131.12 | 136.09 | 141.06 |
| | Nov | 806 | 173.59 | 178.56 | 183.53 | | Nov | 830 | 176.5 | 181.47 | 186.44 |
| | Dec | 807 | 252.58 | 257.55 | 262.52 | | Dec | 831 | 256.78 | 261.75 | 266.73 |
| | Jan | 808 | 279.19 | 284.17 | 289.14 | | Jan | 832 | 283.83 | 288.81 | 293.78 |
| | Feb | 809 | 422.35 | 427.32 | 432.29 | | Feb | 833 | 429.33 | 434.3 | 439.27 |
| | Mar | 810 | 669.04 | 674.01 | 678.98 | | Mar | 834 | 680.05 | 685.02 | 689.99 |
| | Apr | 811 | 1041.7 | 1046.7 | 1051.6 | | Apr | 835 | 1058.8 | 1063.8 | 1068.7 |
| | May | 812 | 958.98 | 963.95 | 968.92 | | May | 836 | 974.72 | 979.69 | 984.66 |
| | Jun | 813 | 574.27 | 579.24 | 584.21 | | Jun | 837 | 583.73 | 588.7 | 593.67 |
| | Jul | 814 | 290.97 | 295.95 | 300.92 | | Jul | 838 | 295.81 | 300.78 | 305.75 |
| | Aug | 815 | 166.11 | 171.08 | 176.05 | | Aug | 839 | 168.9 | 173.87 | 178.84 |
| | Sep | 816 | 130.26 | 135.23 | 140.2 | | Sep | 840 | 132.47 | 137.44 | 142.41 |

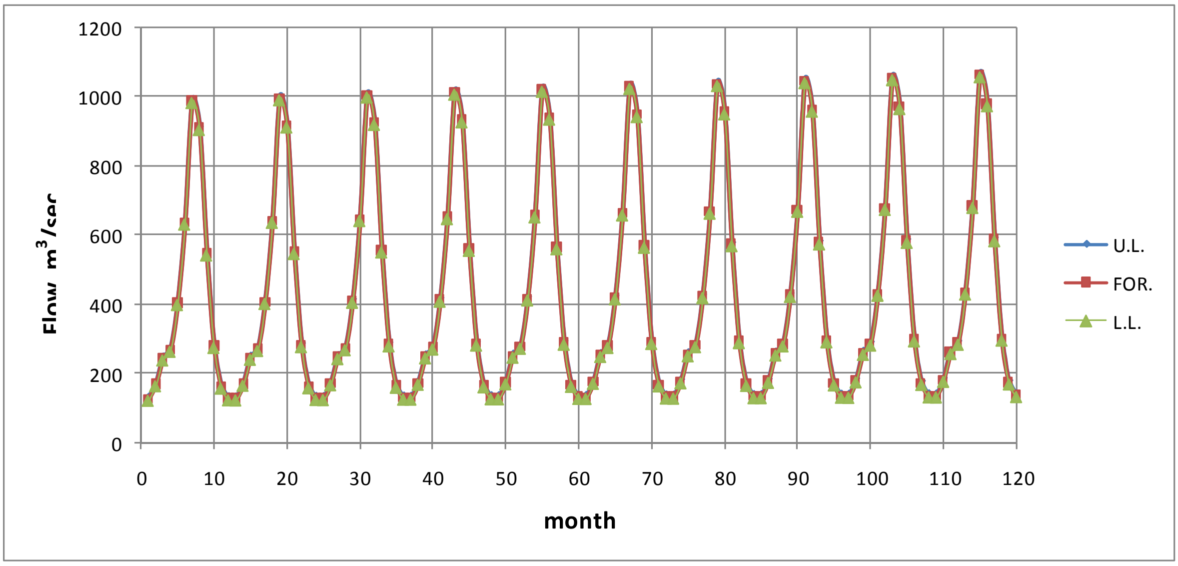



**Figure 5.3: Probability limits of forecasted monthly flow data for Greater Zab river obtained by seasonal ARIMA (0,1,1) $(0,1,1)_{12}$model from 1993-2002 .**

## 5.3 Comparison between $PARMA_{12}$ (1,0) Model and SARIMA $(0,1,1)X(0,1,1)_{12}$ Model

In order to evaluate the accuracy of the forecasting and to compare the forecasting ability of the two modeling methods for minimum forecasted error, the mean absolute relative error (MARE %) which obtained by equation (3-48) and mean absolute error (MAE) which obtained by equation (3-49) and mean square error (MSE) which obtained by equation (3-50) were used. The best method is the one that gives the minimum value for each from (MARE %, MAE and MSE) value. As explained in Table 5.5 for calculation of (MARE %, MAE and MSE) for monthly flow Greater Zab river for period from 1993 to 2002. Results of application of these measures are summarized in Table 5.6 to select the proper model.

Figures 5.4 & 5.5 show the comparison between the forecasted data obtained from the two models ($PARMA_{12}$ (1,0) model and SARIMA $(0,1,1)X(0,1,1)_{12}$ model) and the historical data for monthly flow Greater Zab river for the ten years for period from 1993 to 2002.

Figures 5.6 & 5.7 which represent the comparison between monthly means of the generated data by PARMA model and SARIMA model and observed data from 1993-2002. These figures indicate that the generated data by SARIMA model is best from generated data by PARMA model because it is give monthly means slightly different with those of observed data. From the result in Table 5.6, it is found that the SARIMA $(0,1,1)X(0,1,1)_{12}$ model represents the observed values more closely than the $PARMA_{12}$ (1,0) model because it gives minimum forecasting error by all tests of (MARE %, MAE and MSE).

# Table 5.5:The calculation of (MARE %, MAE and MSE) for Greater Zab river from 1993-2002

| time | Observed data | PARMA model | | | | SARIMA model | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Generated data | MARE% | MAE | MSE | Generated data | MARE% | MAE | MSE |
| 1 | 140 | 94.85 | 32.25 | 45.15 | 2038.31 | 126.52 | 9.628 | 13.48 | 181.69 |
| 2 | 253 | 172.3 | 31.89 | 80.68 | 6508.78 | 168.71 | 33.31 | 84.29 | 7104.01 |
| 3 | 481 | 153.7 | 68.05 | 327.3 | 107125 | 243.35 | 49.41 | 237.6 | 56476.6 |
| 4 | 362 | 246.7 | 31.85 | 115.3 | 13293.4 | 268.5 | 25.83 | 93.5 | 8741.93 |
| 5 | 475 | 534.3 | 12.48 | 59.28 | 3514.12 | 403.77 | 15 | 71.23 | 5074.32 |
| 6 | 510 | 735.9 | 44.3 | 225.9 | 51043.5 | 636.86 | 24.87 | 126.9 | 16093.2 |
| 7 | 1694 | 822.6 | 51.44 | 871.4 | 759345 | 988.98 | 41.62 | 705 | 497056 |
| 8 | 2282 | 1164 | 48.98 | 1118 | 1249522 | 910.82 | 60.09 | 1371 | 1880148 |
| 9 | 1030 | 546.1 | 46.98 | 483.9 | 234116 | 547.31 | 46.86 | 482.7 | 232989 |
| 10 | 282 | 235.3 | 16.58 | 46.75 | 2185.47 | 279.63 | 0.839 | 2.367 | 5.60338 |
| 11 | 147 | 115.3 | 21.55 | 31.68 | 1003.75 | 161.65 | 9.965 | 14.65 | 214.57 |
| 12 | 105 | 136.4 | 29.89 | 31.39 | 985.207 | 127.78 | 21.69 | 22.78 | 518.759 |
| 13 | 122 | 101.4 | 16.87 | 20.59 | 423.783 | 127.55 | 4.549 | 5.55 | 30.799 |
| 14 | 220 | 204.2 | 7.177 | 15.79 | 249.293 | 170.09 | 22.69 | 49.91 | 2491.33 |
| 15 | 165 | 188.5 | 14.22 | 23.47 | 550.7 | 245.33 | 48.69 | 80.33 | 6453.09 |
| 16 | 305 | 428.7 | 40.55 | 123.7 | 15293.8 | 270.69 | 11.25 | 34.31 | 1177.5 |
| 17 | 320 | 668.6 | 108.9 | 348.6 | 121533 | 407.05 | 27.2 | 87.05 | 7577.6 |
| 18 | 437 | 1087 | 148.7 | 649.7 | 422058 | 642.04 | 46.92 | 205 | 42040.7 |
| 19 | 1035 | 985.8 | 4.753 | 49.2 | 2420.34 | 997.02 | 3.669 | 37.98 | 1442.38 |
| 20 | 665 | 881.1 | 32.49 | 216.1 | 46685.8 | 918.22 | 38.08 | 253.2 | 64121.7 |
| 21 | 773 | 633.9 | 17.99 | 139.1 | 19348.8 | 551.76 | 28.62 | 221.2 | 48946.5 |
| 22 | 210 | 370.4 | 76.38 | 160.4 | 25727.2 | 281.91 | 34.24 | 71.91 | 5170.62 |
| 23 | 145 | 213.2 | 47.01 | 68.17 | 4646.74 | 162.96 | 12.39 | 17.96 | 322.663 |
| 24 | 102 | 165.5 | 62.25 | 63.5 | 4032 | 128.82 | 26.29 | 26.82 | 719.068 |
| 25 | 130 | 193.5 | 48.83 | 63.48 | 4029.2 | 128.59 | 1.087 | 1.413 | 1.99657 |
| 26 | 375 | 172.3 | 54.05 | 202.7 | 41087.3 | 171.47 | 54.27 | 203.5 | 41424.4 |
| 27 | 590 | 113.4 | 80.77 | 476.6 | 227108 | 247.33 | 58.08 | 342.7 | 117425 |
| 28 | 483 | 187.9 | 61.1 | 295.1 | 87092.3 | 272.89 | 43.5 | 210.1 | 44147.6 |
| 29 | 765 | 475.7 | 37.82 | 289.3 | 83717.1 | 410.36 | 46.36 | 354.6 | 125770 |
| 30 | 801 | 515.8 | 35.6 | 285.2 | 81322.5 | 647.26 | 19.19 | 153.7 | 23636 |
| 31 | 1727 | 691.9 | 59.94 | 1035 | 1071469 | 1005.1 | 41.8 | 721.9 | 521097 |
| 32 | 1378 | 361 | 73.8 | 1017 | 1034265 | 925.69 | 32.82 | 452.3 | 204584 |
| 33 | 710 | 365.6 | 48.5 | 344.4 | 118584 | 556.25 | 21.66 | 153.8 | 23639.4 |
| 34 | 328 | 200.8 | 38.78 | 127.2 | 16182.9 | 284.2 | 13.35 | 43.8 | 1918.47 |
| 35 | 163 | 115.1 | 29.39 | 47.91 | 2295.46 | 164.29 | 0.79 | 1.288 | 1.65929 |
| 36 | 105 | 123.4 | 17.49 | 18.37 | 337.273 | 129.86 | 23.68 | 24.86 | 618.171 |
| 37 | 131 | 124.5 | 4.935 | 6.465 | 41.7962 | 129.63 | 1.044 | 1.367 | 1.86938 |
| 38 | 160 | 131.4 | 17.88 | 28.62 | 818.818 | 172.86 | 8.04 | 12.86 | 165.496 |
| 39 | 133 | 110.1 | 17.24 | 22.93 | 525.693 | 249.34 | 87.47 | 116.3 | 13534.5 |
| 40 | 317 | 217 | 31.54 | 99.99 | 9998.4 | 275.11 | 13.22 | 41.89 | 1755.11 |
| 41 | 383 | 561.9 | 46.72 | 178.9 | 32021.3 | 413.7 | 8.015 | 30.7 | 942.311 |
| 42 | 503 | 1012 | 101.2 | 509.2 | 259325 | 652.52 | 29.73 | 149.5 | 22357.4 |
| 43 | 867 | 1868 | 115.4 | 1001 | 1001260 | 1013.3 | 16.87 | 146.3 | 21404.9 |
| 44 | 848 | 2113 | 149.2 | 1265 | 1601237 | 933.22 | 10.05 | 85.22 | 7262.17 |
| 45 | 388 | 1119 | 188.3 | 730.7 | 533849 | 560.77 | 44.53 | 172.8 | 29850.3 |
| 46 | 195 | 543.5 | 178.7 | 348.5 | 121437 | 286.51 | 46.93 | 91.51 | 8374.25 |
| 47 | 110 | 293.3 | 166.6 | 183.3 | 33602.2 | 165.62 | 50.57 | 55.62 | 3094.05 |
| 48 | 100 | 197.4 | 97.4 | 97.4 | 9487.34 | 130.92 | 30.92 | 30.92 | 955.995 |
| 49 | 105 | 160.8 | 53.18 | 55.83 | 3117.44 | 130.69 | 24.46 | 25.69 | 659.822 |
| 50 | 125 | 144.7 | 15.72 | 19.65 | 386.28 | 174.27 | 39.42 | 49.27 | 2427.57 |
| 51 | 306 | 241.3 | 21.14 | 64.69 | 4184.28 | 251.37 | 17.85 | 54.63 | 2984.93 |
| 52 | 365 | 467.3 | 28.02 | 102.3 | 10462.8 | 277.34 | 24.02 | 87.66 | 7683.7 |
| 53 | 325 | 523.7 | 61.14 | 198.7 | 39484.1 | 417.06 | 28.33 | 92.06 | 8475.32 |
| 54 | 635 | 817.3 | 28.72 | 182.3 | 33249.3 | 657.83 | 3.595 | 22.83 | 521.231 |
| 55 | 1278 | 1083 | 15.28 | 195.2 | 38114.8 | 1021.5 | 20.07 | 256.5 | 65769.3 |
| 56 | 1253 | 1124 | 10.32 | 129.3 | 16723.7 | 940.81 | 24.92 | 312.2 | 97463.9 |
| 57 | 695 | 727.8 | 4.724 | 32.83 | 1078.07 | 565.33 | 18.66 | 129.7 | 16813.5 |
| 58 | 355 | 389.9 | 9.838 | 34.92 | 1219.69 | 288.84 | 18.64 | 66.16 | 4377.01 |
| 59 | 188 | 218.3 | 16.11 | 30.28 | 917 | 166.97 | 11.19 | 21.03 | 442.211 |
| 60 | 130 | 174.6 | 34.31 | 44.6 | 1989.52 | 131.98 | 1.526 | 1.984 | 3.93578 |

# Table 5.5:Cont.

| time | Observed data | PARMA model | | | | SARIMA model | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Generated data | MARE% | MAE | MSE | Generated data | MARE% | MAE | MSE |
| **61** | 145 | 168.3 | 16.05 | 23.27 | 541.4 | 131.75 | 9.138 | 13.25 | 175.567 |
| **62** | 170 | 156.6 | 7.854 | 13.35 | 178.276 | 175.69 | 3.346 | 5.688 | 32.3494 |
| **63** | 325 | 247 | 24.01 | 78.04 | 6090.71 | 253.41 | 22.03 | 71.59 | 5125.17 |
| **64** | 300 | 261.2 | 12.95 | 38.84 | 1508.47 | 279.6 | 6.8 | 20.4 | 416.208 |
| **65** | 525 | 375 | 28.56 | 150 | 22488 | 420.45 | 19.91 | 104.5 | 10930 |
| **66** | 645 | 482.1 | 25.26 | 162.9 | 26538.4 | 663.18 | 2.819 | 18.18 | 330.525 |
| **67** | 1360 | 629.6 | 53.7 | 730.4 | 533433 | 1029.9 | 24.28 | 330.1 | 108997 |
| **68** | 790 | 732.6 | 7.26 | 57.36 | 3289.6 | 948.46 | 20.06 | 158.5 | 25109.3 |
| **69** | 435 | 439.6 | 1.054 | 4.584 | 21.0131 | 569.93 | 31.02 | 134.9 | 18206.3 |
| **70** | 210 | 268.6 | 27.93 | 58.65 | 3439.24 | 291.19 | 38.66 | 81.19 | 6591.82 |
| **71** | 110 | 143 | 29.96 | 32.96 | 1086.1 | 168.33 | 53.03 | 58.33 | 3402.28 |
| **72** | 100 | 98.52 | 1.477 | 1.477 | 2.18242 | 133.06 | 33.06 | 33.06 | 1092.78 |
| **73** | 107 | 87.98 | 17.78 | 19.02 | 361.738 | 132.82 | 24.13 | 25.82 | 666.739 |
| **74** | 124 | 163 | 31.45 | 39 | 1520.69 | 177.12 | 42.84 | 53.12 | 2821.36 |
| **75** | 104 | 345 | 231.7 | 241 | 58078.1 | 255.47 | 145.6 | 151.5 | 22943.3 |
| **76** | 123 | 398.7 | 224.2 | 275.7 | 76034.2 | 281.87 | 129.2 | 158.9 | 25240.5 |
| **77** | 245 | 430 | 75.5 | 185 | 34213.9 | 423.87 | 73.01 | 178.9 | 31995.4 |
| **78** | 231 | 578.8 | 150.6 | 347.8 | 120973 | 668.57 | 189.4 | 437.6 | 191471 |
| **79** | 440 | 1014 | 130.5 | 574 | 329476 | 1038.2 | 136 | 598.2 | 357877 |
| **80** | 335 | 1090 | 225.3 | 754.8 | 569678 | 956.17 | 185.4 | 621.2 | 385855 |
| **81** | 178 | 592.2 | 232.7 | 414.2 | 171522 | 574.57 | 222.8 | 396.6 | 157264 |
| **82** | 84 | 328.8 | 291.5 | 244.8 | 59948.1 | 293.56 | 249.5 | 209.6 | 43914.6 |
| **83** | 65 | 199.3 | 206.6 | 134.3 | 18037.3 | 169.7 | 161.1 | 104.7 | 10961.7 |
| **84** | 65 | 165.6 | 154.8 | 100.6 | 10121 | 134.14 | 106.4 | 69.14 | 4780.25 |
| **85** | 86 | 190.4 | 121.4 | 104.4 | 10908.1 | 133.9 | 55.7 | 47.9 | 2294.55 |
| **86** | 75 | 119.4 | 59.24 | 44.43 | 1974.02 | 178.56 | 138.1 | 103.6 | 10724 |
| **87** | 105 | 83.29 | 20.67 | 21.71 | 471.124 | 257.55 | 145.3 | 152.5 | 23271 |
| **88** | 165 | 105.8 | 35.86 | 59.17 | 3501.44 | 284.17 | 72.22 | 119.2 | 14200.3 |
| **89** | 205 | 130.1 | 36.53 | 74.88 | 5606.57 | 427.32 | 108.4 | 222.3 | 49426.1 |
| **90** | 285 | 386.4 | 35.58 | 101.4 | 10283.2 | 674.01 | 136.5 | 389 | 151330 |
| **91** | 513 | 681.1 | 32.77 | 168.1 | 28256.9 | 1046.7 | 104 | 533.7 | 284805 |
| **92** | 346 | 698.4 | 101.8 | 352.4 | 124184 | 963.95 | 178.6 | 617.9 | 381861 |
| **93** | 162 | 406.7 | 151 | 244.7 | 59877.6 | 579.24 | 257.6 | 417.2 | 174088 |
| **94** | 102 | 238.7 | 134 | 136.7 | 18673.5 | 295.95 | 190.1 | 193.9 | 37614.9 |
| **95** | 73 | 140.6 | 92.63 | 67.62 | 4572.06 | 171.08 | 134.4 | 98.08 | 9619.32 |
| **96** | 71 | 105.7 | 48.9 | 34.72 | 1205.48 | 135.23 | 90.47 | 64.23 | 4125.53 |
| **97** | 65 | 148.4 | 128.3 | 83.38 | 6951.89 | 134.99 | 107.7 | 69.99 | 4898.66 |
| **98** | 80 | 101.7 | 27.1 | 21.68 | 470.152 | 180.01 | 125 | 100 | 10001.8 |
| **99** | 235 | 101.2 | 56.93 | 133.8 | 17897.1 | 259.64 | 10.49 | 24.64 | 607.267 |
| **100** | 155 | 139.8 | 9.828 | 15.23 | 232.044 | 286.48 | 84.82 | 131.5 | 17286 |
| **101** | 200 | 357 | 78.49 | 157 | 24644.9 | 430.8 | 115.4 | 230.8 | 53266.4 |
| **102** | 440 | 968.5 | 120.1 | 528.5 | 279337 | 679.49 | 54.43 | 239.5 | 57356.6 |
| **103** | 520 | 1316 | 153 | 795.5 | 632852 | 1055.2 | 102.9 | 535.2 | 286422 |
| **104** | 345 | 810.9 | 135 | 465.9 | 217055 | 971.79 | 181.7 | 626.8 | 392863 |
| **105** | 240 | 601.3 | 150.5 | 361.3 | 130549 | 583.95 | 143.3 | 343.9 | 118301 |
| **106** | 160 | 308.9 | 93.09 | 148.9 | 22183.4 | 298.35 | 86.47 | 138.4 | 19141.4 |
| **107** | 115 | 170.6 | 48.32 | 55.56 | 3087.25 | 172.47 | 49.97 | 57.47 | 3302.74 |
| **108** | 80 | 149.9 | 87.42 | 69.94 | 4891.46 | 136.33 | 70.41 | 56.33 | 3173.07 |
| **109** | 85 | 123.5 | 45.24 | 38.46 | 1478.94 | 136.09 | 60.1 | 51.09 | 2610.01 |
| **110** | 85 | 137.6 | 61.85 | 52.58 | 2764.13 | 181.47 | 113.5 | 96.47 | 9307.03 |
| **111** | 225 | 216 | 4.02 | 9.045 | 81.812 | 261.75 | 16.34 | 36.75 | 1350.88 |
| **112** | 485 | 226.1 | 53.38 | 258.9 | 67027.1 | 288.81 | 40.45 | 196.2 | 38492.1 |
| **113** | 365 | 312.2 | 14.46 | 52.76 | 2783.83 | 434.3 | 18.99 | 69.3 | 4802.3 |
| **114** | 600 | 526.7 | 12.22 | 73.32 | 5375.53 | 685.02 | 14.17 | 85.02 | 7228.14 |
| **115** | 1375 | 1050 | 23.61 | 324.7 | 105424 | 1063.8 | 22.64 | 311.2 | 96867.2 |
| **116** | 1085 | 1295 | 19.34 | 209.8 | 44032.8 | 979.69 | 9.706 | 105.3 | 11089.9 |
| **117** | 560 | 747.1 | 33.42 | 187.1 | 35021 | 588.7 | 5.125 | 28.7 | 823.586 |
| **118** | 260 | 328.2 | 26.25 | 68.24 | 4656.97 | 300.78 | 15.68 | 40.78 | 1662.91 |
| **119** | 150 | 184 | 22.7 | 34.05 | 1159.06 | 173.87 | 15.91 | 23.87 | 569.876 |
| **120** | 123 | 122.8 | 0.171 | 0.21 | 0.0441 | 137.44 | 11.74 | 14.44 | 208.478 |
| | | **Sum =** | **7497** | **25318** | **1.4E+07** | | **6598** | **19644** | **8007874** |

**Table 5.6: Result of minumim forcasting error by PARMA & SARIMA models**

| Type of model | MARE % | MAE | MSE |
|---|---|---|---|
| $PARMA_{12}(1,0)$ | 62.467 | 210.983 | 116666.667 |
| $SARIMA(0,1,1)(0,1,1)_{12}$ | 54.983* | 163.700* | 66732.283* |

* Minimum value

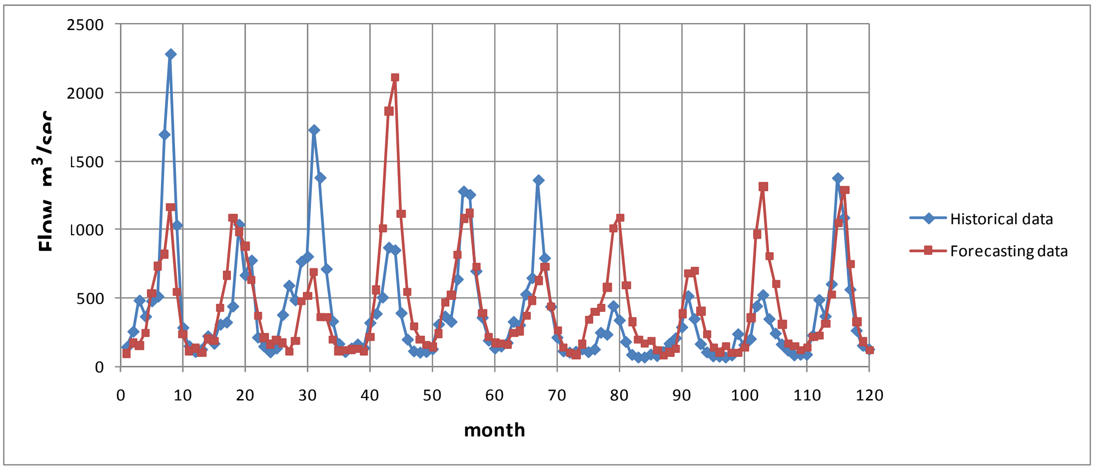



**Figure 5.4: Compared between Historical data and Forecasted data for monthly flow Greater Zab river obtained by $PARMA_{12}(1,0)$ model for period (1993-2002).**

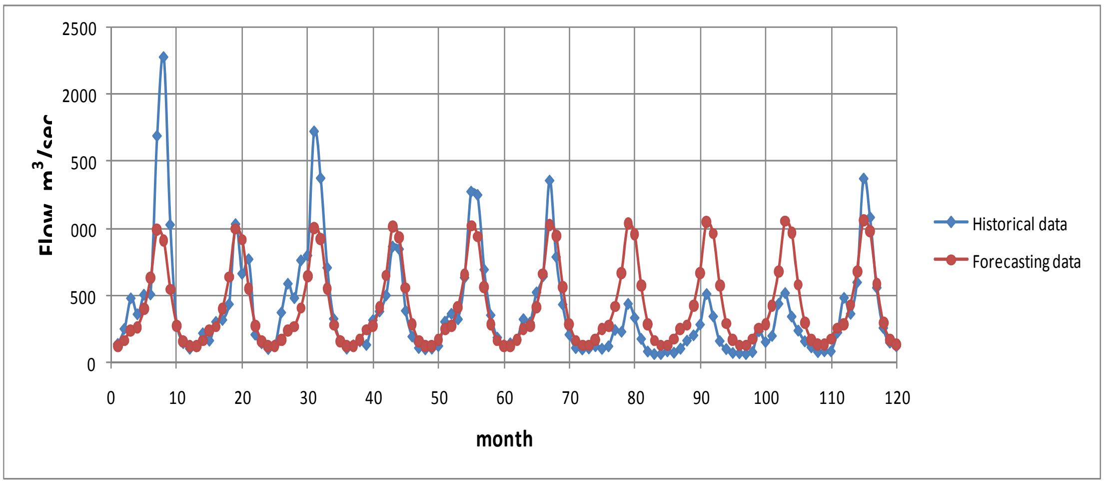



**Figure 5.5: Compared between Historical data and Forecasted data for monthly flow Greater Zab river obtained by SARIMA model (0, 1, 1) × $(0, 1, 1)_{12}$ for period (1993-2002).**

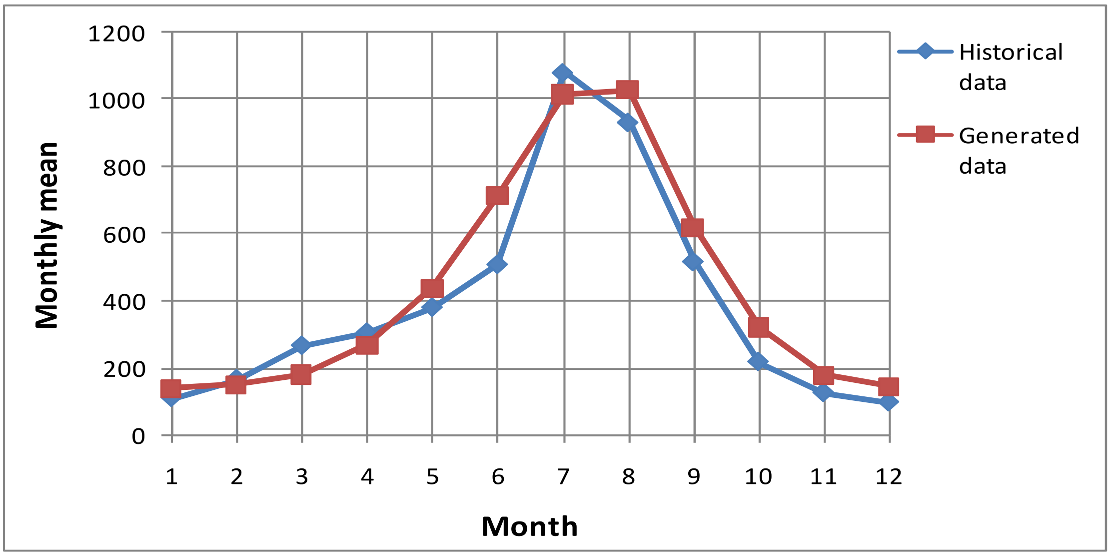



**Figure 5.6: Comparison between monthly means of the generated data by PARMA$_{12}$(1,0) model and historical data from 1993-2002.**

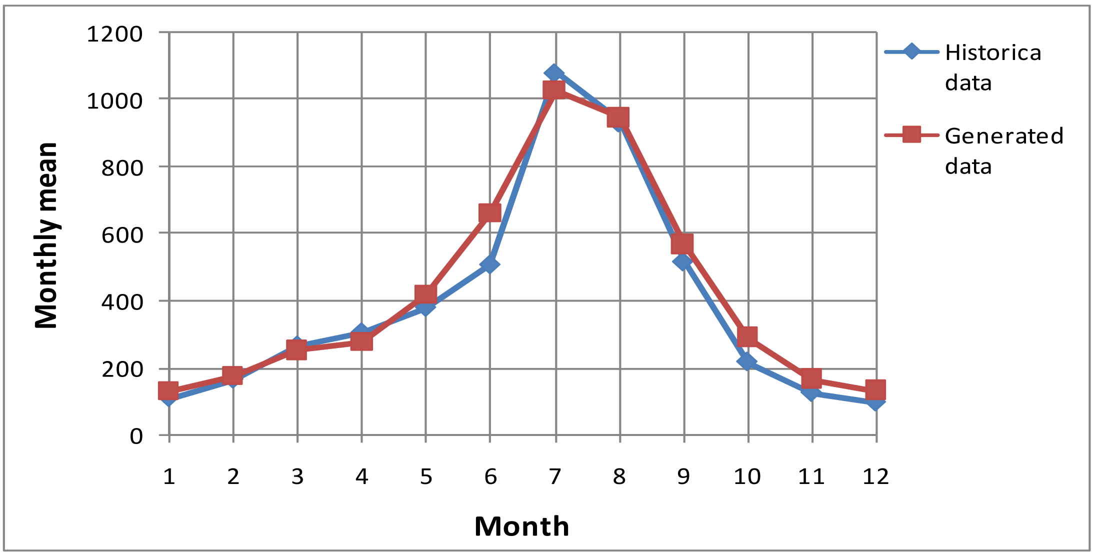



**Figure 5.7: Comparison between monthly means of the generated data by SARIMA (0,1,1)(0,1,1)$_{12}$ model and historical data from 1993-2002.**

## 5.4 Forecasting Future Values by SARIMA (0, 1, 1) (0, 1, 1)$_{12}$ Model

This section includes making forecasting by SARIMA (0, 1, 1) (0, 1, 1)$_{12}$ model for ten years extends from 2003 to 2012. Table 5.7 shows a calculation of future values with upper and the lower probability limits for forecasted series at 95% confidence limits for monthly flow Greater Zab river. Also, see Figure 5.8 which shows future values.

**Table 5.7: Future values with upper and the lower probability limits of monthly flow for Greater Zab river from 2003-2012**

| Year | Month | t | $F_t$ (-) | $F_t$ † | $F_t$ (+) | Year | Month | t | $F_t$ (-) | $F_t$ † | $F_t$ (+) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2003 | Oct | 1 | 132.2 | 137.2 | 142.2 | 2006 | Oct | 37 | 135.6 | 140.6 | 145.5 |
| | Nov | 2 | 178 | 182.9 | 187.9 | | Nov | 38 | 182.5 | 187.4 | 192.4 |
| | Dec | 3 | 258.9 | 263.9 | 268.9 | | Dec | 39 | 265.4 | 270.4 | 275.3 |
| | Jan | 4 | 286.2 | 291.2 | 296.1 | | Jan | 40 | 293.3 | 298.3 | 303.3 |
| | Feb | 5 | 432.9 | 437.8 | 442.8 | | Feb | 41 | 443.6 | 448.6 | 453.6 |
| | Mar | 6 | 685.6 | 690.6 | 695.6 | | Mar | 42 | 702.6 | 707.6 | 712.5 |
| | Apr | 7 | 1067 | 1072 | 1077 | | Apr | 43 | 1094 | 1099 | 1104 |
| | May | 8 | 982.7 | 987.7 | 992.6 | | May | 44 | 1007 | 1012 | 1017 |
| | Jun | 9 | 588.5 | 593.5 | 598.5 | | Jun | 45 | 603.1 | 608.1 | 613.1 |
| | Jul | 10 | 298.3 | 303.2 | 308.2 | | Jul | 46 | 305.7 | 310.7 | 315.7 |
| | Aug | 11 | 170.3 | 175.3 | 180.3 | | Aug | 47 | 174.6 | 179.6 | 184.6 |
| | Sep | 12 | 133.6 | 138.6 | 143.5 | | Sep | 48 | 137 | 142 | 146.9 |
| 2004 | Oct | 13 | 133.3 | 138.3 | 143.3 | 2007 | Oct | 49 | 136.7 | 141.7 | 146.7 |
| | Nov | 14 | 179.5 | 184.4 | 189.4 | | Nov | 50 | 184 | 189 | 193.9 |
| | Dec | 15 | 261.1 | 266 | 271 | | Dec | 51 | 267.6 | 272.6 | 277.5 |
| | Jan | 16 | 288.6 | 293.5 | 298.5 | | Jan | 52 | 295.8 | 300.7 | 305.7 |
| | Feb | 17 | 436.4 | 441.4 | 446.4 | | Feb | 53 | 447.3 | 452.2 | 457.2 |
| | Mar | 18 | 691.2 | 696.2 | 701.2 | | Mar | 54 | 708.4 | 713.3 | 718.3 |
| | Apr | 19 | 1076 | 1081 | 1086 | | Apr | 55 | 1103 | 1108 | 1113 |
| | May | 20 | 990.7 | 995.7 | 1001 | | May | 56 | 1015 | 1020 | 1025 |
| | Jun | 21 | 593.3 | 598.3 | 603.3 | | Jun | 57 | 608.1 | 613 | 618 |
| | Jul | 22 | 300.7 | 305.7 | 310.7 | | Jul | 58 | 308.2 | 313.2 | 318.2 |
| | Aug | 23 | 171.7 | 176.7 | 181.7 | | Aug | 59 | 176.1 | 181.1 | 186 |
| | Sep | 24 | 134.7 | 139.7 | 144.7 | | Sep | 60 | 138.1 | 143.1 | 148.1 |
| 2005 | Oct | 25 | 134.5 | 139.4 | 144.4 | 2008 | Oct | 61 | 137.9 | 142.9 | 147.8 |
| | Nov | 26 | 181 | 185.9 | 190.9 | | Nov | 62 | 185.5 | 190.5 | 195.5 |
| | Dec | 27 | 263.2 | 268.2 | 273.2 | | Dec | 63 | 269.8 | 274.8 | 279.8 |
| | Jan | 28 | 290.9 | 295.9 | 300.9 | | Jan | 64 | 298.2 | 303.2 | 308.2 |
| | Feb | 29 | 440 | 445 | 450 | | Feb | 65 | 451 | 455.9 | 460.9 |
| | Mar | 30 | 696.9 | 701.9 | 706.8 | | Mar | 66 | 714.2 | 719.1 | 724.1 |
| | Apr | 31 | 1085 | 1090 | 1095 | | Apr | 67 | 1112 | 1117 | 1122 |
| | May | 32 | 998.8 | 1004 | 1009 | | May | 68 | 1024 | 1028 | 1033 |
| | Jun | 33 | 598.2 | 603.2 | 608.1 | | Jun | 69 | 613 | 618 | 623 |
| | Jul | 34 | 303.2 | 308.2 | 313.1 | | Jul | 70 | 310.8 | 315.8 | 320.7 |
| | Aug | 35 | 173.2 | 178.1 | 183.1 | | Aug | 71 | 177.6 | 182.5 | 187.5 |
| | Sep | 36 | 135.8 | 140.8 | 145.8 | | Sep | 72 | 139.3 | 144.3 | 149.3 |

**Table 5.7: Cont.**

| Year | Month | t | $F_t$ (-) | $F_t$ † | $F_t$ (+) | Year | Month | t | $F_t$ (-) | $F_t$ † | $F_t$ (+) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2009 | Oct | 73 | 139.1 | 144 | 149 | 2011 | Oct | 97 | 141.4 | 146.4 | 151.4 |
| | Nov | 74 | 187.1 | 192.1 | 197 | | Nov | 98 | 190.2 | 195.2 | 200.2 |
| | Dec | 75 | 272.1 | 277 | 282 | | Dec | 99 | 276.6 | 281.5 | 286.5 |
| | Jan | 76 | 300.7 | 305.7 | 310.6 | | Jan | 100 | 305.7 | 310.6 | 315.6 |
| | Feb | 77 | 454.7 | 459.6 | 464.6 | | Feb | 101 | 462.2 | 467.1 | 472.1 |
| | Mar | 78 | 720 | 725 | 730 | | Mar | 102 | 731.8 | 736.8 | 741.8 |
| | Apr | 79 | 1121 | 1126 | 1131 | | Apr | 103 | 1139 | 1144 | 1149 |
| | May | 80 | 1032 | 1037 | 1042 | | May | 104 | 1049 | 1054 | 1059 |
| | Jun | 81 | 618.1 | 623 | 628 | | Jun | 105 | 628.2 | 633.2 | 638.2 |
| | Jul | 82 | 313.4 | 318.3 | 323.3 | | Jul | 106 | 318.6 | 323.5 | 328.5 |
| | Aug | 83 | 179 | 184 | 189 | | Aug | 107 | 182 | 187 | 192 |
| | Sep | 84 | 140.5 | 145.5 | 150.4 | | Sep | 108 | 142.9 | 147.8 | 152.8 |
| 2010 | Oct | 85 | 140.2 | 145.2 | 150.2 | 2012 | Oct | 109 | 142.6 | 147.6 | 152.5 |
| | Nov | 86 | 188.7 | 193.6 | 198.6 | | Nov | 110 | 191.8 | 196.8 | 201.8 |
| | Dec | 87 | 274.3 | 279.3 | 284.2 | | Dec | 111 | 278.9 | 283.8 | 288.8 |
| | Jan | 88 | 303.2 | 308.1 | 313.1 | | Jan | 112 | 308.2 | 313.2 | 318.1 |
| | Feb | 89 | 458.4 | 463.4 | 468.3 | | Feb | 113 | 466 | 470.9 | 475.9 |
| | Mar | 90 | 725.9 | 730.9 | 735.8 | | Mar | 114 | 737.8 | 742.8 | 747.8 |
| | Apr | 91 | 1130 | 1135 | 1140 | | Apr | 115 | 1149 | 1154 | 1158 |
| | May | 92 | 1040 | 1045 | 1050 | | May | 116 | 1057 | 1062 | 1067 |
| | Jun | 93 | 623.1 | 628.1 | 633.1 | | Jun | 117 | 633.4 | 638.4 | 643.3 |
| | Jul | 94 | 315.9 | 320.9 | 325.9 | | Jul | 118 | 321.2 | 326.2 | 331.1 |
| | Aug | 95 | 180.5 | 185.5 | 190.5 | | Aug | 119 | 183.6 | 188.5 | 193.5 |
| | Sep | 96 | 141.7 | 146.6 | 151.6 | | Sep | 120 | 144.1 | 149 | 154 |

**† Future values ($m^3/sec$)**

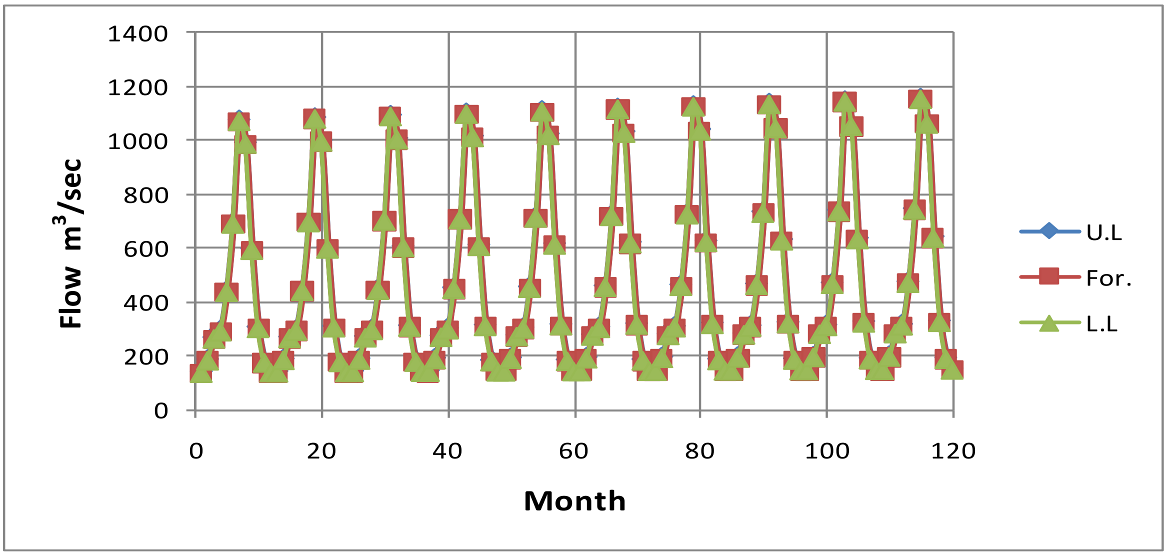



**Figure 5.8: Future monthly flow data for Greater Zab river with upper and the lower probability limits obtained by SARIMA (0, 1, 1) × (0, 1, 1)$_{12}$ model from 2003-2012.**

# Chapter Six

# CONCLUSIONS AND RECOMMENDATIONS

## 6.1 Conclusions

The following conclusions are drawn from this study:

1. All months show periodic patterns, which are attributed to the influence of the annual cyclic pattern of the hydrological inputs to the river.

2. The periodic autoregressive PAR (1) model of first order fits all months. There is also an indication that the correlation between the consecutive observations is periodic. The values of the parameter of the model for monthly flow are found equal to 0.775, 0.516, 0.452, 0.455, 0.711, 0.646, 0.66, 0.784, 0.849, 0.908, 0.941, and 0.933 for the 12 months from October to September.

3. The Cox- Box transformation is most suitable for normalizing data. It uses periodic transformation for each season which is more accurate than non-periodic transformation for normalizing monthly data.

4. The SARIMA model is also applied to the same data of monthly flow. It is found that the seasonal ARIMA (0,1,1) $(0,1,1)_{12}$ model with positive parameters fits these data very well. The values of the parameters of the model $\theta$ and $\Theta$ are found equal to 0.361 and 0.95 respectively.

5. Monthly mean of the data generated by PARMA model and SARIMA model are compared with observed data from 1993-2002. It is found that the generated data by SARIMA model is better than the data generated by

PARMA model because it gives monthly mean slightly different from those of the observed data.

6. The minimum forecasted error using MARE%, MAE and MSE of forecast series of the two methods are compared and it is found that seasonal ARIMA (0,1,1) ( 0,1,1)$_{12}$ model provides more satisfactory forecast for monthly flow Greater Zab river than PARMA model .

## 6.2 Recommendations

The following suggestions are put forward for further research:

1. Multisite and multivariate using SARIMA models are recommended as future studies to account for the correlations between the stations and the parameters respectively.

2. The method of moments and maximum likelihood estimation are suggested to be used instead of least squares estimation method for estimation of parameters of PARMA and SARIMA models and the results should be compared with least squares results in future studies.

3. Fourier analysis for estimating periodic parameters of PARMA model should be used for small time interval series, such as daily and weekly series. These applications result in increasing number of seasons which in turn leads to increase in the number of parameters to be estimated.

4. The PARMA model should be used for other applications such as reservoir operation, flood analysis and river flow forecasting in future work.

# REFERENCES


1. Abdul-Razaq, T. T., 1984. ***"Multivariate Stochastic Generation of Daily Rainfall Data"***, M. Sc. Thesis, College of Engineering, University of Baghdad, Iraq.
2. Abed, Z. A., (2007). "***Stochastic Models of Some Properties of Water in The Middle of Euphrates Region in Iraq***", M. Sc. Thesis, College of Engineering, University of Babylon, Iraq.
3. Ahmad, S., Khan, I. H., and Parida, B. P., 2001. ***"Performance of Stochastic Approaches for Forecasting River Water Quality"***, Water Resources Research, Vol. 35, No. 18, PP. 4261-4266.
4. Akgun, B., 2003. "***Identification of Periodic Autoregressive Moving Average Models",*** M. Sc. Thesis, the Graduate School of Natural and Applied Sciences, University of Middle East Technical.
5. AL-Husseini , A. H., 2000." ***Hydrochemical Forecasting Model of Shutt AL-Hilla*** ", M. Sc. Thesis, College of Engineering, University of Babylon.
6. AL-Mousawi ,E. M., 2003." ***Multisite Stochastic Model of Water Quality Properties at Selected Regions*** ", M. Sc. Thesis, College of Engineering, University of Babylon.
7. Al-Ta'ee, M. H. Ali, 2009.***"Analysis of Records of Rainfall and Evaporation in Babylon***", M.Sc. thesis, College of Engineering, University of Babylon, Iraq.
8. AL-Tikriti ,H. A., 2001. "***Forecasting of Pollution Levels in Accordance with Discharge Reduction in selected Area on Euphrates River"***, M. Sc. Thesis, College of Engineering, University of Baghdad.

9. Anderson, O.D., DeGooijer and K.D.C. Stoodley, 1982.***"Time series Analysis: Theory and Practice 1",*** proceedings of the international conference held at Valencia, Spain, June 1981, north-holand publishing company**.**

10. Anderson, P.L., Meerschaert, M.M., and Vecchia, A.V., 1999. ***"Innovations Algorithm for Periodically Stationary Time Series",*** Stochastic Processes and their Applications, Vol. 83, pp. 149- 169.

11. Basawa, I.V., and Lund, R., 1999. ***"Large Sample Properties of Parameter Estimates for Periodic ARMA Models"***, Journal of Time Series Analysis, Vol. 22, No. 6, pp. 651-663.

12. Batchelor, R., 2004. "***Box-Jenkins Analysis***", ESCP-EAP, Paris, 39 Pages.

13. Bell, W. R., 1993. "Empirical Comparisons of Seasonal ARIMA and ARIMA Component (Structural) Time Series Models", Research Report, No. RR-93/10, Statistical Research Division, Bureau of the Census, Washington, D. C.

14. Box, G.E., and Jenkins, G.M. 1976. "***Time Series Analysis: Forecasting and Control***", 2nd edition, San Francisco: Holden-Day.

15. Brockwell, P. J. and Davis, R. A., 2002. "***Introduction to Time Series and Forecasting"***, Second Edition, Springer-Verlag New York, Inc.

16. Caldwell, J. G., 2006. "***The Box-Jenkins Forecasting Technique***". Article, Posted at internet websites http://www.foundationwebsite.org

17. Chandra, S., Spaila, S.K. and Kumar, A., 1978***." Flood Frequency Analysis by Power Transformation "***, Journal of Hydraulic Division, ASCE, Vol. 104, No. HY11, Pages 1945-1505.Chapman and Hill, 4nd Edition, London.

18. Chatfield, C., 1982. "***The Analysis of Time Series: An Introduction*** ", Chapman and Hill, 2nd Edition.

19. Chatfield, C., 2000. ***"time series forecasting***". Chapman & hall.

20. Chen, H.-L., And, Rao ,A.R., 2002. "***Testing hydrologic time series for stationary***". Journal of Hydrologic Engineering.

21. Cortez, P., Rocha, M., and Neves, J., 2004. "***Evolving Time Series Forecasting ARMA Models***", revmetaarma, 12, 46, PP 1-25.

22. Cryer, J. D. and Chan K.S., 2008. ***"Time Series Analysis with Applications in R",*** Springer Science Business Media, LLC, Second Edition.

23. Gautier, A., 200***7."Estimation of Periodic ARMA Models and Application to Motorway Traffic***", International Conference on Applied Statistics for Development in Africa

24. Haghaparast, A., 2007. ***"ARIMA Model of Time Series***", Helsinki University of Technology, Laboratory of Acoustics Audio and Signal Processing, Lecture.

25. Hipel, K. W., and McLeod, A. I. 1994. "***Time series modeling of water resources and environmental systems"***. Elsevier, Amsterdam, The Netherlands.

26. Hipel, K. W., McLeod, A.I. and Lennox, W.C., 1977. "Advance in Box-Jenkins Modeling: 1-Model Construction ", Journal of Water Resource Research, Vol.13, No.3.

27. Hussein, wael Abdul-Bari, 2006. ***"Reliability of Iraqi Reservoirs***". M.Sc. thesis, College of Engineering, University of Babylon, Iraq.

28. Jayawardena, A. W. and Lai, F., 1989. ***"Time Series Analysis of Water Quality Data in Pearl River, China"***, Journal of Environmental Engineering, ASCE, Vol. 115, No.3, PP. 590-607.

29. Kihoro, J. M., Otieno, R. O., and Wafula, C., 2004. ***"Seasonal Time Series Forecasting: A Comparative Study of ARIMA and ANN Models",*** African

Journal of Science and Technology (AJST), Science and Engineering Series, Vol. 5, No. 2, PP. 41-49.

30. Kurunc, A., Yureki, k . and Cevik, O., 2005. " ***Performance of Two Stochastic Approaches for Forecasting Water Quality and Streamflow Data From Yesilirmak River, Turkey*** ", Journal of Environmental Modeling and Software, 20, Pages 1195-1200 .
31. Lehmann, A. and Rode, M., 2001. ***"Long-Term Behaviour and Cross-Correlation Water Quality Analysis of the River Elbe, Germany"***, Water Resources Research, Vol. 35, No. 9, PP. 2153-2160.
32. Li, W. Keung, 2004."***Diagnostic Checks in Time Series***", Chapman & Hall/CRC
33. Lohani, B. N. and Wang, M. M., 1987. ***"Water Quality Data Analysis in Chang Kang River",*** Journal of Environmental Engineering, ASCE, Vol. 113, No.1, PP. 186-195.
34. Lund, R. B. and Basawa, I. V., 1999. ***"Modeling and inference for periodically correlated time series. Asymptotics, Nonparameterics, and Time series"*** (ed. S. Ghosh), 37–62. Marcel Dekker, New York.
35. Mahmood, H. S., 2000. ***"Time Series Analysis of Hydrochemical Pollutant: Application of Box-Jenkins Models on Euphrates River at Kufa "***. M. Sc. Thesis, College of Engineering, University of Babylon.
36. Mahpol, K. S. B., 2005. "***Box-Jenkins and Genetic Algorithm Hybrid Model for Electricity Forecasting System"***, M. Sc. Thesis, Faculty of Science (Mathematics), University Technology Malaysia, Malaysia.
37. Matalas, N.C., 1967. "***Time Series Analysis*** ". Journal of Water Resource Research, Vol. 3, No.3, Pages 817-829.

38. McLeod, A. I., Noakes, D. J., Hipel, K. W., and Thompston, R. M., 1987. ***"Combining Hydrologic Forecasts"***, Journal of Water Resources Planning and Management, ASCE, Vol. 113, No.1, PP. 29-39.

39. Nochai, R. and Nochai, T., 2006. ***"ARIMA Model for Forecasting Oil Palm Price"***, The 2nd IMT-Gt Regional Conference on Mathematics, Statistics, and Applications, University Sains Malysia, Penange, PP. 1-7.

40. Ooms, M. and Franses, P. H., 1998. ***"A Seasonal Periodic Long Memory Model for Monthly River Flows"***, Econometric Institute Report EI9842/A, PP., Erasms University Rotterdam, Netherlands, http://www.eur.nl\few\ei\papers.

41. Palit, A. K. and Popovic, D., 2005. "***Computational Intelligence in Time Series Forecasting Theory and Engineering Applications"***, Springer-Verlag London Limited.

42. Rosovsky A., Copaken M., Soffer N., 1999."***Rivers of Fire: The Conflict over Water in the Middle East",*** rowman & Littlefield publishers, Inc.

43. Sabry, M., Abd-EL- Latif, H., Yousef, S., and Badra, N., 2007. "***Use of Box and Jenkins Time Series Technique in Traffic Volume Forecasting***", Ain Shams University, Cairo, Egypt, Research Journal of Social Sciences, 2, PP. 83-90.

44. Salas, J.D, Tabios, G.Q. and Bartolini, P., 1985. ***"Approaches to multivariate modeling of water resources time series***". Water Resources Bulletin, 21, 683–708.

45. Salas, J.D, Delleur, J.W., Yevjevich, V. and Lane, W.L., 1980. "***Applied Modeling of Hydrologic Time Series***", Water Resource Publications, Littleton, Colorado.

46. Salas, J.D., 1993. "***Analysis and modeling of hydrologic time series***", The Mc Graw-Hill Handbook of Hydrology, D.R. Maidment, ed., 19.1–19.71

47. Salas, J.D., Chung, C. and Cancelliere, A., 2005. ***"Correlations and Crossing Rates of Periodic-Stochastic Hydrologic Processes"***, Journal of Hydrologic Engineering, Vol. 10, No. 4, ©ASCE.
48. Shaker, R. H., 1985. ***"Stochastic Analysis of Daily Streamflow of Tigris River", M. Sc. Thesis, College of Engineering, University of Baghdad, Iraq.***
49. Shao, Q. and Lund, R., 2004. "***Computation and Characterization of Autocorrelation and partial autocorrelation in Periodic ARMA model***", Journal of time series analysis Vol.25, No.3, 359 – 372 , Blackwell Publishing Ltd
50. Smadi, A.A., 1994. ***"Analysis and Development of Statistical Properties of Periodic Autoregressive Moving Average Processes",*** Ph.D. Dissertation, Dept. of Statistics, Middle East Technical University, Ankara, Turkey, 124 pp.
51. Srikanthan, R. and McMahon, T.A. 1982. ***"Stochastic generation of monthly streamflows"***, J. of Hyd. Div., ASCE, Vol.108, No. hy3, Pages 419-441
52. Sveinsson ,O. G. B., Salas, J. D., Lane, W. L. and Frevert, D. K., 2007. "***Stochastic Analysis, Modeling, and Simulation (SAMS)***", Version 2007, USER's MANUAL, Colorado State University.
53. Tesfaye, Y.G., 2005. "***Seasonal Time Series Models and Their Application to the Modeling of River Flows***". Ph.D. thesis, University of Nevada, Reno.
54. Trajkovic, S., 1999. ***"Comparison of Prediction Models of Reference Crop Evaportranspiration***", University of Nis, Yugoslavia, The Scientific Journal Facta Universitatis, Series: Arichitecture and Civil Engineering Vol. 1, No. 5, PP. 617-625.

55. Ula, T.A., and Smadi, A.A., 1997. ***"Periodic Stationarity Conditions for Periodic Autoregressive Moving Average Processes as Eigenvalue Problems"***, Water Resources Research, Vol. 33, No. 8, pp. 1929-1934.

56. Ula, T.A., and Smadi, A.A., 2003. ***"Identification of Periodic Moving Average Models"***, Commune. Stat.-Theory and Meth. ,Vol. 32, No. 12.

57. Valenca, M, Valença, A. and Ludermir, T., 2005**.** ***"River Flow Forecasting for Reservoir management through Neural Networks"***, Proceedings of the Fifth International Conference on Hybrid Intelligent Systems (HIS'05) 0-7695-2457-5/05 , IEEE computer society

58. Yaffee, R.A., McGee, M., 1999. ***"Introduction to Time Series Analysis and Forecasting with Applications of SAS and SPSS",*** **A**cademic Press, INC**.**

59. Yurekli, K., Kurung, A., and Ozturk, F., 2005. ***"Testing The Residuals of An ARIMA Model on The Cekerek Stream Watershed in Turkey"***, Turkish Journal of Engineering Environmental Science, 29, PP. 61-74.

# A.1 DEFINITION OF STATISTICAL CHARACTERISTICS

A time series process can be characterized by a number of statistical properties such as the mean, standard deviation, coefficient of variation, skewness coefficient, season-to-season correlations, and autocorrelations. These statistics are defined for both annual and seasonal data as shown below.

## A.1.1 Annual Data

The mean and the standard deviation of a time series $Y_t$ are estimated by

$$\overline{Y}=\frac{1}{N}\sum_{t=1}^{N} Y_t \quad ,\ldots\ldots,,,,(A\text{-}1)$$

And

$$Sd=\sqrt{\frac{1}{N}\sum_{t=1}^{N}(Y_t-\overline{Y})^2} \quad ,\ldots\ldots.(A\text{-}2)$$

Respectively, where N is the sample size. The coefficient of variation is defined as

$$cv= Sd / \overline{Y} \quad ,\ldots\ldots(A\text{-}3)$$

Likewise, the skewness coefficient is estimated by

$$Cs=\frac{\frac{1}{N}\sum_{t=1}^{N}(Y_t-\overline{Y})^3}{(Sd)^3} \quad ,\ldots\ldots..(A\text{-}4)$$

The sample autocorrelation coefficients $r_k$ of a time series may be estimated by

$$r_k=\frac{\sigma_k}{\sigma_0} \quad ,\ldots\ldots...(A\text{-}5)$$

Where k = time lag. and

$$\sigma_k=\sqrt{\frac{1}{N}\sum_{t=1}^{N-k}(Y_t-\overline{Y})(Y_{t+k}-\overline{Y})} \quad ,\ldots\ldots....(A\text{-}6)$$

## A.1.2 Seasonal data

Seasonal hydrologic time series, such as monthly flows, are better characterized by seasonal statistics. Let $Y_{r,m}$ be a seasonal time series, where r= 1,...,N represents years with N being the number of years, and m = 1,...,s seasons with s being the number of seasons. The mean and standard deviation for season m can be estimated by

$$\overline{Y}_m = \frac{1}{N}\sum_{r=1}^{N} Y_{r,m} \quad , \ldots\ldots (A\text{-}7)$$

And

$$(Sd)_m = \sqrt{\frac{1}{N}\sum_{r=1}^{N}(Y_{r,m} - \overline{Y}_m)^2} \quad , \ldots\ldots (A\text{-}8)$$

Respectively. The seasonal coefficient of variation is $(cv)_m = (Sd)_m \,/\, \overline{Y}_m$ . Similarly, the seasonal skewness coefficient is estimated by

$$(Cs)_m = \frac{\frac{1}{N}\sum_{r=1}^{N}(Y_{r,m} - \overline{Y}_m)^3}{(Sd_m)^3} \quad , \ldots\ldots (A\text{-}9)$$

The sample lag-k season-to-season correlation coefficient may be estimated by

$$\left(r_{k,m}\right) = \frac{\sigma_{k,m}}{\sqrt{\sigma_{0,m}\sigma_{m-k}}} \quad , \ldots\ldots (A\text{-}10)$$

Where

$$\sigma_{k,m} = \frac{1}{N}\sum_{r=1}^{N}(Y_{r,m} - \overline{Y}_m)\,(Y_{r,m-k} - \overline{Y}_{m-k}) \quad , \ldots\ldots (A\text{-}11)$$

When $m - k < 1$ the terms, r =1, $Y_{r,m-k}$, $\overline{Y}_{m-k}$, $\sigma_{m-k}$ replaced by
r =2, $Y_{r-1,m-k+s}$, $\overline{Y}_{m-k+s}$, $\sigma_{m-k+s}$

## A.2 Periodic Autocorrelation Function (PeACF)

For the univariate periodic stationary PARMA process $Y_{r,m}$ defined by (3-32), in which the white noise terms $a_{r,m}$ are assumed to be independent, the periodic autocovariance function (PeACVF) is defined as

$$\sigma_{k,m} = \text{Cov}\ (Y_{r,m}\ ,\ Y_{r,m-k})$$

$$=\text{E}\ [(Y_{r,m} - \overline{Y}_m)\ (Y_{r,m-k} - \overline{Y}_{m-k})]\ \ ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(A-12)}$$

for season m at backward lag k ≥ 0

Then, the PeACF for season m at backward lag k ≥ 0 is defined as (Akgun, 2003)

$$(r_{k,m}) = \frac{\sigma_{k,m}}{\sqrt{\sigma_{0,m}\sigma_{0,m-k}}}\ \ ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(A-13)}$$

Where $\sigma_{0,m}$ is the variance for the m season and $Y_{r,m}$ denotes the periodically standardized time series. For a season m which follows a pure PMA process, successive values for q(m) is checked initially with q(m) = 1 according to the appropriate band limits. Therefore, the final lag of which the value of sample PeACF falls outside the band limits indicates the order of the MA process of the corresponding season. so that the approximate 95% band (−1.96 / N ,1.96 / N ) is applied to the sample PeACF where -1.96 and 1.96 are the upper α/2-point and lower α/2- point of standard normal distribution when significance level α is equal to 0.05.

## A.3 Periodic Partial Autocorrelation Function (PePACF)

In time series analysis, the partial autocorrelation function is well adapted to the identification of pure AR processes. The partial autocorrelation function, like the autocorrelation function, carries vital information about the dependence structure of a stationary series.

It was mentioned that for a pure PMA process, the PeACF is zero for lags beyond q(m) and the order of the process can be decided according to the sample PeACF. Likewise, for pure PAR processes, the PePACF becomes zero for lags beyond p(m), so if the correct order is p(m) for season m, then $\phi_{kk}(m) = 0$ for all k > p(m). This is the cut-off property of periodic partial autocorrelation function for PAR processes.

For the computation of the PePACF, $\phi_{kk}(m)$, Sakai's (1982) algorithm for calculating $\phi_{kk}(m)$ iteratively is used. This approach is as follows: (Akgun, 2003)

**(1) Initial Conditions**

$\delta_m^2(0) = \tau_m^2(0) = \sigma_0(m)$ ,………………………………………………(A-14)

$\alpha_m(p,0) = 1$ where p=1,2,…… m=1,2,……,s

**(2) Order update from** p to p+1 (p=0,1,2,…)

**i. Compute;**

$\Delta_m(p) = \sum_{j=0}^{p} \sigma_{p+1-j}(m-j)\, \alpha_m(p,j)$ ,…………………………………..(A-15)

$\alpha_m(p+1,p+1) = \frac{-\Delta_m(p)}{\tau_{m-1}^2(p)}$ ,…………………………………………………..(A-16)

$\beta_m(p+1,p+1) = \frac{-\Delta_m(p)}{\delta_m^2(p)}$ ,………………………………………………..(A-17)

**ii. Update;**

$\delta_m^2(p+1) = \delta_m^2(p)[1 - \alpha_m(p+1,p+1)\, \beta_m(p+1,p+1)]$ ,…………………(A-18)

And for i=1,2,….p

$\alpha_m(p+1,i) = \alpha_m(p,i) + \alpha_m(p+1,p+1)\, \beta_{m-1}(p, p+1-i)$ ,………………...(A-19)

$\beta_m(p+1,i) = \beta_{m-1}(p,i) + \beta_m(p+1,p+1)\,\alpha_m(p\,,p+1-i)$ ,…………………(A-20)

**iii. Calculate;**

$\phi_{kk}(m) = \frac{\Delta_m(p)}{\delta_m(p)\,\tau_{m-1}(p)}$ ,………………………………………………(A-21)

Where $\sigma_k(m)$ is the periodic autocovariance function m is season and lag k and the subscript m-k = 0 is always replaced by s=12.

Sakai (1982) also derived the distribution of $\phi_{kk}(m)$: If a season m follows a PAR (p(m)) process, then for all m and k > p(m), $\phi_{kk}(m)$ are asymptotically independent, and normally distributed with zero mean and variance 1/N. Therefore, for large N, for order identification, the approximate 95% band (−1.96 / N, 1.96 / N) should be applied for $\phi_{kk}(m)$ for all k > p(m), again -1.96 and 1.96 representing the upper α/2-point and lower α/2-point of standard normal distribution when significance level α is equal to 0.05. Hence, the final lag of which the value of sample PePACF falls outside the band limits indicates the order of the AR process of the corresponding season. (Akgun, 2003)

## A.4 Derivative of Multiplicative SARIMA Models

These models which applied in the current study is multiplicative of an ARIMA (p, d, q) ×ARIMA (P, D, Q)s and derived as the flowing:

Where (p,P,q,Q) are order of autoregressive and moving average for nonseasonal and seasonal model respectively ,d and D is degree of differencing for non seasonal and seasonal model respectively ,S is length of periodic cycle S=12 for monthly data .

**1- SARIMA (0,1,1)x(0,1,1)$_{12}$**

The form nonseasonal model is

$W_t = (1-\theta\beta)a_t$ , …....………………………………………………..(A-22)

And form seasonal model is

$W_t = (1-\Theta\beta^{12})a_t$ ,……………………………………………………(A-23)

Multiplicative equation (A-22) x (A-23) as the flowing

$W_t = (1-\theta\beta)(1-\Theta\beta^{12})\,a_t$ ,…………………………………….....(A-24)

$W_t = a_t - \Theta\beta^{12} a_t - \theta\beta a_t + \theta\Theta\beta^{13} a_t$ ,………………………………....(A-15)

Where

$\beta^j a_t = a_{t-j}$ (Chatfield ,1982) ,..……………………………………...(A-25)

Then substitute the (A-25) in (A-24) and result

$[W_t] = [a_t] - \theta[a_{t-1}] - \Theta[a_{t-12}] + \theta\Theta[a_{t-13}]$ ,………………………..(A-26)

Then the model may be written in either forward form

$[a_t] = [W_t] + \theta[a_{t-1}] + \Theta[a_{t-12}] - \theta\Theta[a_{t-13}]$ ,……………………….(A-27)

or backward form

$[e_t] = [W_t] + \theta[e_{t+1}] + \Theta[e_{t+12}] - \theta\Theta[e_{t+13}]$ ,……………………….(A-28)

where : $Z = \ln X_t$ ,……………………………………………………...(A-29)

$W_t = \nabla\nabla_{12} Z_t = Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13}$ ,……………(A-30)

**2- <u>SARIMA (0, 1, 2) x (0, 1, 1)$_{12}$</u>**

The form nonseasonal model is

$W_t = (1-\theta_1\beta - \theta_2\beta^2)a_t$ ,………………………………………………..(A-31)

And form seasonal model is

$W_t = (1-\Theta\beta^{12})a_t$ ,……………………………………………………...(A-32)

Multiplicative equation (A-31) x (A-32) as the flowing

$W_t = (1-\theta_1\beta - \theta_2\beta^2)(1-\Theta\beta^{12})\,a_t$ ,……………………………………(A-33)

$W_t = a_t - \Theta\beta^{12} a_t - \theta_1\beta a_t + \theta_1\Theta\beta^{13} a_t - \theta_2\beta^2 a_t + \theta_2\Theta\beta^{14} a_t$ ,………...(A-34)

Where

$\beta^j a_t = a_{t-j}$ (Chatfield ,1982) ,..……………………………………..(A-35)

Then substitute the (A-35) in (A-34) and result

$[W_t]= [a_t]- \theta[a_{t-1}] - \Theta[a_{t-12}] +\theta\Theta[a_{t-13}]$ ,………………………..(A-36)

Then the model may be written in either forward form

$[a_t]= [W_t]+ \theta[a_{t-1}] + \Theta[a_{t-12}] -\theta\Theta[a_{t-13}]$ ,……………………….(A-37)

or backward form

$[e_t]= [W_t]+ \theta[e_{t+1}] + \Theta[e_{t+12}] -\theta\Theta[e_{t+13}]$ ,……………………….(A-38)

where : $Z= \ln X_t$ ,…………………………………………………(A-39)

$W_t= \nabla \nabla_{12} Z_t= Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13}$ ,…………….(A-40)

**3- <u>SARIMA (1, 1, 0) x (1, 1, 0)$_{12}$</u>**

The form nonseasonal model is

$a_t= (1-\phi\beta)W_t$, …..……………………………………………..(A-41)

And form seasonal model is

$a_t= (1- \Phi\beta^{12})W_t$ ,……………………………………………….(A-42)

Multiplicative equation (A-41) x (A-42) as the flowing

$a_t= (1-\phi\beta) (1- \Phi\beta^{12})W_t$ ,…………………………………....(A-43)

$a_t=W_t - \phi\beta W_t - \Phi\beta^{12}W_t + \phi\Phi\beta^{12}W_t$ ,…………………………........(A-44)

Where

$\beta^j w_t= w_{t-j}$ (Chatfield ,1982) ,..…………………………………...(A-45)

Then substitute the (A-45) in (A-44) and the model may be written in either forward form

$[a_t]= [W_t] - \phi[W_{t-1}] - \Phi[W_{t-12}] + \phi\Phi[W_{t-13}]$ ,…………………(A-46)

or backward form

$[e_t]= [W_t] - \phi[W_{t+1}] - \Phi[W_{t+12}] + \phi\Phi[W_{t+13}]$ ,…………………..(A-47)

where : $Z= \ln X_t$ ,………………………………………………(A-48)

$W_t= \nabla \nabla_{12} Z_t= Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13}$ ,…………….(A-49)

**4- <u>SARIMA (2, 1, 0) x (1, 1, 0)$_{12}$</u>**

The form nonseasonal model is

$a_t = (1-\phi_1\beta - \phi_2\beta^2)W_t$ ,.........................................................(A-50)

And form seasonal model is

$a_t = (1-\Phi\beta^{12})W_t$ ,.........................................................(A-51)

Multiplicative equation (A-50) x (A-51) as the flowing

$a_t = (1-\phi_1\beta - \phi_2\beta^2)(1-\Phi\beta^{12})W_t$ ,.........................................................(A-52)

$a_t = W_t - \phi_1\beta W_t - \phi_2\beta^2 W_t - \Phi\beta^{12}W_t + \phi_1\Phi\beta^{13}W_t + \phi_2\Phi\beta^{14}W_t$ ,........(A-53)

Where

$\beta^j w_t = w_{t-j}$ (Chatfield ,1982) ,.........................................................(A-54)

Then substitute the (A-54) in (A-53) and the model may be written in either forward form

$[a_t] = [W_t] - \phi_1[W_{t-1}] - \phi_2[W_{t-2}] - \Phi[W_{t-12}] + \phi_1\Phi[W_{t-13}] + \phi_2\Phi[W_{t-14}]$

,......................................................................... (A-55)

or backward form

$[a_t] = [W_t] - \phi_1[W_{t+1}] - \phi_2[W_{t+2}] - \Phi[W_{t+12}] + \phi_1\Phi[W_{t+13}] + \phi_2\Phi[W_{t+14}]$

,......................................................................... (A-56)

Where: $Z = \ln X_t$ ,.........................................................(A-57)

$W_t = \nabla\nabla_{12} Z_t = Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13}$ ,................(A-58)

# TableB.1: Mean monthly discharge for Greater Zab river (1933-2002)

| mean monthly discharge (m3/s) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| year | Oct | Nov | Dec | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Average |
| 1933 | 74 | 136 | 93 | 146 | 238 | 555 | 751 | 1000 | 545 | 291 | 152 | 97 | 340 |
| 1934 | 81 | 73 | 273 | 150 | 277 | 371 | 767 | 682 | 477 | 195 | 108 | 84 | 295 |
| 1935 | 83 | 85 | 94 | 186 | 414 | 445 | 771 | 748 | 417 | 175 | 103 | 84 | 300 |
| 1936 | 83 | 145 | 147 | 101 | 345 | 348 | 1030 | 993 | 505 | 251 | 122 | 93 | 347 |
| 1937 | 85 | 352 | 214 | 195 | 636 | 631 | 1310 | 856 | 692 | 215 | 115 | 90 | 449 |
| 1938 | 133 | 201 | 167 | 301 | 411 | 397 | 1060 | 911 | 550 | 286 | 155 | 117 | 391 |
| 1939 | 105 | 166 | 214 | 236 | 345 | 571 | 1090 | 1130 | 637 | 300 | 145 | 113 | 421 |
| 1940 | 100 | 137 | 215 | 568 | 661 | 592 | 1420 | 955 | 613 | 324 | 172 | 123 | 490 |
| 1941 | 144 | 118 | 220 | 382 | 1010 | 864 | 1020 | 1130 | 511 | 263 | 152 | 120 | 495 |
| 1942 | 115 | 105 | 131 | 415 | 394 | 865 | 1180 | 1310 | 611 | 273 | 157 | 122 | 473 |
| 1943 | 130 | 394 | 251 | 245 | 276 | 395 | 790 | 1060 | 650 | 355 | 183 | 127 | 405 |
| 1944 | 125 | 117 | 132 | 285 | 322 | 724 | 1220 | 1090 | 535 | 324 | 192 | 142 | 434 |
| 1945 | 131 | 301 | 166 | 455 | 403 | 391 | 730 | 754 | 522 | 288 | 155 | 114 | 368 |
| 1946 | 110 | 134 | 215 | 232 | 533 | 1020 | 1440 | 1770 | 1040 | 551 | 296 | 155 | 625 |
| 1947 | 202 | 161 | 155 | 411 | 472 | 776 | 790 | 620 | 355 | 184 | 117 | 96 | 362 |
| 1948 | 104 | 205 | 205 | 180 | 264 | 415 | 1010 | 1300 | 911 | 401 | 194 | 136 | 444 |
| 1949 | 144 | 161 | 224 | 201 | 308 | 733 | 1220 | 1550 | 936 | 385 | 194 | 140 | 516 |
| 1950 | 144 | 153 | 168 | 322 | 307 | 910 | 1060 | 1440 | 730 | 370 | 195 | 142 | 495 |
| 1951 | 150 | 147 | 151 | 241 | 297 | 405 | 556 | 564 | 382 | 216 | 135 | 117 | 280 |
| 1952 | 195 | 195 | 237 | 217 | 707 | 813 | 1200 | 1250 | 635 | 330 | 171 | 127 | 506 |
| 1953 | 128 | 127 | 145 | 246 | 443 | 755 | 1240 | 1140 | 675 | 361 | 207 | 155 | 469 |
| 1954 | 141 | 212 | 167 | 315 | 538 | 1240 | 1670 | 1420 | 908 | 480 | 267 | 210 | 631 |
| 1955 | 187 | 188 | 196 | 187 | 225 | 303 | 592 | 621 | 362 | 193 | 130 | 107 | 274 |
| 1956 | 102 | 145 | 325 | 251 | 503 | 681 | 1180 | 894 | 726 | 390 | 198 | 144 | 462 |
| 1957 | 110 | 117 | 171 | 177 | 335 | 818 | 776 | 1110 | 720 | 366 | 203 | 148 | 421 |
| 1958 | 142 | 175 | 192 | 237 | 294 | 453 | 646 | 596 | 355 | 190 | 132 | 105 | 293 |
| 1959 | 71 | 66 | 153 | 100 | 98 | 293 | 1010 | 806 | 462 | 226 | 136 | 113 | 295 |
| 1960 | 106 | 112 | 115 | 214 | 343 | 451 | 760 | 847 | 386 | 164 | 115 | 95 | 309 |
| 1961 | 84 | 191 | 145 | 175 | 277 | 347 | 834 | 876 | 462 | 192 | 102 | 91 | 315 |
| 1962 | 76 | 191 | 308 | 332 | 516 | 618 | 745 | 723 | 501 | 255 | 135 | 93 | 374 |
| 1963 | 87 | 115 | 310 | 628 | 675 | 721 | 1760 | 1610 | 1050 | 607 | 324 | 204 | 674 |
| 1964 | 232 | 225 | 220 | 194 | 452 | 1240 | 1030 | 946 | 663 | 275 | 165 | 141 | 482 |
| 1965 | 143 | 152 | 157 | 193 | 341 | 544 | 812 | 855 | 538 | 218 | 140 | 112 | 350 |
| 1966 | 207 | 153 | 155 | 217 | 350 | 527 | 680 | 611 | 374 | 185 | 135 | 125 | 310 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1967 | 145 | 143 | 156 | 225 | 280 | 492 | 784 | 1540 | 702 | 344 | 194 | 165 | 431 |
| 1968 | 166 | 245 | 361 | 295 | 325 | 761 | 1360 | 1180 | 793 | 375 | 220 | 175 | 521 |
| 1969 | 168 | 307 | 835 | 604 | 554 | 1650 | 1780 | 1630 | 697 | 352 | 227 | 180 | 749 |
| 1970 | 175 | 173 | 236 | 295 | 298 | 377 | 595 | 475 | 305 | 183 | 151 | 135 | 283 |
| 1971 | 142 | 156 | 186 | 134 | 140 | 356 | 1230 | 843 | 500 | 234 | 134 | 125 | 348 |
| 1972 | 132 | 154 | 187 | 168 | 300 | 574 | 1210 | 1380 | 841 | 447 | 290 | 223 | 492 |
| 1973 | 178 | 223 | 182 | 176 | 383 | 426 | 855 | 926 | 526 | 313 | 188 | 143 | 377 |
| 1974 | 137 | 155 | 178 | 183 | 221 | 741 | 997 | 718 | 505 | 324 | 185 | 143 | 374 |
| 1975 | 134 | 146 | 152 | 155 | 366 | 376 | 562 | 527 | 386 | 270 | 214 | 164 | 288 |
| 1976 | 110 | 110 | 171 | 232 | 510 | 438 | 1170 | 1137 | 715 | 477 | 256 | 162 | 457 |
| 1977 | 205 | 224 | 230 | 253 | 407 | 608 | 697 | 727 | 588 | 394 | 242 | 177 | 396 |
| 1978 | 171 | 161 | 403 | 518 | 621 | 726 | 831 | 746 | 688 | 445 | 225 | 166 | 475 |
| 1979 | 155 | 155 | 288 | 362 | 535 | 524 | 765 | 677 | 565 | 300 | 164 | 130 | 385 |
| 1980 | 195 | 264 | 320 | 320 | 384 | 631 | 1155 | 924 | 678 | 378 | 227 | 180 | 471 |
| 1981 | 135 | 256 | 204 | 390 | 564 | 837 | 900 | 898 | 725 | 448 | 228 | 178 | 480 |
| 1982 | 168 | 208 | 250 | 350 | 424 | 522 | 908 | 1004 | 682 | 366 | 208 | 183 | 439 |
| 1983 | 216 | 231 | 196 | 227 | 321 | 280 | 858 | 932 | 612 | 235 | 173 | 161 | 370 |
| 1984 | 130 | 174 | 240 | 167 | 288 | 545 | 671 | 653 | 570 | 265 | 131 | 105 | 328 |
| 1985 | 105 | 335 | 228 | 352 | 780 | 852 | 1252 | 1055 | 545 | 226 | 148 | 130 | 501 |
| 1986 | 114 | 133 | 204 | 307 | 437 | 455 | 701 | 586 | 425 | 230 | 102 | 91 | 315 |
| 1987 | 130 | 288 | 365 | 365 | 580 | 645 | 1215 | 1726 | 670 | 356 | 198 | 144 | 557 |
| 1988 | 182 | 264 | 798 | 693 | 695 | 1635 | 1690 | 1558 | 700 | 404 | 173 | 96 | 741 |
| 1989 | 66 | 83 | 103 | 82 | 61 | 242 | 313 | 243 | 95 | 37 | 34 | 35 | 116 |
| 1990 | 45 | 122 | 271 | 145 | 261 | 335 | 666 | 553 | 322 | 117 | 56 | 50 | 245 |
| 1991 | 48 | 55 | 82 | 276 | 406 | 622 | 998 | 411 | 184 | 116 | 66 | 50 | 276 |
| 1992 | 44 | 45 | 350 | 137 | 406 | 622 | 998 | 963 | 574 | 287 | 163 | 125 | 393 |
| 1993 | 140 | 253 | 481 | 362 | 475 | 510 | 1694 | 2282 | 1030 | 282 | 147 | 105 | 647 |
| 1994 | 122 | 220 | 165 | 305 | 320 | 437 | 1035 | 665 | 773 | 210 | 145 | 102 | 375 |
| 1995 | 130 | 375 | 590 | 483 | 765 | 801 | 1727 | 1378 | 710 | 328 | 163 | 105 | 630 |
| 1996 | 131 | 160 | 133 | 317 | 383 | 503 | 867 | 848 | 388 | 195 | 110 | 100 | 345 |
| 1997 | 105 | 125 | 306 | 365 | 325 | 635 | 1278 | 1253 | 695 | 355 | 188 | 130 | 480 |
| 1998 | 145 | 170 | 325 | 300 | 525 | 645 | 1360 | 790 | 435 | 210 | 110 | 100 | 426 |
| 1999 | 107 | 124 | 104 | 123 | 245 | 231 | 440 | 335 | 178 | 84 | 65 | 65 | 175 |
| 2000 | 86 | 75 | 105 | 165 | 205 | 285 | 513 | 346 | 162 | 102 | 73 | 71 | 182 |
| 2001 | 65 | 80 | 235 | 155 | 200 | 440 | 520 | 345 | 240 | 160 | 115 | 80 | 220 |
| 2002 | 85 | 85 | 225 | 485 | 365 | 600 | 1375 | 1085 | 560 | 260 | 150 | 123 | 450 |
| Average | 128 | 173 | 233 | 277 | 405 | 608 | 1002 | 964 | 574 | 288 | 164 | 125 | 412 |

## Table B.2: Calculation min sum of square for season two with PARMA(1,0) model and $\phi_{1,2}$ = 0.516

| year | $W_{r,2}$ | $W_{r,1}$ | $\phi_{1,2}W_{r,1}$ | $a_{r,2}$ | $a^2_{r,2}$ | year | $W_{r,2}$ | $W_{r,1}$ | $\phi_{1,2}W_{r,1}$ | $a_{r,2}$ | $a^2_{r,2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -0.43901 | -1.30412 | -0.67293 | 0.23392 | 0.05472 | 31 | -0.80293 | -0.98789 | -0.50975 | -0.2932 | 0.08595 |
| 2 | -1.69601 | -1.13281 | -0.58453 | -1.1115 | 1.23537 | 32 | 0.778779 | 2.1889 | 1.12947 | -0.3507 | 0.12299 |
| 3 | -1.41133 | -1.08431 | -0.55951 | -0.8518 | 0.72559 | 33 | -0.18664 | 0.2996 | 0.154596 | -0.3412 | 0.11645 |
| 4 | -0.29472 | -1.08431 | -0.55951 | 0.26479 | 0.07012 | 34 | -0.17148 | 1.67254 | 0.863029 | -1.0345 | 1.07023 |
| 5 | 2.039534 | -1.03601 | -0.53458 | 2.57412 | 6.6261 | 35 | -0.32625 | 0.34385 | 0.177425 | -0.5037 | 0.25369 |
| 6 | 0.488581 | 0.07684 | 0.039651 | 0.44893 | 0.20154 | 36 | 1.004937 | 0.80268 | 0.41418 | 0.59075 | 0.34899 |
| 7 | 0.019766 | -0.56238 | -0.29019 | 0.30996 | 0.09607 | 37 | 1.634806 | 0.84586 | 0.436465 | 1.19834 | 1.43601 |
| 8 | -0.42267 | -0.67929 | -0.35052 | -0.0721 | 0.0052 | 38 | 0.118557 | 0.99636 | 0.514123 | -0.3956 | 0.15648 |
| 9 | -0.74832 | 0.32174 | 0.166017 | -0.9143 | 0.83602 | 39 | -0.1264 | 0.27745 | 0.143162 | -0.2696 | 0.07266 |
| 10 | -0.99228 | -0.33123 | -0.17091 | -0.8214 | 0.67464 | 40 | -0.15639 | 0.05442 | 0.028081 | -0.1845 | 0.03403 |
| 11 | 2.386679 | 0.00949 | 0.004897 | 2.38178 | 5.67289 | 41 | 0.755422 | 1.06056 | 0.547249 | 0.20817 | 0.04333 |
| 12 | -0.76642 | -0.10333 | -0.05332 | -0.7131 | 0.50851 | 42 | -0.14136 | 0.16626 | 0.085792 | -0.2272 | 0.0516 |
| 13 | 1.577858 | 0.03197 | 0.016496 | 1.56136 | 2.43785 | 43 | -0.27907 | 0.09924 | 0.051208 | -0.3303 | 0.10908 |
| 14 | -0.47195 | -0.44638 | -0.23033 | -0.2416 | 0.05838 | 44 | -0.89615 | -0.44638 | -0.23033 | -0.6658 | 0.44331 |
| 15 | -0.05254 | 1.56808 | 0.809128 | -0.8617 | 0.7425 | 45 | 0.767118 | 1.6308 | 0.841494 | -0.0744 | 0.00553 |
| 16 | 0.538524 | -0.58569 | -0.30222 | 0.84074 | 0.70685 | 46 | -0.05254 | 0.91048 | 0.46981 | -0.5224 | 0.27286 |
| 17 | -0.05254 | 0.32174 | 0.166017 | -0.2186 | 0.04777 | 47 | -0.14136 | 0.5636 | 0.290817 | -0.4322 | 0.18678 |
| 18 | -0.17148 | 0.32174 | 0.166017 | -0.3375 | 0.11391 | 48 | 1.208452 | 1.42112 | 0.733298 | 0.47515 | 0.22576 |
| 19 | -0.26349 | 0.45402 | 0.234275 | -0.4978 | 0.24777 | 49 | 1.124015 | 0.12161 | 0.06275 | 1.06127 | 1.12628 |
| 20 | 0.412387 | 1.42112 | 0.733298 | -0.3209 | 0.10299 | 50 | 0.575548 | 0.84586 | 0.436465 | 0.13908 | 0.01934 |
| 21 | -0.58994 | -0.03555 | -0.01835 | -0.5716 | 0.32672 | 51 | 0.848012 | 1.85954 | 0.959521 | -0.1115 | 0.01244 |
| 22 | 0.624353 | 0.25526 | 0.131715 | 0.49264 | 0.24269 | 52 | 0.132447 | 0.00949 | 0.004897 | 0.12755 | 0.01627 |
| 23 | 0.321454 | 1.25211 | 0.64609 | -0.3246 | 0.10539 | 53 | 1.891011 | -0.56238 | -0.29019 | 2.18121 | 4.75766 |
| 24 | -0.29472 | -0.63242 | -0.32633 | 0.03161 | 0.001 | 54 | -0.48855 | -0.35419 | -0.18276 | -0.3058 | 0.0935 |
| 25 | -0.76642 | -0.44638 | -0.23033 | -0.5361 | 0.28739 | 55 | 1.451773 | 0.00949 | 0.004897 | 1.44688 | 2.09345 |
| 26 | 0.146283 | 0.27745 | 0.143162 | 0.00312 | 9.7E-06 | 56 | 1.208452 | 1.14589 | 0.591277 | 0.61717 | 0.3809 |
| 27 | -1.87705 | -1.37834 | -0.71122 | -1.1658 | 1.35914 | 57 | -1.45679 | -1.50319 | -0.77565 | -0.6811 | 0.46394 |
| 28 | -0.85852 | -0.53911 | -0.27818 | -0.5803 | 0.33679 | 58 | -0.67695 | -2.04646 | -1.05597 | 0.37903 | 0.14367 |
| 29 | 0.360702 | -1.06014 | -0.54703 | 0.90774 | 0.82399 | 59 | -2.18999 | -1.96667 | -1.0148 | -1.1752 | 1.38104 |
| 30 | 0.360702 | -1.25492 | -0.64754 | 1.00825 | 1.01656 | 60 | -2.51389 | -2.07325 | -1.0698 | -1.4441 | 2.08538 |
| | | | | | | | | | | SS= | 43.2741 |

**Table B.3: Calculation min sum of square for season three with PARMA(1,0) model and $\phi_{1,3}$ = 0.452**

| year | $W_{r,3}$ | $W_{r,2}$ | $\phi_{1,3}W_{r,2}$ | $a_{r,3}$ | $a^2_{r,3}$ | year | $W_{r,3}$ | $W_{r,2}$ | $\phi_{1,3}W_{r,2}$ | $a_{r,3}$ | $a^2_{r,3}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -2.15263 | -0.43901 | -0.19843 | -1.9542 | 3.81888 | 31 | 1.014052 | -0.80293 | -0.36293 | 1.376984 | 1.89608 |
| 2 | 0.747867 | -1.69601 | -0.7666 | 1.514475 | 2.29363 | 32 | 0.263217 | 0.778779 | 0.352008 | -0.0888 | 0.00788 |
| 3 | -2.11716 | -1.41133 | -0.63792 | -1.47923 | 2.18813 | 33 | -0.58452 | -0.18664 | -0.08436 | -0.50016 | 0.25016 |
| 4 | -0.76382 | -0.29472 | -0.13321 | -0.63061 | 0.39767 | 34 | -0.61908 | -0.17148 | -0.07751 | -0.54157 | 0.29329 |
| 5 | 0.198012 | 2.039534 | 0.92187 | -0.72387 | 0.52399 | 35 | -0.60172 | -0.32625 | -0.14746 | -0.45425 | 0.20635 |
| 6 | -0.42058 | 0.488581 | 0.220839 | -0.64143 | 0.41143 | 36 | 1.315492 | 1.004937 | 0.454232 | 0.861254 | 0.74176 |
| 7 | 0.198012 | 0.019766 | 0.008934 | 0.189077 | 0.03575 | 37 | 2.679736 | 1.634806 | 0.738932 | 1.940792 | 3.76667 |
| 8 | 0.209057 | -0.42267 | -0.19104 | 0.400104 | 0.16008 | 38 | 0.425524 | 0.118557 | 0.053588 | 0.371935 | 0.13834 |
| 9 | 0.263217 | -0.74832 | -0.33824 | 0.601464 | 0.36176 | 39 | -0.14414 | -0.1264 | -0.05713 | -0.08701 | 0.00757 |
| 10 | -1.08952 | -0.99228 | -0.44851 | -0.641 | 0.41088 | 40 | -0.1307 | -0.15639 | -0.07069 | -0.06001 | 0.0036 |
| 11 | 0.564233 | 2.386679 | 1.078779 | -0.51456 | 0.26478 | 41 | -0.19895 | 0.755422 | 0.341451 | -0.54041 | 0.29204 |
| 12 | -1.06755 | -0.76642 | -0.34642 | -0.72112 | 0.52002 | 42 | -0.25548 | -0.14136 | -0.06389 | -0.19158 | 0.0367 |
| 13 | -0.43635 | 1.577858 | 0.713192 | -1.14955 | 1.32147 | 43 | -0.6721 | -0.27907 | -0.12614 | -0.54596 | 0.29807 |
| 14 | 0.209057 | -0.47195 | -0.21332 | 0.422382 | 0.17841 | 44 | -0.35882 | -0.89615 | -0.40506 | 0.046243 | 0.00214 |
| 15 | -0.61908 | -0.05254 | -0.02375 | -0.59533 | 0.35441 | 45 | 0.366521 | 0.767118 | 0.346737 | 0.019778 | 0.00039 |
| 16 | 0.095233 | 0.538524 | 0.243413 | -0.14818 | 0.02196 | 46 | 1.522032 | -0.05254 | -0.02375 | 1.545782 | 2.38944 |
| 17 | 0.305316 | -0.05254 | -0.02375 | 0.329066 | 0.10828 | 47 | 0.86156 | -0.14136 | -0.06389 | 0.925455 | 0.85647 |
| 18 | -0.40495 | -0.17148 | -0.07751 | -0.32744 | 0.10722 | 48 | 1.078432 | 1.208452 | 0.54622 | 0.532203 | 0.28324 |
| 19 | -0.6901 | -0.26349 | -0.1191 | -0.57101 | 0.32605 | 49 | 0.083422 | 1.124015 | 0.508055 | -0.42464 | 0.18032 |
| 20 | 0.435153 | 0.412387 | 0.186399 | 0.248751 | 0.06188 | 50 | 0.555351 | 0.575548 | 0.260148 | 0.2952 | 0.08714 |
| 21 | -0.80175 | -0.58994 | -0.26665 | -0.53509 | 0.28632 | 51 | -0.01409 | 0.848012 | 0.383302 | -0.3974 | 0.15792 |
| 22 | -0.42058 | 0.624353 | 0.282208 | -0.7028 | 0.49392 | 52 | 0.4637 | 0.132447 | 0.059866 | 0.403833 | 0.16308 |
| 23 | -0.01409 | 0.321454 | 0.145297 | -0.15939 | 0.0254 | 53 | 0.346371 | 1.891011 | 0.854737 | -0.50838 | 0.25845 |
| 24 | 1.109571 | -0.29472 | -0.13321 | 1.242786 | 1.54452 | 54 | 0.083422 | -0.48855 | -0.22082 | 0.304249 | 0.09257 |
| 25 | -0.35882 | -0.76642 | -0.34642 | -0.01239 | 0.00015 | 55 | 1.336589 | 1.451773 | 0.656201 | 0.680377 | 0.46291 |
| 26 | -0.06497 | 0.146283 | 0.06612 | -0.13109 | 0.01718 | 56 | 2.617241 | 1.208452 | 0.54622 | 2.071012 | 4.28909 |
| 27 | -0.65426 | -1.87705 | -0.84843 | 0.194179 | 0.03771 | 57 | -1.82014 | -1.45679 | -0.65847 | -1.16166 | 1.34946 |
| 28 | -1.47646 | -0.85852 | -0.38805 | -1.08841 | 1.18463 | 58 | 0.732044 | -0.67695 | -0.30598 | 1.038032 | 1.07751 |
| 29 | -0.80175 | 0.360702 | 0.163037 | -0.96479 | 0.93082 | 59 | -2.58182 | -2.18999 | -0.98988 | -1.59193 | 2.53425 |
| 30 | 1.000825 | 0.360702 | 0.163037 | 0.837786 | 0.70188 | 60 | 1.255743 | -2.51389 | -1.13628 | 2.392041 | 5.72186 |
| | | | | | | | | | | SS= | 46.9339 |

## Table B.4: Calculation min sum of square for season four with PARMA(1,0) model and $\phi_{1,4}$ = 0.455

| year | $W_{r,4}$ | $W_{r,3}$ | $\phi_{1,4}W_{r,3}$ | $a_{r,4}$ | $a^2_{r,4}$ | year | $W_{r,4}$ | $W_{r,3}$ | $\phi_{1,4}W_{r,3}$ | $a_{r,4}$ | $a^2_{r,4}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | -1.15575 | -2.15263 | -0.97945 | -0.17629 | 0.03108 | **31** | 2.004761 | 1.014052 | 0.461394 | 1.54336 | 2.38196 |
| **2** | -1.09344 | 0.747867 | 0.340279 | -1.43372 | 2.05556 | **32** | -0.50774 | 0.263217 | 0.119764 | -0.6275 | 0.39376 |
| **3** | -0.60272 | -2.11716 | -0.96331 | 0.360602 | 0.13003 | **33** | -0.51938 | -0.58452 | -0.26596 | -0.25342 | 0.06422 |
| **4** | -2.02029 | -0.76382 | -0.34754 | -1.67275 | 2.79809 | **34** | -0.25674 | -0.61908 | -0.28168 | 0.024943 | 0.00062 |
| **5** | -0.49617 | 0.198012 | 0.090095 | -0.58626 | 0.3437 | **35** | -0.17616 | -0.60172 | -0.27378 | 0.097623 | 0.00953 |
| **6** | 0.462294 | -0.42058 | -0.19137 | 0.653662 | 0.42727 | **36** | 0.418644 | 1.315492 | 0.598549 | -0.17991 | 0.03237 |
| **7** | -0.07032 | 0.198012 | 0.090095 | -0.16042 | 0.02573 | **37** | 1.925515 | 2.679736 | 1.21928 | 0.706216 | 0.49874 |
| **8** | 1.799984 | 0.209057 | 0.095121 | 1.704862 | 2.90655 | **38** | 0.418644 | 0.425524 | 0.193613 | 0.225028 | 0.05064 |
| **9** | 0.973092 | 0.263217 | 0.119764 | 0.853326 | 0.72817 | **39** | -1.35448 | -0.14414 | -0.06559 | -1.2889 | 1.66126 |
| **10** | 1.148196 | -1.08952 | -0.49573 | 1.643933 | 2.70252 | **40** | -0.83375 | -0.1307 | -0.05947 | -0.77428 | 0.59951 |
| **11** | 0.012362 | 0.564233 | 0.256726 | -0.24437 | 0.05972 | **41** | -0.7279 | -0.19895 | -0.09052 | -0.63738 | 0.40625 |
| **12** | 0.343702 | -1.06755 | -0.48574 | 0.829445 | 0.68798 | **42** | -0.63949 | -0.25548 | -0.11624 | -0.52325 | 0.27379 |
| **13** | 1.341169 | -0.43635 | -0.19854 | 1.539711 | 2.37071 | **43** | -1.01803 | -0.6721 | -0.30581 | -0.71222 | 0.50726 |
| **14** | -0.10818 | 0.209057 | 0.095121 | -0.2033 | 0.04133 | **44** | -0.10818 | -0.35882 | -0.16326 | 0.05509 | 0.00303 |
| **15** | 1.127794 | -0.61908 | -0.28168 | 1.409477 | 1.98663 | **45** | 0.08313 | 0.366521 | 0.166767 | -0.08364 | 0.007 |
| **16** | -0.67692 | 0.095233 | 0.043331 | -0.72026 | 0.51877 | **46** | 1.610478 | 1.522032 | 0.692525 | 0.917943 | 0.84262 |
| **17** | -0.42806 | 0.305316 | 0.138919 | -0.56698 | 0.32147 | **47** | 0.858761 | 0.86156 | 0.39201 | 0.466745 | 0.21785 |
| **18** | 0.607937 | -0.40495 | -0.18425 | 0.792192 | 0.62757 | **48** | 0.594518 | 1.078432 | 0.490686 | 0.103824 | 0.01078 |
| **19** | -0.02397 | -0.6901 | -0.314 | 0.290031 | 0.08412 | **49** | 1.017012 | 0.083422 | 0.037957 | 0.979055 | 0.95855 |
| **20** | -0.25674 | 0.435153 | 0.197994 | -0.45474 | 0.20679 | **50** | 0.786813 | 0.555351 | 0.252685 | 0.534125 | 0.28529 |
| **21** | 0.021344 | -0.80175 | -0.3648 | 0.386146 | 0.14911 | **51** | -0.15651 | -0.01409 | -0.00641 | -0.1501 | 0.02253 |
| **22** | 0.560567 | -0.42058 | -0.19137 | 0.751936 | 0.56541 | **52** | -0.84737 | 0.4637 | 0.210984 | -1.05836 | 1.12012 |
| **23** | -0.59061 | -0.01409 | -0.00641 | -0.5842 | 0.34129 | **53** | 0.798989 | 0.346371 | 0.157599 | 0.641388 | 0.41138 |
| **24** | 0.065668 | 1.109571 | 0.504855 | -0.43919 | 0.19289 | **54** | 0.505008 | 0.083422 | 0.037957 | 0.46705 | 0.21814 |
| **25** | -0.71504 | -0.35882 | -0.16326 | -0.55177 | 0.30445 | **55** | 0.876342 | 1.336589 | 0.608148 | 0.268185 | 0.07192 |
| **26** | -0.06097 | -0.06497 | -0.02956 | -0.0314 | 0.00099 | **56** | 2.203854 | 2.617241 | 1.190845 | 1.01299 | 1.02615 |
| **27** | -2.04403 | -0.65426 | -0.29769 | -1.74634 | 3.04969 | **57** | -2.5218 | -1.82014 | -0.82816 | -1.69362 | 2.86835 |
| **28** | -0.28779 | -1.47646 | -0.67179 | 0.384007 | 0.14746 | **58** | -1.17162 | 0.732044 | 0.33308 | -1.50471 | 2.26415 |
| **29** | -0.74084 | -0.80175 | -0.3648 | -0.37604 | 0.14141 | **59** | 0.273766 | -2.58182 | -1.17473 | 1.448514 | 2.09819 |
| **30** | 0.673699 | 1.000825 | 0.455376 | 0.218317 | 0.04766 | **60** | -1.30304 | 1.255743 | 0.571363 | -1.87441 | 3.51341 |
| | | | | | | | | | | **SS=** | **46.81349** |

## Table B.5: Calculation min sum of square for season five with PARMA(1,0) model and $\phi_{1,5} = 0.71$

| year | $W_{r,5}$ | $W_{r,4}$ | $\phi_{1,5}W_{r,4}$ | $a_{r,5}$ | $a^2_{r,5}$ | year | $W_{r,5}$ | $W_{r,4}$ | $\phi_{1,5}W_{r,4}$ | $a_{r,5}$ | $a^2_{r,5}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | -1.03799 | -1.15575 | -0.82174 | -0.21624 | 0.04676 | **31** | 1.466551 | 2.004761 | 1.425385 | 0.041158 | 0.00169 |
| **2** | -0.74769 | -1.09344 | -0.77743 | 0.029744 | 0.00088 | **32** | 0.350094 | -0.50774 | -0.361 | 0.711098 | 0.50566 |
| **3** | 0.133434 | -0.60272 | -0.42854 | 0.561972 | 0.31581 | **33** | -0.31349 | -0.51938 | -0.36928 | 0.055788 | 0.00311 |
| **4** | -0.2878 | -2.02029 | -1.43643 | 1.148635 | 1.31936 | **34** | -0.2559 | -0.25674 | -0.18254 | -0.07336 | 0.00538 |
| **5** | 1.286748 | -0.49617 | -0.35277 | 1.639524 | 2.68804 | **35** | -0.72626 | -0.17616 | -0.12525 | -0.601 | 0.36121 |
| **6** | 0.115904 | 0.462294 | 0.328691 | -0.21279 | 0.04528 | **36** | -0.41783 | 0.418644 | 0.297656 | -0.71548 | 0.51192 |
| **7** | -0.2878 | -0.07032 | -0.05 | -0.2378 | 0.05655 | **37** | 0.889286 | 1.925515 | 1.369041 | -0.47976 | 0.23017 |
| **8** | 1.402634 | 1.799984 | 1.279789 | 0.122839 | 0.01509 | **38** | -0.60004 | 0.418644 | 0.297656 | -0.8977 | 0.80586 |
| **9** | 2.831666 | 0.973092 | 0.691868 | 2.139794 | 4.57872 | **39** | -1.8996 | -1.35448 | -0.96304 | -0.93655 | 0.87713 |
| **10** | 0.015303 | 1.148196 | 0.816367 | -0.80107 | 0.64171 | **40** | -0.58626 | -0.83375 | -0.5928 | 0.00654 | 4.3E-05 |
| **11** | -0.75487 | 0.012362 | 0.008789 | -0.76365 | 0.58317 | **41** | -0.05098 | -0.7279 | -0.51754 | 0.466561 | 0.21768 |
| **12** | -0.43768 | 0.343702 | 0.244372 | -0.68205 | 0.4652 | **42** | -1.17213 | -0.63949 | -0.45468 | -0.71745 | 0.51473 |
| **13** | 0.068833 | 1.341169 | 0.953571 | -0.88474 | 0.78277 | **43** | -0.15537 | -1.01803 | -0.72382 | 0.568461 | 0.32315 |
| **14** | 0.782728 | -0.10818 | -0.07691 | 0.859641 | 0.73898 | **44** | 0.663525 | -0.10818 | -0.07691 | 0.740438 | 0.54825 |
| **15** | 0.460382 | 1.127794 | 0.801861 | -0.34148 | 0.11661 | **45** | 0.092428 | 0.08313 | 0.059105 | 0.033322 | 0.00111 |
| **16** | -0.84199 | -0.67692 | -0.48129 | -0.36069 | 0.1301 | **46** | 1.216094 | 1.610478 | 1.14505 | 0.071038 | 0.00505 |
| **17** | -0.53161 | -0.42806 | -0.30435 | -0.22725 | 0.05164 | **47** | 0.792968 | 0.858761 | 0.610579 | 0.182386 | 0.03326 |
| **18** | -0.5384 | 0.607937 | 0.432243 | -0.97064 | 0.94215 | **48** | -0.04491 | 0.594518 | 0.422702 | -0.46762 | 0.21867 |
| **19** | -0.60695 | -0.02397 | -0.01704 | -0.58991 | 0.34799 | **49** | 0.939301 | 1.017012 | 0.723096 | 0.216201 | 0.04674 |
| **20** | 1.610144 | -0.25674 | -0.18254 | 1.792688 | 3.21373 | **50** | 0.191404 | 0.786813 | 0.559424 | -0.36802 | 0.13544 |
| **21** | 0.29965 | 0.021344 | 0.015176 | 0.284474 | 0.08093 | **51** | -0.44432 | -0.15651 | -0.11128 | -0.33304 | 0.11092 |
| **22** | 0.808291 | 0.560567 | 0.398563 | 0.409726 | 0.16788 | **52** | -0.66966 | -0.84737 | -0.60248 | -0.06718 | 0.00451 |
| **23** | -1.14011 | -0.59061 | -0.41992 | -0.72018 | 0.51866 | **53** | 1.925739 | 0.798989 | 0.568081 | 1.357655 | 1.84323 |
| **24** | 0.626701 | 0.065668 | 0.04669 | 0.580011 | 0.33641 | **54** | 0.265728 | 0.505008 | 0.35906 | -0.09333 | 0.00871 |
| **25** | -0.35232 | -0.71504 | -0.50839 | 0.15608 | 0.02436 | **55** | 1.018389 | 0.876342 | 0.62308 | 0.395306 | 0.15627 |
| **26** | -0.62774 | -0.06097 | -0.04335 | -0.5844 | 0.34152 | **56** | 1.556695 | 2.203854 | 1.56694 | -0.01025 | 0.00011 |
| **27** | -2.36529 | -2.04403 | -1.4533 | -0.91198 | 0.83171 | **57** | -2.87121 | -2.5218 | -1.793 | -1.0782 | 1.16252 |
| **28** | -0.30063 | -0.28779 | -0.20462 | -0.096 | 0.00922 | **58** | -0.86408 | -1.17162 | -0.83302 | -0.03105 | 0.00096 |
| **29** | -0.74769 | -0.74084 | -0.52674 | -0.22095 | 0.04882 | **59** | 0.08654 | 0.273766 | 0.194648 | -0.10811 | 0.01169 |
| **30** | 0.694882 | 0.673699 | 0.479 | 0.215879 | 0.0466 | **60** | 0.08654 | -1.30304 | -0.92646 | 1.013005 | 1.02618 |
| | | | | | | | | | | **SS=** | **29.158** |

## Table B.6: Calculation min sum of square for season six with PARMA(1,0) model and $\phi_{1,6}$ = 0.646

| year | $W_{r,6}$ | $W_{r,5}$ | $\phi_{1,6}W_{r,5}$ | $a_{r,6}$ | $a^2_{r,6}$ | year | $W_{r,6}$ | $W_{r,5}$ | $\phi_{1,6}W_{r,5}$ | $a_{r,6}$ | $a^2_{r,6}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | -0.00626 | -1.03799 | -0.67054 | 0.66429 | 0.44128 | **31** | 0.60933 | 1.466551 | 0.947392 | -0.3381 | 0.11429 |
| **2** | -1.05942 | -0.74769 | -0.48301 | -0.5764 | 0.33224 | **32** | 1.733388 | 0.350094 | 0.226161 | 1.50723 | 2.27173 |
| **3** | -0.56718 | 0.133434 | 0.086199 | -0.6534 | 0.4269 | **33** | -0.05549 | -0.31349 | -0.20252 | 0.14703 | 0.02162 |
| **4** | -1.2395 | -0.2878 | -0.18592 | -1.0536 | 1.11002 | **34** | -0.13421 | -0.2559 | -0.16531 | 0.0311 | 0.00097 |
| **5** | 0.302041 | 1.286748 | 0.831239 | -0.5292 | 0.28006 | **35** | -0.30734 | -0.72626 | -0.46916 | 0.16182 | 0.02619 |
| **6** | -0.87277 | 0.115904 | 0.074874 | -0.9476 | 0.89803 | **36** | 0.73012 | -0.41783 | -0.26992 | 1.00004 | 1.00008 |
| **7** | 0.063102 | -0.2878 | -0.18592 | 0.24902 | 0.06201 | **37** | 2.252294 | 0.889286 | 0.574479 | 1.67781 | 2.81505 |
| **8** | 0.150347 | 1.402634 | 0.906101 | -0.7558 | 0.57117 | **38** | -1.01485 | -0.60004 | -0.38763 | -0.6272 | 0.3934 |
| **9** | 1.006084 | 2.831666 | 1.829256 | -0.8232 | 0.67763 | **39** | -1.17513 | -1.8996 | -1.22714 | 0.05202 | 0.00271 |
| **10** | 1.008548 | 0.015303 | 0.009886 | 0.99866 | 0.99733 | **40** | 0.075822 | -0.58626 | -0.37872 | 0.45455 | 0.20662 |
| **11** | -0.88655 | -0.75487 | -0.48764 | -0.3989 | 0.15912 | **41** | -0.6827 | -0.05098 | -0.03293 | -0.6498 | 0.4222 |
| **12** | 0.618692 | -0.43768 | -0.28274 | 0.90144 | 0.81259 | **42** | 0.670799 | -1.17213 | -0.7572 | 1.428 | 2.03919 |
| **13** | -0.91438 | 0.068833 | 0.044466 | -0.9589 | 0.9194 | **43** | -1.02221 | -0.15537 | -0.10037 | -0.9218 | 0.8498 |
| **14** | 1.350578 | 0.782728 | 0.505642 | 0.84493 | 0.71391 | **44** | -0.60897 | 0.663525 | 0.428637 | -1.0376 | 1.07663 |
| **15** | 0.773281 | 0.460382 | 0.297407 | 0.47587 | 0.22645 | **45** | 0.214129 | 0.092428 | 0.059708 | 0.15442 | 0.02385 |
| **16** | -0.75273 | -0.84199 | -0.54392 | -0.2088 | 0.0436 | **46** | 0.624906 | 1.216094 | 0.785597 | -0.1607 | 0.02582 |
| **17** | 0.646476 | -0.53161 | -0.34342 | 0.9899 | 0.97989 | **47** | -0.14845 | 0.792968 | 0.512257 | -0.6607 | 0.43654 |
| **18** | 1.115695 | -0.5384 | -0.3478 | 1.4635 | 2.14184 | **48** | 0.302041 | -0.04491 | -0.02901 | 0.33106 | 0.1096 |
| **19** | -0.81855 | -0.60695 | -0.39209 | -0.4265 | 0.18187 | **49** | 0.938101 | 0.939301 | 0.606788 | 0.33131 | 0.10977 |
| **20** | 0.875201 | 1.610144 | 1.040153 | -0.165 | 0.02721 | **50** | -0.158 | 0.191404 | 0.123647 | -0.2817 | 0.07933 |
| **21** | 0.712542 | 0.29965 | 0.193574 | 0.51897 | 0.26933 | **51** | -1.87926 | -0.44432 | -0.28703 | -1.5922 | 2.53519 |
| **22** | 1.733388 | 0.808291 | 0.522156 | 1.21123 | 1.46707 | **52** | -0.05096 | -0.66966 | -0.4326 | 0.38165 | 0.14565 |
| **23** | -1.64185 | -1.14011 | -0.73651 | -0.9053 | 0.81963 | **53** | 0.97622 | 1.925739 | 1.244028 | -0.2678 | 0.07173 |
| **24** | 0.479375 | 0.626701 | 0.404849 | 0.07452 | 0.00555 | **54** | -0.50895 | 0.265728 | 0.171661 | -0.6806 | 0.46323 |
| **25** | 0.888505 | -0.35232 | -0.2276 | 1.1161 | 1.24569 | **55** | 0.353513 | 1.018389 | 0.657879 | -0.3044 | 0.09264 |
| **26** | -0.52046 | -0.62774 | -0.40552 | -0.1149 | 0.01321 | **56** | 2.236421 | 1.556695 | 1.005625 | 1.23079 | 1.51484 |
| **27** | -1.74205 | -2.36529 | -1.52798 | -0.2141 | 0.04582 | **57** | -2.33383 | -2.87121 | -1.8548 | -0.479 | 0.22946 |
| **28** | -0.53204 | -0.30063 | -0.1942 | -0.3378 | 0.11413 | **58** | -1.34837 | -0.86408 | -0.5582 | -0.7902 | 0.62436 |
| **29** | -1.24768 | -0.74769 | -0.48301 | -0.7647 | 0.58471 | **59** | 0.268151 | 0.08654 | 0.055905 | 0.21225 | 0.04505 |
| **30** | 0.252881 | 0.694882 | 0.448894 | -0.196 | 0.03842 | **60** | 0.268151 | 0.08654 | 0.055905 | 0.21225 | 0.04505 |
| | | | | | | | | | | **SS** | **34.39867** |

# Table B.7: Calculation min sum of square for season seven with PARMA(1,0) model and $\phi_{1,7} = 0.66$

| year | $W_{r,7}$ | $W_{r,6}$ | $\phi_{1,7}W_{r,6}$ | $a_{r,7}$ | $a^2_{r,7}$ | year | $W_{r,7}$ | $W_{r,6}$ | $\phi_{1,7}W_{r,6}$ | $a_{r,7}$ | $a^2_{r,7}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -0.72348 | -0.00626 | -0.00413 | -0.71935 | 0.51747 | 31 | 2.110437 | 0.60933 | 0.402158 | 1.70828 | 2.91821 |
| 2 | -0.66276 | -1.05942 | -0.69922 | 0.03646 | 0.00133 | 32 | 0.233585 | 1.733388 | 1.144036 | -0.91046 | 0.82893 |
| 3 | -0.64771 | -0.56718 | -0.37434 | -0.27337 | 0.07473 | 33 | -0.49634 | -0.05549 | -0.03662 | -0.45971 | 0.21134 |
| 4 | 0.233585 | -1.2395 | -0.81807 | 1.05166 | 1.10598 | 34 | -1.00379 | -0.13421 | -0.08858 | -0.91521 | 0.83761 |
| 5 | 1.033987 | 0.302041 | 0.199347 | 0.83464 | 0.69662 | 35 | -0.59915 | -0.30734 | -0.20285 | -0.3963 | 0.15705 |
| 6 | 0.325753 | -0.87277 | -0.57603 | 0.90178 | 0.81322 | 36 | 1.164644 | 0.73012 | 0.48188 | 0.68276 | 0.46616 |
| 7 | 0.416212 | 0.063102 | 0.041647 | 0.37456 | 0.1403 | 37 | 2.153808 | 2.252294 | 1.486514 | 0.66728 | 0.44527 |
| 8 | 1.317306 | 0.150347 | 0.099229 | 1.21808 | 1.48371 | 38 | -1.3661 | -1.01485 | -0.6698 | -0.6963 | 0.48484 |
| 9 | 0.202469 | 1.006084 | 0.664015 | -0.46155 | 0.21303 | 39 | 0.817899 | -1.17513 | -0.77558 | 1.59349 | 2.5392 |
| 10 | 0.678086 | 1.008548 | 0.665642 | 0.01244 | 0.00015 | 40 | 0.762433 | 0.075822 | 0.050043 | 0.71239 | 0.5075 |
| 11 | -0.57691 | -0.88655 | -0.58512 | 0.00821 | 6.7E-05 | 41 | -0.34292 | -0.6827 | -0.45058 | 0.10767 | 0.01159 |
| 12 | 0.790241 | 0.618692 | 0.408337 | 0.3819 | 0.14585 | 42 | 0.130128 | 0.670799 | 0.442727 | -0.3126 | 0.09772 |
| 13 | -0.80449 | -0.91438 | -0.60349 | -0.201 | 0.0404 | 43 | -1.51599 | -1.02221 | -0.67466 | -0.84133 | 0.70784 |
| 14 | 1.367245 | 1.350578 | 0.891382 | 0.47586 | 0.22644 | 44 | 0.649655 | -0.60897 | -0.40192 | 1.05158 | 1.10582 |
| 15 | -0.57691 | 0.773281 | 0.510365 | -1.08728 | 1.18218 | 45 | -0.93497 | 0.214129 | 0.141325 | -1.0763 | 1.15842 |
| 16 | 0.17115 | -0.75273 | -0.4968 | 0.66796 | 0.44617 | 46 | -0.4279 | 0.624906 | 0.412438 | -0.84034 | 0.70617 |
| 17 | 0.790241 | 0.646476 | 0.426674 | 0.36356 | 0.13218 | 47 | -0.6703 | -0.14845 | -0.09798 | -0.57232 | 0.32755 |
| 18 | 0.325753 | 1.115695 | 0.736359 | -0.41061 | 0.1686 | 48 | 0.606707 | 0.302041 | 0.199347 | 0.40736 | 0.16594 |
| 19 | -1.54387 | -0.81855 | -0.54024 | -1.00362 | 1.00725 | 49 | -0.18773 | 0.938101 | 0.619147 | -0.80688 | 0.65106 |
| 20 | 0.734472 | 0.875201 | 0.577633 | 0.15684 | 0.0246 | 50 | -0.16068 | -0.158 | -0.10428 | -0.0564 | 0.00318 |
| 21 | 0.845409 | 0.712542 | 0.470277 | 0.37513 | 0.14072 | 51 | -0.33241 | -1.87926 | -1.24031 | 0.90791 | 0.82431 |
| 22 | 1.911128 | 1.733388 | 1.144036 | 0.76708 | 0.58842 | 52 | -1.04068 | -0.05096 | -0.03363 | -1.00705 | 1.01415 |
| 23 | -1.3795 | -1.64185 | -1.08362 | -0.29587 | 0.08754 | 53 | 0.878228 | 0.97622 | 0.644305 | 0.23392 | 0.05472 |
| 24 | 0.678086 | 0.479375 | 0.316388 | 0.3617 | 0.13082 | 54 | -0.91894 | -0.50895 | -0.33591 | -0.58303 | 0.33993 |
| 25 | -0.62897 | 0.888505 | 0.586413 | -1.21538 | 1.47716 | 55 | 0.776356 | 0.353513 | 0.233319 | 0.54304 | 0.29489 |
| 26 | -1.14491 | -0.52046 | -0.3435 | -0.8014 | 0.64224 | 56 | 1.956022 | 2.236421 | 1.476038 | 0.47997 | 0.23037 |
| 27 | 0.17115 | -1.74205 | -1.14975 | 1.32091 | 1.7448 | 57 | -2.89715 | -2.33383 | -1.54033 | -1.35681 | 1.84092 |
| 28 | -0.68922 | -0.53204 | -0.35114 | -0.33807 | 0.11429 | 58 | -1.06132 | -1.34837 | -0.88992 | -0.17139 | 0.02937 |
| 29 | -0.41719 | -1.24768 | -0.82347 | 0.40628 | 0.16507 | 59 | 0.133296 | 0.268151 | 0.17698 | -0.04369 | 0.00191 |
| 30 | -0.74647 | 0.252881 | 0.166902 | -0.91338 | 0.83426 | 60 | 0.133296 | 0.268151 | 0.17698 | -0.04369 | 0.00191 |
| | | | | | | | | | | SS= | 33.30948 |

## Table B.8: Calculation min sum of square for season eight with PARMA(1,0) model and $\phi_{1,8}$ = 0.784

| year | $W_{r,8}$ | $W_{r,7}$ | $\phi_{1,8}W_{r,7}$ | $a_{r,8}$ | $a^2_{r,8}$ | year | $W_{r,8}$ | $W_{r,7}$ | $\phi_{1,8}W_{r,7}$ | $a_{r,8}$ | $a^2_{r,8}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.1784 | -0.72348 | -0.56721 | 0.74561 | 0.55594 | 31 | 1.67871 | 2.110437 | 1.654582 | 0.02412 | 0.00058 |
| 2 | -0.80068 | -0.66276 | -0.5196 | -0.2811 | 0.079 | 32 | 0.02493 | 0.233585 | 0.183131 | -0.1582 | 0.02503 |
| 3 | -0.58067 | -0.64771 | -0.5078 | -0.0729 | 0.00531 | 33 | -0.24433 | -0.49634 | -0.38913 | 0.1448 | 0.02097 |
| 4 | 0.15875 | 0.233585 | 0.183131 | -0.0244 | 0.00059 | 34 | -1.05004 | -1.00379 | -0.78697 | -0.2631 | 0.0692 |
| 5 | -0.2413 | 1.033987 | 0.810646 | -1.0519 | 1.10659 | 35 | 1.52362 | -0.59915 | -0.46973 | 1.99335 | 3.97346 |
| 6 | -0.07698 | 0.325753 | 0.25539 | -0.3324 | 0.11047 | 36 | 0.6612 | 1.164644 | 0.913081 | -0.2519 | 0.06345 |
| 7 | 0.53115 | 0.416212 | 0.32631 | 0.20484 | 0.04196 | 37 | 1.72238 | 2.153808 | 1.688585 | 0.03379 | 0.00114 |
| 8 | 0.05081 | 1.317306 | 1.032768 | -0.982 | 0.96424 | 38 | -1.57364 | -1.3661 | -1.07103 | -0.5026 | 0.25261 |
| 9 | 0.53115 | 0.202469 | 0.158736 | 0.37242 | 0.1387 | 39 | -0.28093 | 0.817899 | 0.641233 | -0.9222 | 0.85038 |
| 10 | 0.98655 | 0.678086 | 0.531619 | 0.45493 | 0.20696 | 40 | 1.15483 | 0.762433 | 0.597747 | 0.55708 | 0.31034 |
| 11 | 0.34399 | -0.57691 | -0.4523 | 0.79629 | 0.63407 | 41 | -0.03306 | -0.34292 | -0.26885 | 0.23579 | 0.0556 |
| 12 | 0.42496 | 0.790241 | 0.619549 | -0.1946 | 0.03787 | 42 | -0.67938 | 0.130128 | 0.10202 | -0.7814 | 0.61058 |
| 13 | -0.56118 | -0.80449 | -0.63072 | 0.06955 | 0.00484 | 43 | -1.36528 | -1.51599 | -1.18854 | -0.1767 | 0.03124 |
| 14 | 2.0206 | 1.367245 | 1.07192 | 0.94868 | 0.89999 | 44 | 0.54954 | 0.649655 | 0.50933 | 0.04021 | 0.00162 |
| 15 | -1.01763 | -0.57691 | -0.4523 | -0.5653 | 0.31959 | 45 | -0.64954 | -0.93497 | -0.73302 | 0.08348 | 0.00697 |
| 16 | 0.96213 | 0.17115 | 0.134182 | 0.82795 | 0.6855 | 46 | -0.58719 | -0.4279 | -0.33547 | -0.2517 | 0.06336 |
| 17 | 1.546 | 0.790241 | 0.619549 | 0.92644 | 0.8583 | 47 | -0.81779 | -0.6703 | -0.52552 | -0.2923 | 0.08542 |
| 18 | 1.2956 | 0.325753 | 0.25539 | 1.04021 | 1.08203 | 48 | -0.0389 | 0.606707 | 0.475658 | -0.5146 | 0.26477 |
| 19 | -1.22344 | -1.54387 | -1.21039 | -0.013 | 0.00017 | 49 | -0.11534 | -0.18773 | -0.14718 | 0.03184 | 0.00101 |
| 20 | 0.83857 | 0.734472 | 0.575826 | 0.26274 | 0.06903 | 50 | 0.1896 | -0.16068 | -0.12598 | 0.31557 | 0.09959 |
| 21 | 0.5574 | 0.845409 | 0.6628 | -0.1054 | 0.01111 | 51 | -0.0156 | -0.33241 | -0.26061 | 0.24501 | 0.06003 |
| 22 | 1.24902 | 1.911128 | 1.498325 | -0.2493 | 0.06216 | 52 | -0.90083 | -1.04068 | -0.81589 | -0.0849 | 0.00721 |
| 23 | -1.01404 | -1.3795 | -1.08153 | 0.06749 | 0.00455 | 53 | 0.33037 | 0.878228 | 0.68853 | -0.3582 | 0.12828 |
| 24 | -0.12721 | 0.678086 | 0.531619 | -0.6588 | 0.43405 | 54 | -1.14138 | -0.91894 | -0.72045 | -0.4209 | 0.17718 |
| 25 | 0.4783 | -0.62897 | -0.49311 | 0.97142 | 0.94365 | 55 | 1.92823 | 0.776356 | 0.608663 | 1.31957 | 1.74126 |
| 26 | -1.10461 | -1.14491 | -0.89761 | -0.207 | 0.04285 | 56 | 1.56384 | 1.956022 | 1.533521 | 0.03032 | 0.00092 |
| 27 | -0.39548 | 0.17115 | 0.134182 | -0.5297 | 0.28054 | 57 | -2.69342 | -2.89715 | -2.27136 | -0.422 | 0.17812 |
| 28 | -0.2687 | -0.68922 | -0.54035 | 0.27165 | 0.07379 | 58 | -1.2651 | -1.06132 | -0.83207 | -0.433 | 0.18751 |
| 29 | -0.18093 | -0.41719 | -0.32708 | 0.14615 | 0.02136 | 59 | -1.847 | 0.133296 | 0.104504 | -1.9515 | 3.80837 |
| 30 | -0.66278 | -0.74647 | -0.58524 | -0.0775 | 0.00601 | 60 | 0.07372 | 0.133296 | 0.104504 | -0.0308 | 0.00095 |
| | | | | | | | | | | SS= | 22.75837 |

## Table B.9: Calculation min sum of square for season nine with PARMA(1,0) model and $\phi_{1,9}$ = 0.849

| year | $W_{r,9}$ | $W_{r,8}$ | $\phi_{1,9}W_{r,8}$ | $a_{r,9}$ | $a^2_{r,9}$ | year | $W_{r,9}$ | $W_{r,8}$ | $\phi_{1,9}W_{r,8}$ | $a_{r,9}$ | $a^2_{r,9}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -0.18774 | 0.1784 | 0.151463 | -0.3392 | 0.11506 | 31 | 2.397251 | 1.67871 | 1.425226 | 0.972022 | 0.94483 |
| 2 | -0.55208 | -0.80068 | -0.67978 | 0.127697 | 0.01631 | 32 | 0.433355 | 0.02493 | 0.021163 | 0.412192 | 0.1699 |
| 3 | -0.8782 | -0.58067 | -0.49299 | -0.38521 | 0.14838 | 33 | -0.22501 | -0.24433 | -0.20744 | -0.01757 | 0.00031 |
| 4 | -0.40143 | 0.15875 | 0.134781 | -0.53621 | 0.28752 | 34 | -1.11499 | -1.05004 | -0.89148 | -0.2235 | 0.04995 |
| 5 | 0.584105 | -0.2413 | -0.20486 | 0.788967 | 0.62247 | 35 | 0.63593 | 1.52362 | 1.293554 | -0.65763 | 0.43247 |
| 6 | -0.16115 | -0.07698 | -0.06536 | -0.09579 | 0.00918 | 36 | 1.104055 | 0.6612 | 0.561359 | 0.542695 | 0.29452 |
| 7 | 0.297599 | 0.53115 | 0.45095 | -0.15335 | 0.02352 | 37 | 0.610028 | 1.72238 | 1.462301 | -0.85228 | 0.72637 |
| 8 | 0.171758 | 0.05081 | 0.043142 | 0.128617 | 0.01654 | 38 | -1.50127 | -1.57364 | -1.33602 | -0.16525 | 0.02731 |
| 9 | -0.36926 | 0.53115 | 0.45095 | -0.82021 | 0.67275 | 39 | -0.42826 | -0.28093 | -0.23851 | -0.18976 | 0.03601 |
| 10 | 0.161248 | 0.98655 | 0.837581 | -0.67633 | 0.45743 | 40 | 1.348626 | 1.15483 | 0.980454 | 0.36817 | 0.13555 |
| 11 | 0.365549 | 0.34399 | 0.292044 | 0.073505 | 0.0054 | 41 | -0.28902 | -0.03306 | -0.02807 | -0.26095 | 0.0681 |
| 12 | -0.241 | 0.42496 | 0.360791 | -0.60179 | 0.36215 | 42 | -0.40143 | -0.67938 | -0.57679 | 0.175364 | 0.03075 |
| 13 | -0.3104 | -0.56118 | -0.47644 | 0.166043 | 0.02757 | 43 | -1.04863 | -1.36528 | -1.15913 | 0.110502 | 0.01221 |
| 14 | 2.347624 | 2.0206 | 1.715489 | 0.632131 | 0.39959 | 44 | 0.703185 | 0.54954 | 0.466557 | 0.236627 | 0.05599 |
| 15 | -1.22053 | -1.01763 | -0.86397 | -0.35656 | 0.12714 | 45 | 0.040112 | -0.64954 | -0.55146 | 0.591577 | 0.34996 |
| 16 | 1.702647 | 0.96213 | 0.816851 | 0.885793 | 0.78463 | 46 | 0.563353 | -0.58719 | -0.49852 | 1.061876 | 1.12758 |
| 17 | 1.828367 | 1.546 | 1.312551 | 0.515813 | 0.26606 | 47 | -0.08153 | -0.81779 | -0.6943 | 0.612773 | 0.37549 |
| 18 | 0.780624 | 1.2956 | 1.099961 | -0.31934 | 0.10198 | 48 | 0.511415 | -0.0389 | -0.03302 | 0.544438 | 0.29641 |
| 19 | -1.07072 | -1.22344 | -1.0387 | -0.03202 | 0.00103 | 49 | 0.75483 | -0.11534 | -0.09793 | 0.852758 | 0.7272 |
| 20 | 0.287132 | 0.83857 | 0.711944 | -0.42481 | 0.18047 | 50 | 0.5322 | 0.1896 | 0.160969 | 0.371231 | 0.13781 |
| 21 | 0.495818 | 0.5574 | 0.473231 | 0.022586 | 0.00051 | 51 | 0.166504 | -0.0156 | -0.01324 | 0.179746 | 0.03231 |
| 22 | 1.687536 | 1.24902 | 1.060416 | 0.627117 | 0.39328 | 52 | -0.05504 | -0.90083 | -0.7648 | 0.709762 | 0.50376 |
| 23 | -1.18158 | -1.01404 | -0.86092 | -0.32065 | 0.10282 | 53 | -0.18774 | 0.33037 | 0.280488 | -0.46823 | 0.21924 |
| 24 | 0.759991 | -0.12721 | -0.108 | 0.86799 | 0.75341 | 54 | -0.83444 | -1.14138 | -0.96903 | 0.134595 | 0.01812 |
| 25 | 0.729017 | 0.4783 | 0.406081 | 0.322936 | 0.10429 | 55 | 0.469806 | 1.92823 | 1.637069 | -1.16727 | 1.36251 |
| 26 | -1.22053 | -1.10461 | -0.93781 | -0.28272 | 0.07993 | 56 | 0.625572 | 1.56384 | 1.327703 | -0.70213 | 0.49299 |
| 27 | -0.63318 | -0.39548 | -0.33576 | -0.29741 | 0.08845 | 57 | -2.75261 | -2.69342 | -2.28671 | -0.4659 | 0.21706 |
| 28 | -1.04863 | -0.2687 | -0.22813 | -0.8205 | 0.67322 | 58 | -1.4053 | -1.2651 | -1.07407 | -0.33123 | 0.10971 |
| 29 | -0.63318 | -0.18093 | -0.15361 | -0.47957 | 0.22998 | 59 | -2.2036 | -1.847 | -1.5681 | -0.63549 | 0.40385 |
| 30 | -0.42289 | -0.66278 | -0.5627 | 0.139807 | 0.01955 | 60 | -0.03387 | 0.07372 | 0.062589 | -0.09646 | 0.0093 |
| | | | | | | | | | | SS= | 16.43818 |

# Table B.10: Calculation min sum of square for season ten with PARMA(1,0) model and $\phi_{1,10}$ = 0.908

| year | $W_{r,10}$ | $W_{r,9}$ | $\phi_{1,10}W_{r,9}$ | $a_{r,10}$ | $a^2_{r,10}$ | year | $W_{r,10}$ | $W_{r,9}$ | $\phi_{1,10}W_{r,9}$ | $a_{r,10}$ | $a^2_{r,10}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -0.03817 | -0.18774 | -0.17047 | 0.13229 | 0.0175 | 31 | 2.606205 | 2.397251 | 2.176704 | 0.4295 | 0.18447 |
| 2 | -0.96548 | -0.55208 | -0.50129 | -0.4642 | 0.21547 | 32 | -0.18686 | 0.433355 | 0.393486 | -0.5804 | 0.33681 |
| 3 | -1.17142 | -0.8782 | -0.7974 | -0.374 | 0.13988 | 33 | -0.73485 | -0.22501 | -0.20431 | -0.5305 | 0.28147 |
| 4 | -0.41389 | -0.40143 | -0.3645 | -0.0494 | 0.00244 | 34 | -1.06778 | -1.11499 | -1.01241 | -0.0554 | 0.00307 |
| 5 | -0.76458 | 0.584105 | 0.530368 | -1.295 | 1.6769 | 35 | 0.441045 | 0.63593 | 0.577425 | -0.1364 | 0.0186 |
| 6 | -0.08442 | -0.16115 | -0.14632 | 0.0619 | 0.00383 | 36 | 0.713005 | 1.104055 | 1.002482 | -0.2895 | 0.0838 |
| 7 | 0.044595 | 0.297599 | 0.27022 | -0.2256 | 0.05091 | 37 | 0.511774 | 0.610028 | 0.553905 | -0.0421 | 0.00178 |
| 8 | 0.262456 | 0.171758 | 0.155957 | 0.1065 | 0.01134 | 38 | -1.0884 | -1.50127 | -1.36315 | 0.27476 | 0.07549 |
| 9 | -0.29975 | -0.36926 | -0.33529 | 0.03554 | 0.00126 | 39 | -0.57787 | -0.42826 | -0.38886 | -0.189 | 0.03572 |
| 10 | -0.2056 | 0.161248 | 0.146413 | -0.352 | 0.12391 | 40 | 1.324778 | 1.348626 | 1.224553 | 0.10022 | 0.01004 |
| 11 | 0.538197 | 0.365549 | 0.331919 | 0.20628 | 0.04255 | 41 | 0.163104 | -0.28902 | -0.26243 | 0.42554 | 0.18108 |
| 12 | 0.262456 | -0.241 | -0.21883 | 0.48128 | 0.23163 | 42 | 0.262456 | -0.40143 | -0.3645 | 0.62695 | 0.39307 |
| 13 | -0.0659 | -0.3104 | -0.28184 | 0.21594 | 0.04663 | 43 | -0.23375 | -1.04863 | -0.95215 | 0.7184 | 0.5161 |
| 14 | 2.168218 | 2.347624 | 2.131643 | 0.03657 | 0.00134 | 44 | 1.572516 | 0.703185 | 0.638492 | 0.93402 | 0.8724 |
| 15 | -1.07808 | -1.22053 | -1.10824 | 0.03016 | 0.00091 | 45 | 0.877002 | 0.040112 | 0.036422 | 0.84058 | 0.70657 |
| 16 | 0.936937 | 1.702647 | 1.546003 | -0.6091 | 0.37096 | 46 | 1.308122 | 0.563353 | 0.511525 | 0.7966 | 0.63457 |
| 17 | 0.799562 | 1.828367 | 1.660157 | -0.8606 | 0.74063 | 47 | 0.044595 | -0.08153 | -0.07403 | 0.11863 | 0.01407 |
| 18 | 0.669519 | 0.780624 | 0.708807 | -0.0393 | 0.00154 | 48 | 0.73903 | 0.511415 | 0.464365 | 0.27466 | 0.07544 |
| 19 | -0.75466 | -1.07072 | -0.97222 | 0.21756 | 0.04733 | 49 | 1.3331 | 0.75483 | 0.685386 | 0.64771 | 0.41953 |
| 20 | 0.316305 | 0.287132 | 0.260715 | 0.05559 | 0.00309 | 50 | 0.634627 | 0.5322 | 0.483238 | 0.15139 | 0.02292 |
| 21 | 0.590883 | 0.495818 | 0.450203 | 0.14068 | 0.01979 | 51 | -0.56815 | 0.166504 | 0.151185 | -0.7193 | 0.51744 |
| 22 | 1.59708 | 1.687536 | 1.532282 | 0.0648 | 0.0042 | 52 | -0.28085 | -0.05504 | -0.04998 | -0.2309 | 0.0533 |
| 23 | -0.98584 | -1.18158 | -1.07287 | 0.08704 | 0.00758 | 53 | -0.65602 | -0.18774 | -0.17047 | -0.4856 | 0.23576 |
| 24 | 0.842637 | 0.759991 | 0.690072 | 0.15257 | 0.02328 | 54 | -0.61686 | -0.83444 | -0.75767 | 0.14081 | 0.01983 |
| 25 | 0.634627 | 0.729017 | 0.661948 | -0.0273 | 0.00075 | 55 | 0.546993 | 0.469806 | 0.426584 | 0.12041 | 0.0145 |
| 26 | -1.01647 | -1.22053 | -1.10824 | 0.09178 | 0.00842 | 56 | 0.962547 | 0.625572 | 0.568019 | 0.39453 | 0.15565 |
| 27 | -0.65602 | -0.63318 | -0.57493 | -0.0811 | 0.00658 | 57 | -2.83748 | -2.75261 | -2.49937 | -0.3381 | 0.11431 |
| 28 | -1.28708 | -1.04863 | -0.95215 | -0.3349 | 0.11218 | 58 | -1.80487 | -1.4053 | -1.27601 | -0.5289 | 0.27968 |
| 29 | -0.99603 | -0.63318 | -0.57493 | -0.4211 | 0.17733 | 59 | -1.81638 | -2.2036 | -2.00087 | 0.18449 | 0.03404 |
| 30 | -0.3757 | -0.42289 | -0.38399 | 0.00829 | 6.9E-05 | 60 | -0.07516 | -0.03387 | -0.03075 | -0.0444 | 0.00197 |
| | | | | | | | | | | SS= | 10.38372 |

# Table B.11: Calculation min sum of square for season eleven with PARMA(1,0) model and $\phi_{1,11}$ = 0.941

| year | $W_{r,11}$ | $W_{r,10}$ | $\phi_{1,11}W_{r,10}$ | $a_{r,11}$ | $a^2_{r,11}$ | year | $W_{r,11}$ | $W_{r,10}$ | $\phi_{1,11}W_{r,10}$ | $a_{r,11}$ | $a^2_{r,11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -0.28009 | -0.03817 | -0.03592 | -0.24416 | 0.05962 | 31 | 2.530034 | 2.606205 | 2.452439 | 0.07759 | 0.00602 |
| 2 | -1.10252 | -0.96548 | -0.90852 | -0.194 | 0.03764 | 32 | -0.04827 | -0.18686 | -0.17584 | 0.12757 | 0.01627 |
| 3 | -1.20046 | -1.17142 | -1.1023 | -0.09816 | 0.00963 | 33 | -0.49817 | -0.73485 | -0.69149 | 0.19332 | 0.03737 |
| 4 | -0.83364 | -0.41389 | -0.38947 | -0.44417 | 0.19728 | 34 | -0.5903 | -1.06778 | -1.00478 | 0.41448 | 0.17179 |
| 5 | -0.96713 | -0.76458 | -0.71947 | -0.24766 | 0.06133 | 35 | 0.454321 | 0.441045 | 0.415023 | 0.0393 | 0.00154 |
| 6 | -0.2262 | -0.08442 | -0.07944 | -0.14675 | 0.02154 | 36 | 0.890271 | 0.713005 | 0.670938 | 0.21933 | 0.04811 |
| 7 | -0.4068 | 0.044595 | 0.041964 | -0.44876 | 0.20139 | 37 | 1.005561 | 0.511774 | 0.481579 | 0.52398 | 0.27456 |
| 8 | 0.074794 | 0.262456 | 0.246971 | -0.17218 | 0.02965 | 38 | -0.2981 | -1.0884 | -1.02418 | 0.72608 | 0.52719 |
| 9 | -0.28009 | -0.29975 | -0.28207 | 0.00198 | 3.9E-06 | 39 | -0.60882 | -0.57787 | -0.54377 | -0.06504 | 0.00423 |
| 10 | -0.1904 | -0.2056 | -0.19347 | 0.00306 | 9.4E-06 | 40 | 2.009577 | 1.324778 | 1.246616 | 0.76296 | 0.58211 |
| 11 | 0.265875 | 0.538197 | 0.506444 | -0.24057 | 0.05787 | 41 | 0.351848 | 0.163104 | 0.153481 | 0.19837 | 0.03935 |
| 12 | 0.420245 | 0.262456 | 0.246971 | 0.17327 | 0.03002 | 42 | 0.300329 | 0.262456 | 0.246971 | 0.05336 | 0.00285 |
| 13 | -0.2262 | -0.0659 | -0.06201 | -0.16418 | 0.02696 | 43 | 0.790775 | -0.23375 | -0.21996 | 1.01074 | 1.02159 |
| 14 | 2.102405 | 2.168218 | 2.040294 | 0.06211 | 0.00386 | 44 | 1.474795 | 1.572516 | 1.479738 | -0.00494 | 2.4E-05 |
| 15 | -0.9288 | -1.07808 | -1.01447 | 0.08567 | 0.00734 | 45 | 1.249893 | 0.877002 | 0.825258 | 0.42463 | 0.18031 |
| 16 | 0.454321 | 0.936937 | 0.881658 | -0.42734 | 0.18262 | 46 | 0.972706 | 1.308122 | 1.230943 | -0.25824 | 0.06669 |
| 17 | 0.454321 | 0.799562 | 0.752388 | -0.29807 | 0.08884 | 47 | -0.06595 | 0.044595 | 0.041964 | -0.10791 | 0.01165 |
| 18 | 0.471328 | 0.669519 | 0.630017 | -0.15869 | 0.02518 | 48 | 1.005561 | 0.73903 | 0.695427 | 0.31013 | 0.09618 |
| 19 | -0.5903 | -0.75466 | -0.71013 | 0.11984 | 0.01436 | 49 | 1.021963 | 1.3331 | 1.254447 | -0.23248 | 0.05405 |
| 20 | 0.057285 | 0.316305 | 0.297643 | -0.24036 | 0.05777 | 50 | 0.690633 | 0.634627 | 0.597184 | 0.09345 | 0.00873 |
| 21 | 0.673878 | 0.590883 | 0.556021 | 0.11786 | 0.01389 | 51 | 0.092279 | -0.56815 | -0.53462 | 0.6269 | 0.39301 |
| 22 | 1.649513 | 1.59708 | 1.502852 | 0.14666 | 0.02151 | 52 | -0.66457 | -0.28085 | -0.26428 | -0.40029 | 0.16023 |
| 23 | -0.68322 | -0.98584 | -0.92767 | 0.24445 | 0.05976 | 53 | -0.35232 | -0.65602 | -0.61732 | 0.26499 | 0.07022 |
| 24 | 0.522229 | 0.842637 | 0.792922 | -0.27069 | 0.07328 | 54 | -1.22018 | -0.61686 | -0.58047 | -0.63971 | 0.40923 |
| 25 | 0.60667 | 0.634627 | 0.597184 | 0.00949 | 9E-05 | 55 | 0.522229 | 0.546993 | 0.514721 | 0.00751 | 5.6E-05 |
| 26 | -0.64596 | -1.01647 | -0.9565 | 0.31054 | 0.09644 | 56 | 0.092279 | 0.962547 | 0.905757 | -0.81348 | 0.66175 |
| 27 | -0.57181 | -0.65602 | -0.61732 | 0.04551 | 0.00207 | 57 | -2.71372 | -2.83748 | -2.67007 | -0.04365 | 0.00191 |
| 28 | -0.96713 | -1.28708 | -1.21115 | 0.24402 | 0.05954 | 58 | -2.18751 | -1.80487 | -1.69838 | -0.48913 | 0.23925 |
| 29 | -1.22018 | -0.99603 | -0.93727 | -0.28291 | 0.08004 | 59 | -1.96513 | -1.81638 | -1.70921 | -0.25592 | 0.06549 |
| 30 | -0.5903 | -0.3757 | -0.35354 | -0.23676 | 0.05606 | 60 | -0.08365 | -0.07516 | -0.07072 | -0.01293 | 0.00017 |
| | | | | | | | | | | SS= | 6.72751 |

# Table B.12: Calculation min sum of square for season twelve with PARMA(1,0) model and $\phi_{1,12} = 0.933$

| year | $W_{r,12}$ | $W_{r,11}$ | $\phi_{1,12}W_{r,11}$ | $a_{r,12}$ | $a^2_{r,12}$ | year | $W_{r,12}$ | $W_{r,11}$ | $\phi_{1,12}W_{r,11}$ | $a_{r,12}$ | $a^2_{r,12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | -0.86624 | -0.28009 | -0.26132 | -0.60492 | 0.36592 | **31** | 1.93828 | 2.530034 | 2.360522 | -0.42224 | 0.17829 |
| **2** | -1.21165 | -1.10252 | -1.02865 | -0.183 | 0.03349 | **32** | 0.294091 | -0.04827 | -0.04504 | 0.339128 | 0.11501 |
| **3** | -1.21165 | -1.20046 | -1.12003 | -0.09162 | 0.00839 | **33** | -0.46928 | -0.49817 | -0.46479 | -0.00449 | 2E-05 |
| **4** | -0.97237 | -0.83364 | -0.77778 | -0.19459 | 0.03786 | **34** | -0.12647 | -0.5903 | -0.55075 | 0.424278 | 0.18001 |
| **5** | -1.05205 | -0.96713 | -0.90233 | -0.14972 | 0.02242 | **35** | 0.922473 | 0.454321 | 0.423881 | 0.498592 | 0.24859 |
| **6** | -0.33731 | -0.2262 | -0.21104 | -0.12627 | 0.01594 | **36** | 1.183521 | 0.890271 | 0.830623 | 0.352898 | 0.12454 |
| **7** | -0.44288 | -0.4068 | -0.37954 | -0.06333 | 0.00401 | **37** | 1.313887 | 1.005561 | 0.938188 | 0.375698 | 0.14115 |
| **8** | -0.17914 | 0.074794 | 0.069783 | -0.24893 | 0.06196 | **38** | 0.136545 | -0.2981 | -0.27813 | 0.414677 | 0.17196 |
| **9** | -0.2582 | -0.28009 | -0.26132 | 0.003123 | 9.8E-06 | **39** | -0.12647 | -0.60882 | -0.56803 | 0.441557 | 0.19497 |
| **10** | -0.20549 | -0.1904 | -0.17765 | -0.02784 | 0.00078 | **40** | 2.43112 | 2.009577 | 1.874936 | 0.556183 | 0.30934 |
| **11** | -0.07382 | 0.265875 | 0.248062 | -0.32188 | 0.10361 | **41** | 0.346564 | 0.351848 | 0.328274 | 0.01829 | 0.00033 |
| **12** | 0.32033 | 0.420245 | 0.392089 | -0.07176 | 0.00515 | **42** | 0.346564 | 0.300329 | 0.280207 | 0.066357 | 0.0044 |
| **13** | -0.41647 | -0.2262 | -0.21104 | -0.20543 | 0.0422 | **43** | 0.896344 | 0.790775 | 0.737793 | 0.158551 | 0.02514 |
| **14** | 0.660982 | 2.102405 | 1.961544 | -1.30056 | 1.69147 | **44** | 0.844074 | 1.474795 | 1.375984 | -0.53191 | 0.28293 |
| **15** | -0.89276 | -0.9288 | -0.86657 | -0.02618 | 0.00069 | **45** | 1.23568 | 1.249893 | 1.16615 | 0.069529 | 0.00483 |
| **16** | 0.162816 | 0.454321 | 0.423881 | -0.26107 | 0.06816 | **46** | 0.948598 | 0.972706 | 0.907534 | 0.041062 | 0.00169 |
| **17** | 0.267847 | 0.454321 | 0.423881 | -0.15603 | 0.02435 | **47** | 0.005108 | -0.06595 | -0.06153 | 0.066638 | 0.00444 |
| **18** | 0.32033 | 0.471328 | 0.439749 | -0.11942 | 0.01426 | **48** | 1.313887 | 1.005561 | 0.938188 | 0.375698 | 0.14115 |
| **19** | -0.33731 | -0.5903 | -0.55075 | 0.213442 | 0.04556 | **49** | 1.261753 | 1.021963 | 0.953492 | 0.308261 | 0.09502 |
| **20** | -0.07382 | 0.057285 | 0.053447 | -0.12727 | 0.0162 | **50** | 1.392058 | 0.690633 | 0.64436 | 0.747697 | 0.55905 |
| **21** | 0.660982 | 0.673878 | 0.628728 | 0.032253 | 0.00104 | **51** | 0.817931 | 0.092279 | 0.086096 | 0.731835 | 0.53558 |
| **22** | 2.094047 | 1.649513 | 1.538996 | 0.55505 | 0.30808 | **52** | -0.65433 | -0.66457 | -0.62005 | -0.03428 | 0.00118 |
| **23** | -0.60143 | -0.68322 | -0.63745 | 0.036021 | 0.0013 | **53** | 0.005108 | -0.35232 | -0.32872 | 0.333826 | 0.11144 |
| **24** | 0.372793 | 0.522229 | 0.487239 | -0.11445 | 0.0131 | **54** | -1.02549 | -1.22018 | -1.13843 | 0.112944 | 0.01276 |
| **25** | 0.477658 | 0.60667 | 0.566023 | -0.08837 | 0.00781 | **55** | 0.372793 | 0.522229 | 0.487239 | -0.11445 | 0.0131 |
| **26** | -0.65433 | -0.64596 | -0.60268 | -0.05165 | 0.00267 | **56** | -0.89276 | 0.092279 | 0.086096 | -0.97885 | 0.95816 |
| **27** | -0.44288 | -0.57181 | -0.5335 | 0.090625 | 0.00821 | **57** | -2.53007 | -2.71372 | -2.5319 | 0.001831 | 3.4E-06 |
| **28** | -0.91929 | -0.96713 | -0.90233 | -0.01695 | 0.00029 | **58** | -2.12294 | -2.18751 | -2.04095 | -0.08199 | 0.00672 |
| **29** | -1.02549 | -1.22018 | -1.13843 | 0.112944 | 0.01276 | **59** | -2.12294 | -1.96513 | -1.83346 | -0.28947 | 0.0838 |
| **30** | -0.97237 | -0.5903 | -0.55075 | -0.42162 | 0.17776 | **60** | -0.12647 | -0.08365 | -0.07805 | -0.04842 | 0.00234 |
| | | | | | | | | | | **SS=** | **7.60338** |

**Table B.13: Calculation min. sum of squares of monthly flow for Greater Zab river with θ = 0.361 and Θ = 0.95 for model (0, 1, 1) (0, 1, 1)$_{12.}$ Calculations start from the bottom of the table.**

| t | $X_t$ | ln $X_t$ | $a_t$ | θ $a_{t-1}$ | Θ $a_{t-12}$ | θΘ $a_{t-13}$ | $W_t$ | θΘ $e_{t+13}$ | Θ $e_{t+12}$ | θ $e_{t+1}$ | $e_t$ | $a_t^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -12 | 74 | 4.304065 | -0.0508617 | 0 | 0 | 0 | -0.05086168 | -0.05086168 | 0 | 0 | 0 | 0.002586911 |
| -11 | 137 | 4.919981 | 0.31381877 | -0.01831021 | 0 | 0 | 0.33212898 | 0.190846528 | -0.14128245 | 0 | 0 | 0.098482223 |
| -10 | 93 | 4.532599 | -0.523788 | 0.11297476 | 0 | 0 | -0.63676277 | -0.10663353 | 0.530129245 | 0 | 0 | 0.274353883 |
| -9 | 147 | 4.990433 | 0.06017241 | -0.18856368 | 0 | 0 | 0.248736097 | -0.04746815 | -0.29620424 | 0 | 0 | 3.62E-03 |
| -8 | 239 | 5.476464 | 0.00882204 | 0.02166207 | 0 | 0 | -0.01284003 | -0.14469599 | -0.13185596 | 0 | 0 | 7.78E-05 |
| -7 | 556 | 6.320768 | 0.45497529 | 0.00317593 | 0 | 0 | 0.451799357 | 0.04986606 | -0.4019333 | 0 | 0 | 0.207002516 |
| -6 | 751 | 6.621406 | -0.0728003 | 0.1637911 | 0 | 0 | -0.23659136 | -0.09807452 | 0.138516833 | 0 | 0 | 0.005299877 |
| -5 | 1000 | 6.907755 | 0.27431582 | -0.02620809 | 0 | 0 | 0.300523908 | 0.028094677 | -0.27242923 | 0 | 0 | 0.075249168 |
| -4 | 546 | 6.302619 | 0.01138204 | 0.09875369 | 0 | 0 | -0.08737165 | -0.00933088 | 0.07804077 | 0 | 0 | 0.000129551 |
| -3 | 291 | 5.673323 | 0.08539444 | 0.00409754 | 0 | 0 | 0.081296907 | 0.055377791 | -0.02591912 | 0 | 0 | 0.007292211 |
| -2 | 152 | 5.023881 | -0.0364265 | 0.030742 | 0 | 0 | -0.06716848 | 0.086658713 | 0.153827197 | 0 | 0 | 0.001326889 |
| -1 | 98 | 4.584967 | -0.1736846 | -0.01311353 | 0 | 0 | -0.1605711 | 0.080147544 | 0.240718648 | 0 | 0 | 0.030166354 |
| 0 | 81 | 4.394449 | -0.2456601 | -0.06252647 | -0.0483186 | 0 | -0.13481501 | 0.034278441 | 0.222632067 | -0.053538613 | 0 | 0.060348875 |
| 1 | 73 | 4.290459 | -0.4928206 | -0.08843763 | 0.298127836 | -0.0173947 | -0.71990555 | -0.2750782 | 0.095217891 | 0.200891082 | -0.148718371 | 0.242872187 |
| 2 | 273 | 5.609472 | 0.92405372 | -0.17741543 | -0.49759861 | 0.107326021 | 1.706393787 | 0.272011068 | -0.76410612 | -0.112245818 | 0.558030784 | 0.85387528 |
| 3 | 150 | 5.010635 | -0.487711 | 0.33265934 | 0.057163792 | -0.1791355 | -1.0566696 | -0.03925582 | 0.755586301 | -0.049966469 | -0.31179394 | 0.237861983 |
| 4 | 278 | 5.627621 | -0.0568191 | -0.17557595 | 0.008380939 | 0.020578965 | 0.130954854 | 0.008395083 | -0.10904395 | -0.152311565 | -0.138795746 | 0.003228412 |
| 5 | 371 | 5.916202 | -0.1469693 | -0.02045488 | 0.432226527 | 0.003017138 | -0.55572379 | -0.05682585 | 0.023319676 | 0.052490589 | -0.423087681 | 0.021599971 |
| 6 | 768 | 6.64379 | 0.14927958 | -0.05290894 | -0.06916024 | 0.15560155 | 0.426950313 | 0.020057205 | -0.15784958 | -0.10323634 | 0.145807193 | 0.022284393 |
| 7 | 682 | 6.52503 | -0.0658713 | 0.05374065 | 0.260600027 | -0.02489769 | -0.4051097 | -0.03305429 | 0.055714457 | 0.029573344 | -0.286767611 | 0.004339034 |
| 8 | 478 | 6.169611 | 0.14300063 | -0.02371368 | 0.01081294 | 0.09381601 | 0.249717378 | 0.065929745 | -0.09181747 | -0.009821981 | 0.082148179 | 0.020449179 |
| 9 | 196 | 5.278115 | -0.1334881 | 0.05148023 | 0.08112472 | 0.003892658 | -0.26220037 | 0.006513508 | 0.183138181 | 0.058292412 | -0.02728328 | 0.017819067 |
| 10 | 109 | 4.691348 | -0.0491898 | -0.04805571 | -0.03460516 | 0.029204899 | 0.062675969 | 0.01006538 | 0.018093078 | 0.091219698 | 0.161923365 | 0.002419636 |
| 11 | 84 | 4.430817 | 0.00813108 | -0.01770833 | -0.16500041 | -0.01245786 | 0.178381959 | 0.037319133 | 0.027959389 | 0.084365836 | 0.253388051 | 6.61E-05 |
| 12 | 83 | 4.418841 | 0.00749239 | 0.00292719 | -0.23337708 | -0.05940015 | 0.178542133 | 0.083939417 | 0.103664258 | 0.036082569 | 0.234349544 | 5.61E-05 |
| 13 | 86 | 4.454347 | -0.2419702 | 0.00269726 | -0.46817961 | -0.08401575 | 0.139496402 | -0.01712391 | 0.233165046 | -0.289556002 | 0.100229359 | 0.058549578 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | 94 | 4.543295 | -0.2707784 | -0.08710927 | 0.877851035 | -0.16854466 | -1.23006487 | -0.18698162 | -0.04756642 | 0.28632744 | -0.804322227 | 0.073320966 |
| 15 | 187 | 5.231109 | 0.40981831 | -0.09748024 | -0.46332541 | 0.316026373 | 1.286650336 | -0.06941899 | -0.5193934 | -0.041321919 | 0.795354001 | 0.167951046 |
| 16 | 414 | 6.025866 | 0.43812511 | 0.14753459 | -0.05397816 | -0.16679715 | 0.177771537 | 0.10856106 | -0.19283051 | 0.00883693 | -0.114783108 | 0.191953616 |
| 17 | 446 | 6.100319 | -0.1765916 | 0.15772504 | -0.13962082 | -0.01943214 | -0.21412797 | 0.003066821 | 0.301558501 | -0.059816681 | 0.024547027 | 0.031184598 |
| 18 | 771 | 6.647688 | -0.0517121 | -0.06357298 | 0.141815602 | -0.0502635 | -0.18021825 | 0.015570994 | 0.008518948 | 0.021112847 | -0.166157448 | 0.002674145 |
| 19 | 749 | 6.618739 | -0.0424371 | -0.01861637 | -0.06257777 | 0.051053617 | 0.089810685 | 0.039622659 | 0.043252761 | -0.03479399 | 0.058646797 | 0.001800905 |
| 20 | 418 | 6.035481 | -0.0847374 | -0.01527735 | 0.135850595 | -0.022528 | -0.22783863 | 0.048274019 | 0.110062942 | 0.069399732 | -0.096649971 | 0.007180423 |
| 21 | 176 | 5.170484 | -0.1797267 | -0.03050546 | -0.12681367 | 0.048906214 | 0.026498636 | -0.02532757 | 0.134094498 | 0.006856324 | 0.192777033 | 0.03230169 |
| 22 | 103 | 4.634729 | -0.0147672 | -0.06470162 | -0.04673031 | -0.04565292 | 0.05101177 | -0.02779281 | -0.07035437 | 0.010595137 | 0.019045345 | 0.000218071 |
| 23 | 84 | 4.430817 | 0.07585013 | -0.0053162 | 0.007724529 | -0.01682291 | 0.056618894 | -0.010731 | -0.07720226 | 0.039283298 | 0.029430936 | 0.005753242 |
| 24 | 83 | 4.418841 | 0.03164299 | 0.02730605 | 0.007117773 | 0.00278083 | 0 | -0.05057132 | -0.02980833 | 0.088357281 | 0.109120272 | 0.001001279 |
| 25 | 146 | 4.983607 | 0.30821671 | 0.01139148 | -0.22987169 | 0.002562398 | 0.529259325 | 0.125321371 | -0.14047589 | -0.018025171 | 0.245436891 | 0.094997542 |
| 26 | 148 | 4.997212 | -0.1388695 | 0.11095802 | -0.25723952 | -0.08275381 | -0.07534183 | 0.126020243 | 0.34811492 | -0.196822762 | -0.05006992 | 0.019284747 |
| 27 | 101 | 4.615121 | -0.637965 | -0.04999303 | 0.389327393 | -0.09260623 | -1.06990559 | -0.24619208 | 0.350056232 | -0.073072616 | -0.546729896 | 0.406999344 |
| 28 | 346 | 5.846439 | 0.4829545 | -0.2296674 | 0.416218859 | 0.140157862 | 0.436560901 | 0.0699483 | -0.68386689 | 0.1142748 | -0.202979489 | 0.233245047 |
| 29 | 349 | 5.855072 | -0.209557 | 0.17386362 | -0.16776203 | 0.149838789 | -0.06581983 | -0.18572077 | 0.194300834 | 0.003228233 | 0.317430001 | 0.043914151 |
| 30 | 1030 | 6.937314 | 0.47070001 | -0.07544053 | -0.04912652 | -0.06039433 | 0.534872737 | 0.026404929 | -0.51589101 | 0.01639052 | 0.008967313 | 0.221558501 |
| 31 | 993 | 6.900731 | 0.13918831 | 0.169452 | -0.04031522 | -0.01768555 | -0.00763403 | 0.061891839 | 0.073347026 | 0.041708062 | 0.045529222 | 0.019373385 |
| 32 | 506 | 6.226537 | -0.1068157 | 0.05010779 | -0.08050051 | -0.01451348 | -0.09093644 | 0.01594436 | 0.171921775 | 0.050814757 | 0.115855729 | 0.011409591 |
| 33 | 251 | 5.525453 | -0.0163001 | -0.03845365 | -0.17074037 | -0.02898018 | 0.163913707 | 0.040390887 | 0.044289888 | -0.026660605 | 0.141152103 | 0.000265694 |
| 34 | 122 | 4.804021 | -0.1441073 | -0.00586805 | -0.01402887 | -0.06146653 | -0.18567689 | -0.02867834 | 0.112196908 | -0.029255591 | -0.074057235 | 0.020766905 |
| 35 | 93 | 4.532599 | -0.04228 | -0.05187862 | 0.072057624 | -0.00505039 | -0.06750936 | -0.07720166 | -0.07966204 | -0.011295788 | -0.081265532 | 0.001787595 |
| 36 | 86 | 4.454347 | -0.0773767 | -0.01522079 | 0.03006084 | 0.025940745 | -0.06627601 | -0.30258085 | -0.21444906 | -0.053232969 | -0.031377189 | 0.005987153 |
| 37 | 352 | 5.863631 | 1.09864623 | -0.02785561 | 0.292805877 | 0.010821902 | 0.844517865 | 0.283802098 | -0.84050236 | 0.131917233 | -0.14786936 | 1.207023535 |
| 38 | 214 | 5.365976 | -0.3530843 | 0.39551264 | -0.13192605 | 0.105410116 | -0.51126081 | 0.043294479 | 0.788339162 | 0.132652888 | 0.366436758 | 0.124668552 |
| 39 | 195 | 5.273 | -0.3965684 | -0.12711036 | -0.60606675 | -0.04749338 | 0.2891153 | -0.21825206 | 0.120262442 | -0.259149558 | 0.368480244 | 0.157266524 |
| 40 | 636 | 6.455199 | 0.48510691 | -0.14276464 | 0.458806773 | -0.21818403 | -0.04911925 | 0.138114698 | -0.60625572 | 0.07362979 | -0.719859884 | 0.235328718 |
| 41 | 631 | 6.447306 | -0.206137 | 0.17463849 | -0.19907918 | 0.165170438 | -0.01652585 | -0.03289664 | 0.383651939 | -0.195495542 | 0.204527193 | 0.042492454 |
| 42 | 1310 | 7.177782 | 0.0928586 | -0.07420931 | 0.447165011 | -0.07166851 | -0.35176561 | 0.127692663 | -0.09137957 | 0.027794662 | -0.543043173 | 0.008622719 |
| 43 | 856 | 6.75227 | -0.38425 | 0.0334291 | 0.132228891 | 0.160979404 | -0.38892862 | -0.04628487 | 0.354701842 | 0.065149304 | 0.077207395 | 0.147648093 |
| 44 | 692 | 6.539586 | 0.17410226 | -0.13833001 | -0.1014749 | 0.047602401 | 0.461509574 | 0.168753733 | -0.12856909 | 0.016783536 | 0.18097029 | 0.030311596 |
| 45 | 215 | 5.370638 | -0.3841415 | 0.06267681 | -0.01548512 | -0.03653096 | -0.4678642 | -0.00320804 | 0.468760368 | 0.042516723 | 0.046620935 | 0.147564724 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 46 | 115 | 4.744932 | -0.1738922 | -0.13829096 | -0.13690191 | -0.00557464 | 0.095725995 | -0.06147496 | -0.00891122 | -0.030187722 | 0.118102009 | 0.030238505 |
| 47 | 90 | 4.49981 | -0.0271834 | -0.0626012 | -0.04016596 | -0.04928469 | 0.026299094 | -0.14187481 | -0.17076377 | -0.081264908 | -0.083854783 | 0.000738936 |
| 48 | 133 | 4.890349 | 0.39995752 | -0.00978602 | -0.07350786 | -0.01445975 | 0.468791655 | -0.01807533 | -0.39409668 | -0.318506157 | -0.225735856 | 0.159966019 |
| 49 | 201 | 5.303305 | 0.21783336 | 0.14398471 | 1.043713917 | -0.02646283 | -0.9963281 | 0.136941031 | -0.05020924 | 0.298739051 | -0.884739325 | 0.047451371 |
| 50 | 167 | 5.117994 | -0.3204031 | 0.07842001 | -0.33543012 | 0.37573701 | 0.312344065 | -0.09152174 | 0.380391754 | 0.045573136 | 0.829830697 | 0.102658122 |
| 51 | 301 | 5.70711 | 0.31076264 | -0.1153451 | -0.37674001 | -0.12075484 | 0.682092909 | 0.071534794 | -0.25422706 | -0.229739011 | 0.126592044 | 0.096573417 |
| 52 | 411 | 6.018593 | -0.1623635 | 0.11187455 | 0.460851568 | -0.1356264 | -0.87071606 | 0.111539517 | 0.198707761 | 0.145383893 | -0.638163918 | 2.64E-02 |
| 53 | 397 | 5.983936 | -0.4469518 | -0.05845087 | -0.19583013 | 0.165906564 | -0.02676423 | -0.15540443 | 0.309831991 | -0.034628046 | 0.403844147 | 0.199765911 |
| 54 | 1060 | 6.966024 | 0.24942322 | -0.16090265 | 0.088215669 | -0.07049885 | 0.251611353 | 0.050534715 | -0.43167899 | 0.13441333 | -0.096189018 | 6.22E-02 |
| 55 | 911 | 6.814543 | -0.0329721 | 0.08979236 | -0.36503754 | 0.031757641 | 0.27403075 | -0.00768632 | 0.140374207 | -0.048720918 | 0.37337036 | 1.09E-03 |
| 56 | 550 | 6.309918 | -0.0069995 | -0.01186994 | 0.165397145 | -0.13141351 | -0.2919402 | -0.0003197 | -0.02135089 | 0.177635508 | -0.135335883 | 4.90E-05 |
| 57 | 286 | 5.655992 | 0.08802421 | -0.00251981 | -0.36493447 | 0.059542972 | 0.51502146 | 0.017324554 | -0.00088806 | -0.003376885 | 0.493431967 | 7.75E-03 |
| 58 | 155 | 5.043425 | 0.01100672 | 0.03168871 | -0.16519761 | -0.13137641 | 0.013139206 | 0.005932718 | 0.048123761 | -0.064710483 | -0.009380235 | 0.000121148 |
| 59 | 117 | 4.762174 | 0.00148062 | 0.00396242 | -0.02582421 | -0.05947114 | -0.03612872 | 0.01076049 | 0.016479773 | -0.1493419 | -0.179751342 | 2.19E-06 |
| 60 | 105 | 4.65396 | -0.1089637 | 0.00053302 | 0.379959645 | -0.00929672 | -0.49875304 | -0.07305084 | 0.029890251 | -0.019026661 | -0.414838612 | 0.011873079 |
| 61 | 166 | 5.111988 | 0.07600096 | -0.03922692 | 0.206941687 | 0.136785472 | 0.045071658 | 0.039152945 | -0.202919 | 0.144148454 | -0.052851836 | 0.005776145 |
| 62 | 214 | 5.365976 | 0.08777775 | 0.02736034 | -0.30438291 | 0.074499007 | 0.439299322 | 0.051306454 | 0.108758181 | -0.096338675 | 0.400412373 | 0.007704933 |
| 63 | 236 | 5.463832 | -0.0548583 | 0.03159999 | 0.295224506 | -0.10957785 | -0.49126066 | -0.00583552 | 0.142517929 | 0.075299783 | -0.267607432 | 0.003009435 |
| 64 | 345 | 5.843544 | -0.2120455 | -0.01974899 | -0.15424536 | 0.106280822 | 0.068229662 | -0.03973616 | -0.01620977 | 0.117410018 | 0.209166064 | 0.044963299 |
| 65 | 571 | 6.347389 | 0.09308946 | -0.07633638 | -0.42460421 | -0.05552833 | 0.538501726 | -0.06159904 | -0.11037821 | -0.163583616 | 0.326138938 | 0.008665648 |
| 66 | 1090 | 6.993933 | 0.08777764 | 0.03351221 | 0.23695206 | -0.15285752 | -0.33554414 | 0.000940776 | -0.17110845 | 0.053194436 | -0.454398933 | 0.007704914 |
| 67 | 1130 | 7.029973 | 0.10249497 | 0.03159995 | -0.03132347 | 0.085302742 | 0.187521226 | 0.034281304 | 0.002613266 | -0.008090864 | 0.147762323 | 0.010505219 |
| 68 | 637 | 6.45677 | -0.0270535 | 0.03689819 | -0.00664951 | -0.01127645 | -0.06857864 | 0.048785304 | 0.095225845 | -0.000336526 | -0.022474622 | 0.000731892 |
| 69 | 300 | 5.703782 | -0.0227832 | -0.00973926 | 0.083622998 | -0.00239382 | -0.09906071 | 0.055625187 | 0.135514733 | 0.018236372 | -0.000934795 | 0.000519072 |
| 70 | 145 | 4.976734 | -0.1423319 | -0.00820194 | 0.010456381 | 0.030104279 | -0.11448204 | -0.00437925 | 0.154514407 | 0.006244967 | 0.05065659 | 0.020258362 |
| 71 | 113 | 4.727388 | -0.0216919 | -0.05123947 | 0.001406589 | 0.003764297 | 0.031905258 | 0.013720366 | -0.01216459 | 0.011326832 | 0.017347129 | 0.00047054 |
| 72 | 100 | 4.60517 | -0.125835 | -0.00780909 | -0.10351547 | 0.000506372 | -0.01400405 | -0.08425097 | 0.038112127 | -0.076895622 | 0.031463422 | 0.015834444 |
| 73 | 137 | 4.919981 | -0.0790508 | -0.0453006 | 0.072200908 | -0.03726557 | -0.1432167 | -0.12243458 | -0.23403046 | 0.041213626 | -0.213598951 | 0.006249031 |
| 74 | 215 | 5.370638 | 0.22560712 | -0.02845829 | 0.083388863 | 0.025992327 | 0.196668876 | -0.20390268 | -0.34009605 | 0.054006794 | 0.114482295 | 0.050898572 |
| 75 | 568 | 6.342121 | 0.87271077 | 0.08121856 | -0.0521154 | 0.030019991 | 0.873627601 | 0.151069749 | -0.56639633 | -0.006142651 | 0.150018872 | 0.761624089 |
| 76 | 661 | 6.493754 | -0.096586 | 0.31417588 | -0.20144324 | -0.01876154 | -0.22808019 | 0.166793384 | 0.419638191 | -0.041827534 | -0.017062919 | 0.009328856 |
| 77 | 592 | 6.383507 | -0.4879084 | -0.03477096 | 0.088434987 | -0.07251956 | -0.614092 | -0.09943054 | 0.463314957 | -0.064841098 | -0.116187595 | 0.238054614 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 78 | 1420 | 7.258412 | 0.10426689 | -0.17564703 | 0.083388758 | 0.031836595 | 0.22836175 | 1.33E-01 | -0.27619596 | 0.00099029 | -0.18011416 | 1.09E-02 |
| 79 | 955 | 6.861711 | -0.3278544 | 0.03753608 | 0.097370221 | 0.030019953 | -0.43274075 | -0.02921085 | 3.70E-01 | 0.036085583 | 0.002750806 | 0.107488508 |
| 80 | 613 | 6.418365 | -0.0489248 | -0.11802758 | -0.02570084 | 0.035053279 | 0.129856852 | -0.00016917 | -0.08114124 | 0.051352952 | 0.100237731 | 2.39E-03 |
| 81 | 324 | 5.780744 | 0.08536112 | -0.01761294 | -0.021644 | -0.0092523 | 0.115365761 | 0.030801584 | -0.00046992 | 0.058552828 | 0.142647088 | 0.00728652 |
| 82 | 172 | 5.147494 | -0.0028937 | 0.03073 | -0.13521528 | -0.00779184 | 0.093799693 | 0.012103163 | 0.085559956 | -0.004609741 | 0.162646745 | 8.37E-06 |
| 83 | 123 | 4.812184 | -0.0589358 | -0.00104175 | -0.02060733 | -0.0486775 | -0.0859642 | -0.02509697 | 0.033619897 | 0.01444249 | -0.012804837 | 0.003473425 |
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| 85 | 118 | 4.770685 | -0.4932603 | 0.05274184 | -0.07509827 | -0.04303557 | -0.51393942 | -0.17055478 | 0.225915292 | -0.128878504 | -0.246347851 | 0.243305711 |
| 86 | 220 | 5.393628 | 0.23607426 | -0.1775737 | 0.214326763 | -0.02703538 | 0.17228582 | -0.158116 | -0.47376327 | -0.214634398 | -0.357995846 | 0.055731055 |
| 87 | 382 | 5.945421 | 0.417214 | 0.08498673 | 0.829075232 | 0.077157635 | -0.41969033 | -0.10367399 | -0.43921111 | 0.159020788 | -0.596206662 | 0.174067524 |
| 88 | 1010 | 6.917706 | 0.58062583 | 0.15019704 | -0.0917567 | 0.298467084 | 0.82065258 | 0.266516851 | -0.2879833 | 0.175571984 | 0.441724411 | 0.337126359 |
| 89 | 864 | 6.761573 | -0.2673409 | 0.2090253 | -0.46351299 | -0.03303241 | -0.04588564 | 0.102075264 | 0.740324585 | -1.05E-01 | 0.487699954 | 0.071471162 |
| 90 | 1020 | 6.927558 | -0.5392449 | -0.09624273 | 0.099053542 | -0.16686468 | -0.70892038 | 0.005639075 | 0.2835424 | 0.140284467 | -2.91E-01 | 0.290785049 |
| 91 | 1130 | 7.029973 | -0.0421333 | -0.19412816 | -0.31146168 | 0.035659275 | 0.499115816 | 0.094352578 | 0.015664098 | -0.03074826 | 0.389679076 | 0.001775215 |
| 92 | 511 | 6.23637 | -0.2997773 | -0.01516799 | -0.0464786 | -0.1121262 | -0.35025692 | -0.00293266 | 0.262090495 | -0.000178074 | -0.085411834 | 0.089866432 |
| 93 | 263 | 5.572154 | -0.0366886 | -0.10791983 | 0.081093062 | -0.0167323 | -0.02659414 | -0.00182305 | -0.00814628 | 0.03242272 | -0.00049465 | 0.001346054 |
| 94 | 152 | 5.023881 | 0.03982507 | -0.0132079 | -0.00274906 | 0.029193502 | 0.084975528 | 0.002588555 | -0.00506403 | 0.012740171 | 0.090063111 | 0.001586036 |
| 95 | 120 | 4.787492 | 0.05825904 | 0.01433703 | -0.05598899 | -0.00098966 | 0.098921343 | 0.044304542 | 0.007190432 | -0.026417868 | 0.035389365 | 0.003394116 |
| 96 | 115 | 4.744932 | -0.0198794 | 0.02097326 | 0.139179844 | -0.02015603 | -0.20018856 | 0.081872584 | 0.123068172 | 0.085610005 | -0.073382965 | 0.000395192 |
| 97 | 105 | 4.65396 | -0.4177017 | -0.00715659 | -0.46859727 | 0.050104744 | 0.108156897 | -0.08175617 | 0.227423845 | -0.179531343 | 0.237805571 | 0.174474721 |
| 98 | 131 | 4.875197 | -0.159113 | -0.15037262 | 0.224270545 | -0.16869502 | -0.40170595 | -0.29654614 | -0.22710048 | -0.166437895 | -0.498698176 | 0.025316947 |
| 99 | 415 | 6.028279 | 0.85962336 | -0.05728068 | 0.396353302 | 0.080737396 | 0.601288135 | 0.130745815 | -0.82373929 | -0.109130514 | -0.462327485 | 0.738952321 |
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| 101 | 865 | 6.76273 | 0.37986083 | -0.11010271 | -0.25397386 | 0.198574035 | 0.942511439 | 0.055821655 | -0.21484839 | 0.107447646 | 0.779289037 | 0.144294253 |
| 102 | 1180 | 7.07327 | -0.1395471 | 0.1367499 | -0.51228264 | -0.09143059 | 0.144555073 | 0.007085411 | 0.155060154 | 0.005935869 | 0.298465684 | 0.019473387 |
| 103 | 1310 | 7.177782 | 0.09625586 | -0.05023695 | -0.04002663 | -0.18442175 | 0.002097693 | 0.104609369 | 0.019681698 | 0.099318503 | 0.016488525 | 9.27E-03 |
| 104 | 611 | 6.415097 | -0.2048089 | 0.03465211 | -0.28478844 | -0.01440959 | 0.030917864 | 0.042527702 | 0.290581582 | -0.003087013 | 0.275884731 | 0.041946676 |
| 105 | 273 | 5.609472 | -0.1474711 | -0.0737312 | -0.03485418 | -0.10252384 | -0.14140961 | -0.01662107 | 0.118132505 | -0.001919002 | -0.008575035 | 0.021747737 |
| 106 | 157 | 5.056246 | -0.0076608 | -0.05308961 | 0.037833817 | -0.0125475 | -0.00495248 | -0.04306676 | -0.04616963 | 0.002724795 | -0.005330561 | 5.87E-05 |
| 107 | 122 | 4.804021 | 0.02313206 | -0.00275788 | 0.055346091 | 0.013620174 | -0.01583598 | -0.09639838 | -0.11962988 | 0.04663636 | 0.007568876 | 0.000535092 |
| 108 | 130 | 4.867534 | 0.07559051 | 0.00832754 | -0.01888545 | 0.019924593 | 0.10607302 | -0.20506403 | -0.26777327 | 0.086181668 | 0.129545445 | 0.005713926 |
| 109 | 394 | 5.976351 | 0.83698296 | 0.02721259 | -0.39681663 | -0.00679876 | 1.199788237 | 0.304713295 | -0.56962229 | -0.086059128 | 0.239393521 | 0.700540473 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 110 | 251 | 5.525453 | -0.3791244 | 0.30131387 | -0.15115735 | -0.14285399 | -0.67213494 | 0.101190172 | 0.846425818 | -0.312153836 | -0.239053134 | 0.143735344 |
| 111 | 245 | 5.501258 | -0.4427019 | -0.1364848 | 0.816642192 | -0.05441665 | -1.17727593 | 0.10852905 | 0.281083812 | 0.137627174 | -0.86709399 | 0.195984961 |
| 112 | 276 | 5.620401 | -0.5728424 | -0.15937268 | -0.29054881 | 0.293991189 | 0.171070266 | 0.008825912 | 0.301469583 | -0.081416233 | 0.382297705 | 3.28E-01 |
| 113 | 395 | 5.978886 | -0.1686516 | -0.20622327 | 0.360867792 | -0.10459757 | -0.4278937 | -0.11846144 | 0.024516421 | 0.058759637 | -0.226156203 | 0.028443363 |
| 114 | 790 | 6.672033 | 0.05941026 | -0.06071458 | -0.13256973 | 0.129912405 | 0.38260697 | -0.10221546 | -0.32905955 | 0.007458328 | 0.163221214 | 0.003529579 |
| 115 | 1060 | 6.966024 | 0.35003441 | 0.02138769 | 0.091443069 | -0.0477251 | 0.189478543 | -0.00505575 | -0.28393184 | 0.110115126 | 0.020717577 | 1.23E-01 |
| 116 | 650 | 6.476972 | 0.17215808 | 0.12601239 | -0.19456843 | 0.032919505 | 0.273633633 | -0.00151947 | -0.01404375 | 0.044766002 | 0.305875349 | 0.029638405 |
| 117 | 355 | 5.872118 | 0.19269455 | 0.06197691 | -0.14009758 | -0.07004464 | 0.200770591 | 0.054703979 | -0.00422075 | -0.017495861 | 0.124350005 | 0.037131191 |
| 118 | 183 | 5.209486 | 0.00312179 | 0.06937004 | -0.00727773 | -0.05043513 | -0.10940565 | 0.045816036 | 0.151955497 | -0.045333427 | -0.048599614 | 9.75E-06 |
| 119 | 127 | 4.844187 | -0.087355 | 0.00112384 | 0.021975455 | -0.00261998 | -0.11307431 | 0.038646673 | 0.127266767 | -0.101471975 | -0.125926187 | 0.0076309 |
| 120 | 125 | 4.828314 | -0.0469347 | -0.03144781 | 0.071810989 | 0.007911164 | -0.07938676 | 0.093974843 | 0.107351869 | -0.215856869 | -0.281866598 | 0.00220287 |
| 121 | 117 | 4.762174 | -0.4225709 | -0.01689651 | 0.795133811 | 0.025851956 | -1.17495626 | 0.006438221 | 0.261041232 | 0.320750836 | -0.599602414 | 0.178566176 |
| 122 | 132 | 4.882802 | -0.227016 | -0.15212553 | -0.36016822 | 0.286248172 | 0.571525958 | -0.19504867 | 0.017883946 | 0.106515971 | 0.890974546 | 0.051536248 |
| 123 | 285 | 5.652489 | 0.42125001 | -0.08172575 | -0.42056679 | -0.12966056 | 0.793881986 | 0.070443533 | -0.54180186 | 0.114241105 | 0.295877697 | 0.177451568 |
| 124 | 322 | 5.774552 | -0.2382265 | 0.15165 | -0.54420029 | -0.15140405 | 0.00291971 | -0.10944978 | 0.19567648 | 0.009290433 | 0.317336404 | 0.05675188 |
| 125 | 724 | 6.584791 | 0.40168648 | -0.08576155 | -0.16021902 | -0.1959121 | 0.451754948 | -0.00277523 | -0.30402717 | -0.124696249 | 0.025806759 | 0.161352027 |
| 126 | 1220 | 7.106606 | 0.08739329 | 0.14460713 | 0.056439749 | -0.05767885 | -0.17133244 | 0.059741845 | -0.00770897 | -0.107595225 | -0.34637847 | 0.007637588 |
| 127 | 1090 | 6.993933 | -0.0629884 | 0.03146159 | 0.332532687 | 0.02031831 | -0.4066644 | 0.052838945 | 0.165949568 | -0.005321844 | -0.298875625 | 0.003967544 |
| 128 | 535 | 6.282267 | -0.2014518 | -0.02267584 | 0.163550178 | 0.119711767 | -0.2226144 | -0.0626561 | 0.146774848 | -0.001599441 | -0.014782899 | 0.040582841 |
| 129 | 324 | 5.780744 | 0.15499044 | -0.07252266 | 0.183059825 | 0.058878064 | 0.103331342 | -0.00868735 | -0.17404471 | 0.057583136 | -0.004442891 | 0.024022038 |
| 130 | 192 | 5.257495 | 0.13224422 | 0.05579656 | 0.002965702 | 0.065901537 | 0.139383493 | 0.00352623 | -0.02413152 | 0.048227406 | 0.159953154 | 0.017488533 |
| 131 | 142 | 4.955827 | 0.02718375 | 0.04760792 | -0.08298727 | 0.001067653 | 0.063630752 | -0.01985847 | 0.009795083 | 0.040680708 | 0.133965018 | 7.39E-04 |
| 132 | 131 | 4.875197 | -0.0696828 | 0.00978615 | -0.044588 | -0.02987542 | -0.06475639 | -0.13399989 | -0.05516243 | 0.098920888 | 0.113001967 | 0.004855695 |
| 133 | 301 | 5.70711 | 0.48757624 | -0.02508582 | -0.40144237 | -0.01605168 | 0.898052744 | 0.257827645 | -0.37222193 | 0.006777074 | 0.274780244 | 0.237730592 |
| 134 | 166 | 5.111988 | -0.6113689 | 0.17552745 | -0.21566517 | -0.14451925 | -0.71575046 | -0.22370216 | 0.716187904 | -0.20531439 | 0.018825207 | 0.37377197 |
| 135 | 455 | 6.120297 | 0.49635652 | -0.22009282 | 0.400187506 | -0.07763946 | 0.238622373 | 0.261696329 | -0.62139488 | 0.074151087 | -0.570317749 | 2.46E-01 |
| 136 | 403 | 5.998937 | -0.4351176 | 0.17868835 | -0.2263152 | 0.144067502 | -0.24342322 | 0.162325488 | 0.726934247 | -0.115210295 | 0.205975242 | 0.189327309 |
| 137 | 391 | 5.968708 | -0.5340355 | -0.15664233 | 0.381602154 | -0.08147347 | -0.84046885 | -0.07245741 | 0.450904132 | -0.002921292 | -0.320028597 | 0.285193969 |
| 138 | 730 | 6.593045 | -0.1440837 | -0.1922528 | 0.08302363 | 0.137376776 | 0.102522229 | -0.02774751 | -0.20127059 | 0.062886152 | -0.008114701 | 0.020760117 |
| 139 | 754 | 6.625392 | 0.00342333 | -0.05187014 | -0.05983902 | 0.029888507 | 0.145020996 | -0.05111922 | -0.07707641 | 0.055619942 | 0.174683756 | 1.17E-05 |
| 140 | 522 | 6.257668 | 0.17533665 | 0.0012324 | -0.19137924 | -0.02154205 | 0.343941448 | -0.01851003 | -0.14199785 | -0.065953786 | 0.15449984 | 0.030742942 |
| 141 | 288 | 5.66296 | 0.18607477 | 0.0631212 | 0.147240921 | -0.06889653 | -0.09318388 | 0.029459773 | -0.05141674 | -0.009144574 | -0.183204962 | 0.034623819 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 142 | 155 | 5.043425 | 0.04332497 | 0.06698692 | 0.125632007 | 0.053006732 | -0.09628722 | 0.014658899 | 0.081832702 | 0.003711821 | -0.025401595 | 0.001877053 |
| 143 | 114 | 4.736198 | -0.0093643 | 0.01559699 | 0.025824558 | 0.045227522 | -0.00555835 | 0.003946539 | 0.040719164 | -0.020903658 | 0.010310613 | 8.77E-05 |
| 144 | 110 | 4.70048 | -0.033955 | -0.00337116 | -0.06619868 | 0.009296841 | 0.044911652 | -0.02711254 | 0.01096261 | -0.141052521 | -0.058065716 | 0.001152944 |
| 145 | 134 | 4.89784 | -0.1597484 | -0.01222381 | 0.463197431 | -0.02383152 | -0.63455351 | -0.04665605 | -0.07531262 | 0.271397521 | -0.391812557 | 0.025519539 |
| 146 | 215 | 5.370638 | 0.26285974 | -0.05750941 | -0.58080048 | 0.166751075 | 1.067920705 | -0.05103739 | -0.12960013 | -0.235475955 | 0.753882004 | 0.069095241 |
| 147 | 232 | 5.446737 | -0.1569539 | 0.0946295 | 0.471538697 | -0.20908817 | -0.93221029 | -0.14441111 | -0.14177052 | 0.27546982 | -0.654099874 | 0.02463453 |
| 148 | 533 | 6.278521 | 0.31352587 | -0.05650341 | -0.4133617 | 0.169753931 | 0.95314491 | -0.04232208 | -0.40114198 | 0.170868934 | 0.765193944 | 0.098298471 |
| 149 | 1020 | 6.927558 | 0.43361124 | 0.11286931 | -0.50733377 | -0.14881021 | 0.679265484 | 0.010797265 | -0.11756133 | -0.07627096 | 0.474635929 | 0.188018705 |
| 150 | 1440 | 7.272398 | -0.0776358 | 0.15610005 | -0.13687953 | -0.18264016 | -0.27949649 | -0.06684821 | 0.029992402 | -0.029207902 | -0.211863778 | 0.00602732 |
| 151 | 1770 | 7.478735 | 0.1985685 | -0.02794889 | 0.003252167 | -0.04927663 | 0.173988599 | 0.015622475 | -0.18568947 | -0.05380971 | -0.08113306 | 0.039429451 |
| 152 | 1040 | 6.946976 | 0.07284965 | 0.07148466 | 0.166569822 | 0.00117078 | -0.16403405 | 0.009348892 | 0.043395764 | -0.019484238 | -0.149471418 | 0.005307071 |
| 153 | 551 | 6.311735 | 0.10249769 | 0.02622587 | 0.176771028 | 0.059965136 | -0.04053408 | 0.070568238 | 0.025969144 | 0.031010287 | -0.054122882 | 0.010505777 |
| 154 | 296 | 5.690359 | 0.01258033 | 0.03689917 | 0.041158723 | 0.06363757 | -0.00183999 | 0.123473626 | 0.196022884 | 0.01543042 | 0.086139687 | 0.000158265 |
| 155 | 155 | 5.043425 | -0.358892 | 0.00452892 | -0.00889611 | 0.01481714 | -0.33970767 | -0.0354334 | 0.342982294 | 0.004154252 | 0.042862278 | 0.12880347 |
| 156 | 202 | 5.308268 | 0.14230487 | -0.12920112 | -0.03225728 | -0.0032026 | 0.300560663 | 0.162055439 | -0.09842612 | -0.028539519 | 0.011539589 | 0.020250675 |
| 157 | 161 | 5.081404 | -0.5131413 | 0.05122975 | -0.15176094 | -0.01161262 | -0.42422277 | 0.056096043 | 0.450153998 | -0.04911163 | -0.079276442 | 0.263314034 |
| 158 | 155 | 5.043425 | -0.3911577 | -0.18473088 | 0.249716749 | -0.05463394 | -0.51077748 | -0.27225751 | 0.155822342 | -0.053723565 | -0.136421193 | 0.153004323 |
| 159 | 411 | 6.018593 | 0.51924775 | -0.14081676 | -0.14910621 | 0.08989803 | 0.899068754 | 0.140018332 | -0.75627085 | -0.152011697 | -0.149232126 | 0.269618224 |
| 160 | 472 | 6.156979 | -0.1549413 | 0.18692919 | 0.297849576 | -0.05367824 | -0.69339828 | 0.073246688 | 0.388939811 | -0.044549557 | -0.422254715 | 0.0240068 |
| 161 | 776 | 6.654153 | 0.09706302 | -0.05577886 | 0.411930675 | 0.107225847 | -0.15186295 | 0.186714386 | 0.203463023 | 0.011365542 | -0.123748769 | 0.00942123 |
| 162 | 790 | 6.672033 | -0.5140664 | 0.03494269 | -0.07375402 | 0.148295043 | -0.32696006 | 0.089753526 | 0.518651073 | -0.070366537 | 0.031570949 | 0.264264304 |
| 163 | 620 | 6.429719 | -0.4185223 | -0.18506392 | 0.188640078 | -0.02655145 | -0.4486499 | 0.012572764 | 0.24931535 | 0.016444711 | -0.195462604 | 1.75E-01 |
| 164 | 355 | 5.872118 | -0.1752141 | -0.15066803 | 0.069207167 | 0.067910428 | -0.02584286 | -0.02675732 | 0.034924345 | 0.009840939 | 0.045679752 | 0.030699995 |
| 165 | 184 | 5.214936 | -0.0125597 | -0.06307709 | 0.097372807 | 0.02491458 | -0.02194085 | -0.04932033 | -0.0743259 | 0.074282356 | 0.027335941 | 1.58E-04 |
| 166 | 117 | 4.762174 | 0.14098914 | -0.0045215 | 0.011951314 | 0.03505421 | 0.168613532 | -0.04475503 | -0.13700092 | 0.129972238 | 0.206339878 | 0.019877937 |
| 167 | 96 | 4.564348 | 0.15461481 | 0.05075609 | -0.3409474 | 0.004302473 | 0.449108594 | -0.07354324 | -0.12431952 | -0.037298317 | 0.361033993 | 0.023905739 |
| 168 | 104 | 4.644391 | 0.12879215 | 0.05566133 | 0.135189624 | -0.12274106 | -0.18479987 | -0.11489553 | -0.20428677 | 0.170584673 | -0.103606437 | 1.66E-02 |
| 169 | 205 | 5.32301 | 0.41569505 | 0.04636517 | -0.48748427 | 0.048668265 | 0.905482412 | 0.17153031 | -0.31915425 | 0.059048466 | 0.473846314 | 0.172802373 |
| 170 | 205 | 5.32301 | -0.008476 | 0.14965022 | -0.37159979 | -0.17549434 | 0.037979248 | 0.063841965 | 0.476473084 | -0.286586849 | 0.164023518 | 7.18E-05 |
| 171 | 180 | 5.192957 | -0.4812113 | -3.05E-03 | 0.493285361 | -0.13377592 | -1.10522123 | 0.015579864 | 0.177338793 | 0.147387718 | -0.796074579 | 0.231564311 |
| 172 | 264 | 5.575949 | -0.2534065 | -0.17323607 | -0.14719421 | 0.17758273 | 0.244606481 | -0.04442467 | 0.043277399 | 0.077101777 | 0.409410328 | 6.42E-02 |
| 173 | 415 | 6.028279 | 0.00912932 | -9.12E-02 | 0.092209869 | -0.05298992 | -0.04484412 | -0.18587612 | -0.12340186 | 0.196541459 | 0.214171604 | 8.33E-05 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 174 | 1010 | 6.917706 | 0.35327455 | 0.00328655 | -0.48836312 | 0.033195553 | 0.871546664 | -0.09624701 | -0.51632257 | 0.094477396 | 0.545948498 | 0.124802906 |
| 175 | 1300 | 7.17012 | 0.40012078 | 0.12717884 | -0.39759618 | -0.17581072 | 0.494727401 | -0.02182811 | -0.26735279 | 0.013234489 | 0.26243721 | 1.60E-01 |
| 176 | 911 | 6.814543 | 0.32274971 | 0.14404348 | -0.16645343 | -0.14313462 | 0.202025042 | 0.076463324 | -0.06063365 | -0.028165603 | 0.036762469 | 0.104167378 |
| 177 | 401 | 5.993961 | 0.00078197 | 0.1161899 | -0.01193173 | -0.05992324 | -0.16339944 | 0.075320333 | 0.212398123 | -0.051916138 | -0.078237786 | 6.11E-07 |
| 178 | 194 | 5.267858 | -0.1348248 | 0.00028151 | 0.133939682 | -0.00429542 | -0.27334145 | 0.032982641 | 0.209223148 | -0.047110555 | -0.144211493 | 0.018177736 |
| 179 | 136 | 4.912655 | -0.1072487 | -0.04853694 | 0.146884068 | 0.048218285 | -0.15737753 | -0.01231036 | 0.091618447 | -0.077413934 | -0.130862653 | 0.011502281 |
| 180 | 144 | 4.969813 | 0.00798045 | -0.03860953 | 0.12235254 | 0.052878265 | -0.02288429 | 0.03701629 | -0.03419546 | -0.120942665 | -0.215038704 | 6.37E-05 |
| 181 | 161 | 5.081404 | -0.2132917 | 0.00287296 | 0.394910296 | 4.40E-02 | -0.56702802 | 0.052305081 | 0.102823028 | 0.180558221 | -0.335951847 | 0.045493337 |
| 182 | 224 | 5.411646 | 0.1032368 | -0.076785 | -0.00805218 | 0.142167706 | 0.330241687 | 0.041185033 | 0.145291892 | 0.067202069 | 0.501550614 | 0.010657836 |
| 183 | 201 | 5.303305 | -0.3953747 | 0.03716525 | -0.45715073 | -0.00289879 | 0.021711984 | -0.0341577 | 0.11440287 | 0.016399856 | 0.186672413 | 0.156321165 |
| 184 | 308 | 5.7301 | -0.1746942 | -0.1423349 | -0.2407362 | -1.65E-01 | 0.043802623 | -0.14339785 | -0.09488251 | -0.046762811 | 0.045555157 | 0.030518068 |
| 185 | 733 | 6.597146 | 0.44716447 | -0.06288992 | 0.008672851 | -0.08666503 | 0.414716502 | -0.04937325 | -0.39832737 | -0.195659078 | -0.129896697 | 0.199956063 |
| 186 | 1220 | 7.106606 | 0.11350115 | 0.16097921 | 0.33561082 | 0.003122227 | -0.37996665 | -0.07492977 | -0.13714792 | -0.101312637 | -0.543497438 | 1.29E-02 |
| 187 | 1550 | 7.34601 | 0.2871454 | 0.04086041 | 0.380114744 | 0.120819895 | -0.01300986 | 0.03729892 | -0.20813825 | -0.022976961 | -0.281423992 | 0.082452481 |
| 188 | 936 | 6.841615 | 0.12432518 | 0.10337234 | 0.306612228 | 0.136841308 | -0.14881809 | 0.099102623 | 0.10360811 | 0.08048771 | -0.063824891 | 0.01545675 |
| 189 | 385 | 5.953243 | -0.1326711 | 0.04475706 | 0.000742869 | 0.110380402 | -0.06779067 | 0.063201983 | 0.275285065 | 0.079284561 | 0.223576971 | 0.017601632 |
| 190 | 194 | 5.267858 | -0.1353945 | -0.04776161 | -0.12808359 | 0.000267433 | 0.040718093 | 0.030762835 | 0.175561065 | 0.034718569 | 0.220234892 | 0.018331682 |
| 191 | 140 | 4.941642 | -0.0755307 | -0.04874204 | -0.10188625 | -0.04611009 | 0.028987537 | 0.005041108 | 0.085452319 | -0.012958278 | 0.09644047 | 0.00570488 |
| 192 | 144 | 4.969813 | -0.0119181 | -0.02719104 | 0.007581431 | -0.03667905 | -0.02898754 | 0.059975273 | 0.014003078 | 0.038964516 | -0.035995216 | 0.000142041 |
| 193 | 153 | 5.030438 | -0.2606134 | -0.00429051 | -0.20262709 | 0.002729315 | -0.05096644 | 0.062454749 | 0.166597979 | 0.05505798 | 0.108234766 | 0.067919323 |
| 194 | 168 | 5.123964 | -0.1595157 | -0.09382081 | 0.098074956 | -0.07294575 | -0.23671563 | -0.17281638 | 0.173485415 | 0.043352666 | 0.152938833 | 0.025445269 |
| 195 | 322 | 5.774552 | 0.29059008 | -0.05742566 | -0.37560598 | 0.035306984 | 0.75892871 | 0.122503656 | -0.4800455 | -0.035955477 | 0.120424073 | 0.084442597 |
| 196 | 307 | 5.726848 | -0.4006276 | 0.10461243 | -0.1659595 | -0.13521815 | -0.47449867 | -0.18527952 | 0.340287934 | -0.150945109 | -0.099876325 | 0.160502469 |
| 197 | 910 | 6.813445 | 0.55987667 | -0.14422593 | 0.424806246 | -0.05974542 | 0.219550933 | 0.072205719 | -0.51466534 | -0.051971842 | -0.41929197 | 0.313461882 |
| 198 | 1060 | 6.966024 | -0.2004294 | 0.2015556 | 0.107826091 | 0.152930249 | -0.35688085 | -0.09081662 | 0.200571441 | -0.078873442 | -0.144366229 | 0.040171947 |
| 199 | 1440 | 7.272398 | 0.22878629 | -0.07215459 | 0.272788131 | 0.038817393 | 0.066970133 | 0.073056659 | -0.25226839 | 0.039262021 | -0.219092895 | 0.052343164 |
| 200 | 730 | 6.593045 | -0.0726909 | 0.08236306 | 0.118108919 | 0.098203727 | -0.17495913 | 0.023233422 | 0.202935165 | 0.104318551 | 0.109061169 | 0.005283963 |
| 201 | 370 | 5.913503 | 0.01410511 | -0.02616871 | -0.12603758 | 0.042519211 | 0.208830614 | 0.050122548 | 0.064537284 | 0.066528404 | 0.289773753 | 0.000198954 |
| 202 | 195 | 5.273 | -0.0332917 | 0.00507784 | -0.12862482 | -0.04537353 | 0.044881728 | 0.031691839 | 0.139229301 | 0.032381932 | 0.184801121 | 0.001108339 |
| 203 | 142 | 4.955827 | -0.028391 | -0.01198502 | -0.07175413 | -0.04630493 | 0.009043235 | 0.012432742 | 0.088032887 | 0.005306429 | 0.08994981 | 0.000806047 |
| 204 | 150 | 5.010635 | 0.03092591 | -0.01022075 | -0.01132219 | -0.02583149 | 0.02663736 | 0.109564538 | 0.034535393 | 0.063131866 | 0.014740082 | 0.000956412 |
| 205 | 147 | 4.990433 | -0.3132007 | 0.01113333 | -0.24758269 | -0.00407599 | -0.08082733 | 0.113894156 | 0.304345938 | 0.065741841 | 0.175366294 | 0.098094682 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 206 | 151 | 5.01728 | -0.2418412 | -0.11275225 | -0.15153994 | -0.08912977 | -0.06667881 | -0.11483436 | 0.316372655 | -0.18191198 | 0.182616226 | 0.058487184 |
| 207 | 241 | 5.484797 | 0.06048165 | -0.08706285 | 0.27606058 | -0.05455438 | -0.18307047 | 0.132207474 | -0.31898433 | 0.128951217 | -0.505311056 | 0.003658029 |
| 208 | 297 | 5.693732 | -0.2015656 | 0.02177339 | -0.38059621 | 0.099381809 | 0.256639003 | 0.070653086 | 0.367242985 | -0.195031077 | 0.358197825 | 0.040628702 |
| 209 | 405 | 6.003887 | -0.1801081 | -0.07256363 | 0.531882833 | -0.13701464 | -0.77644192 | 0.037575658 | 0.196258571 | 0.07600602 | -0.541752991 | 0.03243892 |
| 210 | 556 | 6.320768 | -0.282423 | -0.06483891 | -0.19040794 | 0.19147782 | 0.16430164 | -0.03804581 | 0.104376827 | -0.095596442 | 0.211127833 | 0.079762764 |
| 211 | 564 | 6.335054 | -0.1078667 | -0.10167229 | 0.217346971 | -0.06854686 | -0.29208825 | -0.05532363 | -0.1056828 | 0.076901747 | -0.265545673 | 0.011635227 |
| 212 | 382 | 5.945421 | 0.10358696 | -0.03883202 | -0.06905633 | 0.07824491 | 0.289720216 | -0.05311626 | -0.15367674 | 0.024456234 | 0.213615963 | 0.010730259 |
| 213 | 216 | 5.375278 | 0.18495076 | 0.03729131 | 0.013399851 | -0.02486028 | 0.109399328 | -0.05331924 | -0.14754516 | 0.052760577 | 0.067933983 | 0.034206785 |
| 214 | 135 | 4.905275 | 0.20063101 | 0.06658227 | -0.03162713 | 0.004823946 | 0.170499818 | -0.0908065 | -0.14810899 | 0.033359831 | 0.146557159 | 0.040252804 |
| 215 | 117 | 4.762174 | 0.23071316 | 0.07222716 | -0.02697143 | -0.01138577 | 0.174071657 | -0.15774771 | -0.25224027 | 0.013087096 | 0.092666197 | 0.053228564 |
| 216 | 195 | 5.273 | 0.57816345 | 0.08305674 | 0.029379611 | -0.00970971 | 0.456017387 | 0.09680735 | -0.43818808 | 0.115331092 | 0.036353046 | 0.334272976 |
| 217 | 195 | 5.273 | -0.0797758 | 0.20813884 | -0.29754067 | 0.01057666 | 0.020202707 | 0.088636452 | 0.268909305 | 0.119888585 | 0.320364145 | 0.006364175 |
| 218 | 237 | 5.46806 | 0.01685952 | -0.02871928 | -0.22974918 | -0.10711464 | 0.168213333 | -0.03947642 | 0.246212367 | -0.120878272 | 0.333023848 | 2.84E-04 |
| 219 | 217 | 5.379897 | -0.4094432 | 0.00606943 | 0.057457563 | -0.0827097 | -0.55567988 | -0.19039787 | -0.10965672 | 0.139165763 | -0.335772979 | 0.167643727 |
| 220 | 707 | 6.561031 | 0.61262649 | -0.14739955 | -0.19148735 | 0.020684723 | 0.972198107 | 0.131115252 | -0.52888296 | 0.074371669 | 0.386571563 | 0.375311215 |
| 221 | 813 | 6.700731 | -0.0520762 | 0.22054554 | -0.17110268 | -0.06893544 | -0.17045449 | 0.026719902 | 0.364209032 | 0.039553324 | 0.20658797 | 0.002711929 |
| 222 | 1200 | 7.090077 | -0.1529878 | -0.01874742 | -0.26830187 | -0.06159696 | 0.072464499 | -0.00323211 | 0.07422195 | -0.040048219 | 0.109870344 | 0.023405278 |
| 223 | 1250 | 7.130899 | -0.0344243 | -0.05507562 | -0.10247337 | -0.09658867 | 0.026536037 | 0.070567596 | -0.0089781 | -0.058235398 | -0.111245052 | 0.001185031 |
| 224 | 635 | 6.453625 | -0.1647349 | -0.01239274 | 0.098407616 | -0.03689041 | -0.28764019 | 0.014234055 | 0.1960211 | -0.055911849 | -0.161764993 | 0.027137587 |
| 225 | 330 | 5.799093 | -0.0034182 | -0.05930456 | 0.175703225 | 0.035426742 | -0.08439014 | 0.054334078 | 0.03953904 | -0.056125511 | -0.155310692 | 1.17E-05 |
| 226 | 171 | 5.141664 | -0.0613097 | -0.00123056 | 0.190599463 | 0.063253161 | -0.18742547 | 0.023820936 | 0.150927994 | -0.095585785 | -0.155904196 | 0.003758883 |
| 227 | 127 | 4.844187 | -0.0258854 | -0.0220715 | 0.219177505 | 0.068615807 | -0.15437563 | 0.01125949 | 0.066169266 | -0.166050221 | -0.26551607 | 0.000670055 |
| 228 | 128 | 4.85203 | -0.0419498 | -0.00931875 | 0.549255278 | 0.078903902 | -0.50298245 | 0.091447002 | 0.03127636 | 0.101902473 | -0.461250614 | 0.001759788 |
| 229 | 127 | 4.844187 | -0.296464 | -0.01510194 | -0.07578699 | 0.1977319 | -0.00784318 | 0.056415374 | 0.254019449 | 0.093301528 | 0.283062426 | 0.087890907 |
| 230 | 145 | 4.976734 | -0.1259411 | -0.10672704 | 0.016016541 | -0.02728332 | -0.06251393 | -0.20652959 | 0.156709373 | -0.041554127 | 0.259170912 | 0.015861163 |
| 231 | 246 | 5.505332 | 0.17668479 | -0.0453388 | -0.38897103 | 0.005765955 | 0.616760581 | -0.04192341 | -0.57369331 | -0.200418807 | -0.11542813 | 0.031217516 |
| 232 | 443 | 6.09357 | 0.19273618 | 0.06360653 | 0.581995165 | -0.14002957 | -0.59289508 | -0.01461403 | -0.11645391 | 0.138016054 | -0.556718907 | 0.037147237 |
| 233 | 755 | 6.626718 | 0.20384193 | 0.06938503 | -0.04947237 | 0.209518259 | 0.393447536 | -0.00239871 | -0.04059453 | 0.028126213 | 0.383377929 | 0.041551533 |
| 234 | 1240 | 7.122867 | 0.05265789 | 0.0733831 | -0.14533844 | -0.01781005 | 0.106803183 | 0.018609504 | -0.00666309 | -0.003402226 | 0.078128368 | 0.002772853 |
| 235 | 1140 | 7.038784 | -0.0863295 | 0.01895684 | -0.03270307 | -0.05232184 | -0.12490511 | 0.010520262 | 0.051693067 | 0.07428168 | -0.009450627 | 0.007452783 |
| 236 | 675 | 6.514713 | -0.0226007 | -0.03107862 | -0.15649816 | -0.0117731 | 0.153202981 | -0.00892885 | 0.02922295 | 0.014983215 | 0.206338 | 0.000510791 |
| 237 | 361 | 5.888878 | 0.07365339 | -0.00813625 | -0.00324731 | -0.05633934 | 0.028697612 | 0.019468966 | -0.02480237 | 0.057193766 | 0.041620043 | 0.005424821 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 238 | 207 | 5.332719 | 0.07070994 | 0.02651522 | -0.05824424 | -0.00116903 | 0.101269933 | 0.021553491 | 0.054080462 | 0.025074669 | 0.158871573 | 0.004999896 |
| 239 | 155 | 5.043425 | 0.03001514 | 0.02545558 | -0.02459116 | -0.02096793 | 0.008182794 | 0.010253838 | 0.059870809 | 0.011852094 | 0.069651859 | 0.000900909 |
| 240 | 141 | 4.94876 | -0.1227025 | 0.01080545 | -0.03985233 | -0.00885282 | -0.1025084 | -0.010688 | 0.028482884 | 0.096260002 | 0.032922484 | 0.015055896 |
| 241 | 212 | 5.356586 | 0.10420271 | -0.04417289 | -0.28164081 | -0.01434684 | 0.415669562 | 0.177976378 | -0.0296889 | 0.059384605 | 0.267388893 | 0.010858204 |
| 242 | 167 | 5.117994 | -0.3518795 | 0.03751297 | -0.11964406 | -0.10139069 | -0.37113912 | -0.2591171 | 0.494378828 | -0.217399572 | 0.164957235 | 0.123819189 |
| 243 | 315 | 5.752573 | 0.19022682 | -0.12667662 | 0.167850554 | -0.04307186 | 0.105981033 | -0.05403089 | -0.71976971 | -0.044129904 | -0.603887699 | 0.036186245 |
| 244 | 538 | 6.287859 | 0.13820252 | 0.06848166 | 0.183099375 | 0.0604262 | -0.05295231 | -0.09583823 | -0.15008579 | -0.01538319 | -0.122583068 | 0.019099936 |
| 245 | 1240 | 7.122867 | 0.47934709 | 0.04975291 | 0.193649835 | 0.065915775 | 0.301860119 | 0.07584894 | -0.26621731 | -0.002524959 | -0.042731084 | 0.229773629 |
| 246 | 1670 | 7.420579 | -0.0455607 | 0.17256495 | 0.050024994 | 0.069713941 | -0.19843666 | 0.038857562 | 0.210691499 | 0.019588952 | -0.007013774 | 0.002075774 |
| 247 | 1420 | 7.258412 | -0.1945075 | -0.01640184 | -0.08201303 | 0.018008998 | -0.07808364 | -0.01348576 | 0.107937671 | 0.01107396 | 0.054413755 | 0.037833167 |
| 248 | 908 | 6.811244 | 0.01493441 | -0.0700227 | -0.02147066 | -0.02952469 | 0.076903079 | -0.00071716 | -0.03746045 | -0.009398793 | 0.030761 | 2.23E-04 |
| 249 | 480 | 6.173786 | 0.071453 | 0.00537639 | 0.069970717 | -0.00772944 | -0.01162354 | 0.032985746 | -0.00199212 | 0.020493649 | -0.026107757 | 0.005105531 |
| 250 | 267 | 5.587249 | 0.03732979 | 0.02572308 | 0.067174446 | 0.025189458 | -0.03037828 | 0.027009876 | 0.091627073 | 0.022687886 | 0.056926802 | 0.001393513 |
| 251 | 210 | 5.347108 | 0.06692286 | 0.01343872 | 0.028514385 | 0.024182801 | 0.049152549 | 0.071951593 | 0.075027434 | 0.010793514 | 0.063021905 | 0.004478669 |
| 252 | 187 | 5.231109 | -0.124074 | 0.02409223 | -0.11656735 | 0.010265179 | -0.02133369 | 0.137299337 | 0.199865536 | -0.011250529 | 0.029981983 | 0.015394353 |
| 253 | 188 | 5.236442 | -0.3062029 | -0.04466663 | 0.098992572 | -0.04196424 | -0.40249304 | 0.197489033 | 0.381387046 | 0.187343556 | -0.031251469 | 0.093760189 |
| 254 | 196 | 5.278115 | -0.1998907 | -0.11023303 | -0.33428553 | 0.035637326 | 0.280265159 | 0.035692201 | 0.548580646 | -0.272754839 | 0.520398766 | 4.00E-02 |
| 255 | 187 | 5.231109 | -0.4524873 | -0.07196066 | 0.180715483 | -0.12034279 | -0.68158487 | 0.118337846 | 0.099145002 | -0.056874617 | -0.75765233 | 0.204744717 |
| 256 | 225 | 5.4161 | -0.4469547 | -0.16289541 | 0.131292393 | 0.065057574 | -0.35029414 | 0.035524801 | 0.328716238 | -0.100882347 | -0.157985046 | 0.19976853 |
| 257 | 303 | 5.713733 | -0.2901649 | -0.1609037 | 0.455379732 | 0.047265262 | -0.5373757 | -0.07862596 | 0.098680003 | 0.079840989 | -0.280228742 | 0.084195685 |
| 258 | 592 | 6.383507 | 0.06038288 | -0.10445937 | -0.04328263 | 0.163936704 | 0.372061583 | -0.02722169 | -0.21840545 | 0.040902696 | 0.221780526 | 0.003646092 |
| 259 | 621 | 6.431331 | 0.06252866 | 0.02173784 | -0.18478212 | -0.01558175 | 0.209991202 | 0.006561251 | -0.07561581 | -0.014195538 | 0.113618601 | 0.003909833 |
| 260 | 362 | 5.891644 | 0.01070048 | 0.02251032 | 0.014187692 | -0.06652156 | -0.0925191 | -0.03561626 | 0.018225698 | -0.000754908 | -0.039432051 | 0.0001145 |
| 261 | 193 | 5.26269 | 0.0751292 | 0.00385217 | 0.067880349 | 0.005107569 | 0.008504252 | -0.05361099 | -0.09893405 | 0.034721838 | -0.002096966 | 0.005644397 |
| 262 | 130 | 4.867534 | 0.22945459 | 0.02704651 | 0.035463299 | 0.024436926 | 0.191381707 | -0.02555581 | -0.14891942 | 0.028431449 | 0.09644955 | 0.05264941 |
| 263 | 107 | 4.672829 | 0.17884909 | 0.08260365 | 0.063576714 | 0.012766787 | 0.045435512 | -0.02879058 | -0.07098837 | 0.075738519 | 0.078976247 | 0.031986998 |
| 264 | 102 | 4.624973 | -0.0082293 | 0.06438567 | -0.11787028 | 0.022887617 | 0.068142893 | -0.0776901 | -0.07997384 | 0.144525618 | 0.210384774 | 6.77E-05 |
| 265 | 145 | 4.976734 | 0.09500561 | -0.00296256 | -0.29089271 | -0.0424333 | 0.346427583 | -0.06295511 | -0.21580584 | 0.207883192 | 0.401460049 | 0.009026066 |
| 266 | 325 | 5.783825 | 0.71444595 | 0.03420202 | -0.18989619 | -0.10472138 | 0.765418744 | 0.050660861 | -0.17487531 | 0.037570737 | 0.577453312 | 0.510433014 |
| 267 | 251 | 5.525453 | -0.3156659 | 0.25720054 | -0.42986289 | -0.06836263 | -0.2113662 | -0.05043859 | 0.140724615 | 0.124566153 | 0.10436316 | 0.099644975 |
| 268 | 503 | 6.22059 | 0.12664936 | -0.11363973 | -0.42460699 | -0.15475064 | 0.510145446 | 0.061415681 | -0.1401072 | 0.037394527 | 0.346017092 | 0.016040061 |
| 269 | 681 | 6.523562 | -0.0718647 | 0.04559377 | -0.27565668 | -0.15285852 | 0.005339733 | -0.01069901 | 0.170599113 | -0.082764169 | 0.103873687 | 0.005164529 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 270 | 1180 | 7.07327 | 0.01066245 | -0.02587128 | 0.057363736 | -0.09923641 | -0.12006642 | 0.051460165 | -0.02971947 | -0.028654413 | -0.229900469 | 0.000113688 |
| 271 | 894 | 6.795706 | -0.2827986 | 0.00383848 | 0.059402227 | 0.020650945 | -0.32538839 | -0.09594132 | 0.142944903 | 0.00690658 | -0.07959559 | 0.079975063 |
| 272 | 726 | 6.58755 | 0.21850426 | -0.10180751 | 0.010165453 | 0.021384802 | 0.33153111 | 0.008351715 | -0.26650365 | -0.037490796 | 0.019184945 | 0.04774411 |
| 273 | 390 | 5.966147 | 0.15392546 | 0.07866153 | 0.071372744 | 0.003659563 | 0.007550747 | 0.078458435 | 0.023199208 | -0.056432621 | -0.104141101 | 0.023693047 |
| 274 | 198 | 5.288267 | -0.0350231 | 0.05541317 | 0.217981863 | 0.025694188 | -0.28272397 | 0.065072555 | 0.217940098 | -0.026900854 | -0.156757281 | 0.00122662 |
| 275 | 144 | 4.969813 | -0.0449233 | -0.01260833 | 0.169906638 | 0.078473471 | -0.12374812 | 0.101427702 | 0.180757097 | -0.030305875 | -0.074724595 | 0.002018101 |
| 276 | 110 | 4.70048 | -0.3066335 | -0.01617238 | -0.00781787 | 0.06116639 | -0.22147691 | 0.062670636 | 0.281743616 | -0.081779054 | -0.084182987 | 0.094024133 |
| 277 | 117 | 4.762174 | -0.3073857 | -0.11038808 | 0.090255331 | -0.00281443 | -0.29006736 | 0.044913242 | 0.1740851 | -0.066268538 | -0.227164039 | 0.094485953 |
| 278 | 171 | 5.141664 | 0.10797107 | -0.11065884 | 0.678723652 | 0.032491919 | -0.42760182 | -0.06543632 | 0.124759005 | 0.053327223 | -0.184079272 | 0.011657752 |
| 279 | 177 | 5.17615 | -0.2124951 | 0.03886959 | -0.29988263 | 0.244340515 | 0.292858419 | -0.09013356 | -0.18176755 | -0.053093255 | 0.148131174 | 0.045154183 |
| 280 | 335 | 5.814131 | 0.09461996 | -0.07649825 | 0.120316895 | -0.10795775 | -0.05715643 | -0.09539809 | -0.250371 | 0.064648085 | -0.147481264 | 0.008952937 |
| 281 | 818 | 6.706862 | 0.51223735 | 0.03406319 | -0.06827143 | 0.043314082 | 0.589759669 | 0.133924854 | -0.26499469 | -0.011262116 | 0.179578014 | 0.262387098 |
| 282 | 776 | 6.654153 | -0.3833047 | 0.18440544 | 0.010129324 | -0.02457771 | -0.60241723 | -0.14495149 | 0.372013483 | 0.054168595 | -0.031283657 | 0.146922529 |
| 283 | 1110 | 7.012115 | 0.22523176 | -0.13798971 | -0.2686587 | 0.003646556 | 0.635526716 | -0.0185755 | -0.40264304 | -0.100990858 | 0.150468319 | 0.050729344 |
| 284 | 720 | 6.579251 | 0.16067128 | 0.08108343 | 0.207579043 | -0.09671713 | -0.22470832 | 0.013014507 | -0.05159861 | 0.008791279 | -0.280530161 | 0.025815261 |
| 285 | 366 | 5.902633 | 0.07412779 | 0.05784166 | 0.146229187 | 0.074728455 | -0.0552146 | 0.039104412 | 0.036151408 | 0.082587827 | 0.024420219 | 0.005494929 |
| 286 | 203 | 5.313206 | 0.02922388 | 0.026686 | -0.03327197 | 0.052642507 | 0.088452354 | 0.036162518 | 0.108623367 | 0.068497426 | 0.229410629 | 0.000854035 |
| 287 | 148 | 4.997212 | -0.0177186 | 0.0105206 | -0.04267711 | -0.01197791 | 0.002460026 | 0.019406839 | 0.100451439 | 0.106766002 | 0.190270628 | 0.000313948 |
| 288 | 142 | 4.955827 | -0.0543691 | -0.00637869 | -0.29130187 | -0.01536376 | 0.227947718 | 0.051252466 | 0.053907885 | 0.065969091 | 0.296572228 | 0.002955997 |
| 289 | 175 | 5.164786 | -0.0594552 | -0.01957287 | -0.29201639 | -0.10486867 | 0.147265347 | 0.15366293 | 0.14236796 | 0.047277097 | 0.183247474 | 0.003534925 |
| 290 | 192 | 5.257495 | -0.1004857 | -0.02140389 | 0.102572518 | -0.1051259 | -0.28678022 | -0.06014436 | 0.426841472 | -0.068880335 | 0.131325269 | 1.01E-02 |
| 291 | 237 | 5.46806 | -0.0988927 | -0.03617485 | -0.20187038 | 0.036926107 | 0.176078593 | 0.10546777 | -0.16706765 | -0.094877433 | -0.191334265 | 0.009779775 |
| 292 | 294 | 5.68358 | -0.2955003 | -0.03560139 | 0.089888961 | -0.07267334 | -0.42246117 | 0.033634241 | 0.292966029 | -0.100419039 | -0.263548424 | 0.087320405 |
| 293 | 453 | 6.115892 | -0.1125341 | -0.10638009 | 0.486625478 | 0.032360026 | -0.46041945 | 0.052924305 | 0.093428448 | 0.14097353 | -0.278941774 | 0.012663921 |
| 294 | 646 | 6.4708 | -0.1722198 | -0.04051227 | -0.36413951 | 0.175185172 | 0.407617195 | 0.010455495 | 0.147011959 | -0.15258052 | 0.39159314 | 2.97E-02 |
| 295 | 596 | 6.390241 | -0.1554603 | -0.06199911 | 0.213970168 | -0.13109022 | -0.43852161 | -0.00519695 | 0.029043041 | -0.019553158 | -0.423834778 | 0.024167915 |
| 296 | 355 | 5.872118 | -0.0656161 | -0.05596572 | 0.152637719 | 0.07702926 | -0.0852588 | -0.03168096 | -0.01443597 | 0.013699481 | -0.054314328 | 0.004305467 |
| 297 | 190 | 5.247024 | 0.0433742 | -0.02362178 | 0.070421401 | 0.054949579 | 0.051524161 | -0.03337008 | -0.08800267 | 0.041162539 | 0.038054113 | 0.001881321 |
| 298 | 132 | 4.882802 | 0.2432309 | 0.01561471 | 0.027762685 | 0.025351704 | 0.225205205 | 0.056235961 | -0.09269467 | 0.038065808 | 0.114340386 | 0.05916127 |
| 299 | 105 | 4.65396 | 0.14788804 | 0.08756312 | -0.01683265 | 0.009994567 | 0.087152133 | 0.15805303 | 0.156211003 | 0.020428251 | 0.105738357 | 0.021870873 |
| 300 | 71 | 4.26268 | -0.3422464 | 0.05323969 | -0.05165063 | -0.00605975 | -0.34989526 | 0.086345761 | 0.439036196 | 0.053949964 | 0.056745142 | 0.117132623 |
| 301 | 66 | 4.189655 | -0.443081 | -0.12320872 | -0.05648248 | -0.01859423 | -0.28198405 | -0.03024527 | 0.239849336 | 0.161750453 | 0.149861011 | 0.19632079 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 302 | 153 | 5.030438 | 0.5134369 | -0.15950917 | -0.09546141 | -0.02033369 | 0.748073781 | 0.151442472 | -0.08401465 | -0.063309848 | 0.449306813 | 0.263617449 |
| 303 | 100 | 4.60517 | -0.5105772 | 0.18483728 | -0.09394811 | -0.03436611 | -0.63583251 | 0.071720422 | 0.420673533 | 0.111018706 | -0.175860689 | 2.61E-01 |
| 304 | 98 | 4.584967 | -0.6664341 | -0.1838078 | -0.28072525 | -0.03382132 | -0.23572233 | -0.30947977 | 0.199223396 | 0.035404464 | 0.308385293 | 0.444134363 |
| 305 | 293 | 5.680173 | 0.41713021 | -0.23991626 | -0.10690738 | -0.10106109 | 0.662892772 | -0.23940919 | -0.85966602 | 0.055709795 | 0.098345735 | 0.173997615 |
| 306 | 1010 | 6.917706 | 0.90767039 | 0.15016688 | -0.16360877 | -0.03848666 | 0.882625623 | 0.073856456 | -0.66502552 | 0.011005784 | 0.154749431 | 0.823865533 |
| 307 | 806 | 6.692084 | 0.09291015 | 0.32676134 | -0.14768732 | -0.05889916 | -0.14506303 | 0.024051694 | 0.205156821 | -0.005470474 | 0.030571622 | 0.008632296 |
| 308 | 462 | 6.135565 | -0.0141161 | 0.03344765 | -0.06233525 | -0.05316743 | -0.03839597 | 0.01026167 | 0.066810261 | -0.033348379 | -0.015195762 | 0.000199265 |
| 309 | 226 | 5.420535 | -0.0313718 | -0.00508181 | 0.041205492 | -0.02244069 | -0.08993617 | -0.00392355 | 0.028504639 | -0.0351264 | -0.092634386 | 0.00098419 |
| 310 | 136 | 4.912655 | 0.06128356 | -0.01129385 | 0.231069354 | 0.014833977 | -0.14365796 | 0.00221237 | -0.01089875 | 0.059195749 | -0.097573333 | 0.003755675 |
| 311 | 113 | 4.727388 | 0.12294526 | 0.02206208 | 0.140493638 | 0.083184967 | 0.043574505 | 0.051658953 | 0.006145472 | 0.166371611 | 0.164432635 | 0.015115537 |
| 312 | 106 | 4.663439 | -0.0041198 | 0.04426029 | -0.32513411 | 0.05057771 | 0.327331749 | 0.09957575 | 0.143497091 | 0.090890275 | 0.462143364 | 1.70E-05 |
| 313 | 112 | 4.718499 | -0.1772769 | -0.00148312 | -0.42092697 | -0.11704828 | 0.128084912 | 0.120374104 | 0.276599307 | -0.03183713 | 0.252472985 | 0.031427098 |
| 314 | 115 | 4.744932 | -0.2388708 | -0.06381968 | 0.487765054 | -0.15153371 | -0.81434992 | -0.23212781 | 0.33437251 | 0.159413128 | -0.088436472 | 0.057059279 |
| 315 | 214 | 5.365976 | 0.29967434 | -0.0859935 | -0.48504836 | 0.175595419 | 1.046311622 | 0.034193081 | -0.64479948 | 0.075495181 | 0.442814245 | 0.08980471 |
| 316 | 343 | 5.83773 | 0.14134495 | 0.10788276 | -0.63311236 | -0.17461741 | 0.491957139 | 0.051460906 | 0.094980781 | -0.325768176 | 0.209708837 | 0.019978395 |
| 317 | 451 | 6.111467 | -0.1463899 | 0.05088418 | 0.396273703 | -0.22792045 | -0.82146824 | -0.02561935 | 0.142946962 | -0.252009671 | -0.9049116 | 0.021430004 |
| 318 | 760 | 6.633318 | -0.0487539 | -0.05270037 | 0.862286869 | 0.142658533 | -0.71568191 | -0.00907626 | -0.07116485 | 0.077743637 | -0.700026864 | 0.002376946 |
| 319 | 847 | 6.741701 | 0.09429408 | -0.01755142 | 0.088264642 | 0.310423273 | 0.334004129 | 0.118155325 | -0.02521183 | 0.025317573 | 0.215954548 | 0.008891374 |
| 320 | 386 | 5.955837 | -0.2405842 | 0.03394587 | -0.01341033 | 0.031775271 | -0.22934447 | 0.03933993 | 0.328209236 | 0.010801758 | 0.070326591 | 0.057880761 |
| 321 | 164 | 5.099866 | -0.2525269 | -0.08661032 | -0.02980321 | -0.00482772 | -0.14094105 | -0.0657984 | 0.109277582 | -0.004130052 | 0.030004883 | 0.063769813 |
| 322 | 115 | 4.744932 | 0.13098469 | -0.09090967 | 0.058219386 | -0.01072916 | 0.152945814 | -0.01602635 | -0.18277334 | 0.00232881 | -0.011472366 | 0.017156988 |
| 323 | 95 | 4.553877 | 0.13720533 | 0.04715449 | 0.116797996 | 0.020958979 | -0.00578817 | -0.00239688 | -0.04451764 | 0.054377845 | 0.006468917 | 0.018825304 |
| 324 | 84 | 4.430817 | -0.0556785 | 0.04939392 | -0.00391379 | 0.042047279 | -0.05911137 | -0.11200237 | -0.00665801 | 0.104816579 | 0.151049569 | 0.003100098 |
| 325 | 191 | 5.252273 | 0.5793485 | -0.02004427 | -0.16841305 | -0.00140897 | 0.766396852 | 0.290831578 | -0.31111769 | 0.126709583 | 0.291157165 | 0.335644683 |
| 326 | 145 | 4.976734 | -0.2597061 | 0.20856546 | -0.2269273 | -0.0606287 | -0.30197294 | -0.09042358 | 0.807865494 | -0.244345065 | 0.351971063 | 0.06744725 |
| 327 | 175 | 5.164786 | -0.1601014 | -0.09349419 | 0.284690623 | -0.08169383 | -0.43299166 | 0.03056075 | -0.2511766 | 0.035992717 | -0.678736292 | 0.025632457 |
| 328 | 277 | 5.624018 | -0.0383703 | -0.0576365 | 0.134277702 | 0.102488624 | -0.0125229 | 0.026557679 | 0.084890973 | 0.054169375 | 0.099979769 | 0.001472282 |
| 329 | 347 | 5.849325 | -0.2496533 | -0.01381332 | -0.13907041 | 0.048339973 | -0.04842962 | -0.15209651 | 0.073771331 | -0.026967734 | 0.150470487 | 0.062326778 |
| 330 | 834 | 6.726233 | 0.26893144 | -0.08987519 | -0.04631624 | -0.05006535 | 0.355057529 | -0.00207636 | -0.4224903 | -0.009553956 | -0.074910371 | 7.23E-02 |
| 331 | 876 | 6.775366 | 0.14381897 | 0.09681532 | 0.089579377 | -0.01667385 | -0.05924957 | 0.085895563 | -0.00576766 | 0.124374026 | -0.026538767 | 0.020683896 |
| 332 | 462 | 6.135565 | -0.0629666 | 0.05177483 | -0.228555 | 0.032248576 | 0.146062125 | 0.080587956 | 0.238598786 | 0.041410452 | 0.345483407 | 0.003964795 |
| 333 | 192 | 5.257495 | -0.2023873 | -0.02266798 | -0.23990051 | -0.0822798 | -0.02209858 | 0.017466347 | 0.223855434 | -0.069261477 | 0.115029034 | 0.040960609 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 334 | 102 | 4.624973 | -0.139648 | -0.07285942 | 0.124435453 | -0.08636418 | -0.27758826 | -0.05354748 | 0.048517629 | -0.016869843 | -0.19239299 | 1.95E-02 |
| 335 | 91 | 4.51086 | 0.11221694 | -0.05027329 | 0.130345068 | 0.044796763 | 0.07694193 | -0.02746344 | -0.14874301 | -0.002523036 | -0.046860675 | 0.012592642 |
| 336 | 76 | 4.330733 | -0.1164868 | 0.0403981 | -0.05289459 | 0.046924225 | -0.05706607 | -0.2442422 | -0.07628733 | -0.117897231 | -0.007008433 | 0.013569173 |
| 337 | 191 | 5.252273 | 0.62757134 | -0.04193525 | 0.550381074 | -0.01904205 | 0.100083459 | 0.055263725 | -0.67845054 | 0.306138503 | -0.327492307 | 0.393845788 |
| 338 | 308 | 5.7301 | 0.53443376 | 0.22592568 | -0.24672078 | 0.198137187 | 0.753366041 | -0.03869105 | 0.153510348 | -0.095182713 | 0.850384731 | 0.28561944 |
| 339 | 332 | 5.805135 | 0.01610226 | 0.19239615 | -0.15209632 | -0.08881948 | -0.11301705 | 0.076073439 | -0.10747515 | 0.032169211 | -0.264396425 | 0.000259283 |
| 340 | 516 | 6.246107 | 0.00583995 | 0.00579681 | -0.03645181 | -0.05475468 | -0.01825974 | 0.131651905 | 0.211315108 | 0.027955452 | 0.089358919 | 3.41E-05 |
| 341 | 618 | 6.426488 | -0.2668712 | 0.00210238 | -0.23717065 | -0.01312265 | -0.04492558 | 0.083018534 | 0.365699736 | -0.160101588 | 0.077654033 | 0.071220238 |
| 342 | 745 | 6.613384 | -0.4452202 | -0.09607363 | 0.25548487 | -0.08538143 | -0.69001286 | -0.01686483 | 0.230607039 | -0.002185638 | -0.444726632 | 0.198221017 |
| 343 | 723 | 6.583409 | -0.1947335 | -0.16027927 | 0.136628021 | 0.091974553 | -0.07910769 | -0.02946683 | -0.04684675 | 0.090416382 | -0.006071218 | 0.03792113 |
| 344 | 501 | 6.216606 | 0.09388965 | -0.07010405 | -0.05981829 | 0.049186088 | 0.272998079 | 0.024818581 | -0.08185231 | 0.084829428 | 0.251156617 | 8.82E-03 |
| 345 | 255 | 5.541264 | 0.06579391 | 0.03380027 | -0.19226791 | -0.02153458 | 0.202726963 | 0.054415794 | 0.068940502 | 0.018385628 | 0.235637299 | 0.004328839 |
| 346 | 135 | 4.905275 | -0.0432296 | 0.02368581 | -0.13266564 | -0.06921645 | -0.00346621 | 0.040251816 | 0.151154984 | -0.056365771 | 0.051071189 | 0.001868797 |
| 347 | 93 | 4.532599 | -0.1197589 | -0.01556265 | 0.106606094 | -0.04775963 | -0.25856198 | -0.01908867 | 0.1118106 | -0.028908881 | -0.156571586 | 0.014342196 |
| 348 | 87 | 4.465908 | -0.0787191 | -0.04311321 | -0.11066245 | 0.038378194 | 0.113434792 | -0.1163839 | -0.0530241 | -0.257097048 | -0.080302447 | 6.20E-03 |
| 349 | 115 | 4.744932 | -0.0348237 | -0.02833886 | 0.596192774 | -0.03983848 | -0.64251608 | -0.19347389 | -0.32328862 | 0.058172343 | -0.714158467 | 0.001212689 |
| 350 | 310 | 5.736572 | 0.79435996 | -0.01253653 | 0.507712069 | 0.214629399 | 0.513813814 | -0.22593093 | -0.53742748 | -0.040727426 | 0.16158984 | 0.631007743 |
| 351 | 628 | 6.44254 | 0.74942307 | 0.28596959 | 0.015297149 | 0.182776345 | 0.630932683 | 0.196555803 | -0.62758592 | 0.080077304 | -0.113131738 | 0.561634942 |
| 352 | 675 | 6.514713 | -0.098966 | 0.26979231 | 0.00554795 | 0.005506974 | -0.36879927 | 0.093333066 | 0.545988341 | 0.138580953 | 0.222436956 | 0.009794267 |
| 353 | 721 | 6.580639 | -0.4056079 | -0.03562776 | -0.25352764 | 0.001997262 | -0.11445525 | -0.15275589 | 0.259258516 | 0.08738793 | 0.384947091 | 0.164517773 |
| 354 | 1720 | 7.45008 | 0.2048366 | -0.14601885 | -0.42295918 | -0.09126995 | 0.682544672 | -0.00227395 | -0.42432192 | -0.017752451 | 0.242744251 | 0.041958032 |
| 355 | 1610 | 7.383989 | 0.00489455 | 0.07374118 | -0.18499681 | -0.1522653 | -0.03611512 | -0.02413699 | -0.00631652 | -0.031017717 | -0.049312365 | 2.40E-05 |
| 356 | 1050 | 6.956545 | 0.09691516 | 0.00176204 | 0.089195165 | -0.06659885 | -0.06064089 | -0.01540294 | -0.06704719 | 0.026124822 | -0.086160326 | 0.009392549 |
| 357 | 607 | 6.408529 | 0.19260932 | 0.03488946 | 0.062504214 | 0.03211026 | 0.127325904 | 0.069250796 | -0.04278594 | 0.057279783 | 0.072568949 | 0.037098349 |
| 358 | 324 | 5.780744 | 0.01397322 | 0.06933935 | -0.04106811 | 0.022501517 | 0.008203491 | 0.083826637 | 0.192363322 | 0.042370333 | 0.159110509 | 0.000195251 |
| 359 | 204 | 5.31812 | -0.1839043 | 0.00503036 | -0.11377096 | -0.01478452 | -0.08994824 | 0.005114822 | 0.232851769 | -0.020093342 | 0.117695368 | 3.38E-02 |
| 360 | 232 | 5.446737 | 0.09527763 | -0.06620556 | -0.07478311 | -0.04095755 | 0.195308752 | 0.142822055 | 0.014207839 | -0.122509374 | -0.055814838 | 0.009077828 |
| 361 | 225 | 5.4161 | -0.2815216 | 0.03429995 | -0.0330825 | -0.02692192 | -0.30966098 | 0.223714039 | 0.396727931 | -0.203656729 | -0.340303816 | 0.079254417 |
| 362 | 220 | 5.393628 | -0.3489091 | -0.10134778 | 0.75464196 | -0.0119097 | -1.01411303 | -0.06479404 | 0.621427885 | -0.237822034 | -0.565713137 | 0.121737591 |
| 363 | 194 | 5.267858 | -0.5170637 | -0.12560729 | 0.711951919 | 0.271671106 | -0.83173726 | -0.14420309 | -0.17998344 | 0.206900845 | -0.66061676 | 0.267354906 |
| 364 | 452 | 6.113682 | 0.23718817 | -0.18614294 | -0.09401769 | 0.256302691 | 0.773651496 | -0.10339187 | -0.40056413 | 0.098245332 | 0.57472457 | 0.056258228 |
| 365 | 1240 | 7.122867 | 0.67716463 | 0.08538774 | -0.38532751 | -0.03384637 | 0.943258032 | 0.222359019 | -0.28719964 | -0.160795674 | 0.272903701 | 0.458551939 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 366 | 1030 | 6.937314 | -0.4779011 | 0.24377927 | 0.194594767 | -0.1387179 | -1.05499301 | 0.006931952 | 0.617663942 | -0.002393629 | -0.446654649 | 0.228389434 |
| 367 | 946 | 6.852243 | -0.2564301 | -0.17204439 | 0.004649825 | 0.070054116 | -0.0189814 | -0.01848436 | 0.019255423 | -0.025407357 | -0.006648969 | 0.065756384 |
| 368 | 663 | 6.496775 | 0.07005708 | -0.09231483 | 0.092069404 | 0.001673937 | 0.071976436 | 0.07499335 | -0.05134546 | -0.01621362 | -0.070575992 | 0.004907994 |
| 369 | 275 | 5.616771 | -0.1569328 | 0.02522055 | 0.182978852 | 0.033144986 | -0.33198724 | -0.00573897 | 0.208314861 | 0.072895575 | -0.045037834 | 0.024627912 |
| 370 | 165 | 5.105945 | 0.007866 | -0.05649582 | 0.013274556 | 0.065872387 | 0.116959652 | -0.01323108 | -0.01594159 | 0.088238565 | 0.202487708 | 6.19E-05 |
| 371 | 141 | 4.94876 | 0.12878175 | 0.00283176 | -0.17470911 | 0.00477884 | 0.305437938 | 0.028961848 | -0.03675299 | 0.005384023 | 0.245107125 | 0.01658474 |
| 372 | 143 | 4.962845 | 0.08523782 | 0.04636143 | 0.090513753 | -0.06289528 | -0.11453264 | 0.101300324 | 0.080449577 | 0.150339006 | 0.01495562 | 0.007265487 |
| 373 | 152 | 5.023881 | -0.177672 | 0.03068562 | -0.26744553 | 0.032584951 | 0.09167286 | 0.190942763 | 0.28138979 | 0.235488462 | 0.417608349 | 0.031567341 |
| 374 | 157 | 5.056246 | -0.2443071 | -0.06396192 | -0.33146369 | -0.09628039 | 0.05483814 | -0.13710416 | 0.530396563 | -0.068204249 | 0.654134616 | 0.059685948 |
| 375 | 193 | 5.26269 | -0.1276204 | -0.08795055 | -0.49121055 | -0.11932693 | 0.332213771 | -0.0109676 | -0.3808449 | -0.151792722 | -0.189456248 | 0.016286966 |
| 376 | 341 | 5.831882 | 0.07958948 | -0.04594334 | 0.225328762 | -0.1768358 | -0.27663173 | 0.005715618 | -0.03046555 | -0.108833547 | -0.421646451 | 0.006334486 |
| 377 | 544 | 6.298949 | 0.04872255 | 0.02865221 | 0.643306401 | 0.081118354 | -0.54211771 | 0.010136541 | 0.015876716 | 0.234062125 | -0.302315408 | 0.002373887 |
| 378 | 812 | 6.6995 | -0.0819525 | 0.01754012 | -0.45400602 | 0.231590304 | 0.586103671 | -0.02861505 | 0.028157059 | 0.007296792 | 0.65017257 | 6.72E-03 |
| 379 | 855 | 6.751101 | 0.02700332 | -0.02950291 | -0.24360857 | -0.16344217 | 0.136672641 | 0.017460302 | -0.07948625 | -0.019457226 | 0.020268866 | 0.000729179 |
| 380 | 538 | 6.287859 | 0.05619917 | 0.0097212 | 0.066554222 | -0.08769909 | -0.10777533 | 0.073713727 | 0.048500838 | 0.078940368 | -0.054047851 | 0.003158347 |
| 381 | 218 | 5.384495 | -0.1761736 | 0.0202317 | -0.14908619 | 0.02395952 | -0.02335961 | -0.04391907 | 0.204760353 | -0.006041022 | 0.219278801 | 3.10E-02 |
| 382 | 140 | 4.941642 | 0.06569422 | -0.0634225 | 0.007472704 | -0.05367103 | 0.067972984 | -0.05117128 | -0.12199743 | -0.013927448 | -0.016780616 | 0.00431573 |
| 383 | 112 | 4.718499 | 0.07734444 | 0.02364992 | 0.122342666 | 0.002690173 | -0.06595797 | -0.13892689 | -0.14214244 | 0.030486155 | -0.038687356 | 0.005982163 |
| 384 | 207 | 5.332719 | 0.66491175 | 0.027844 | 0.080975933 | 0.04404336 | 0.600135182 | 0.236175303 | -0.38590803 | 0.10663192 | 0.084683765 | 0.442107641 |
| 385 | 153 | 5.030438 | -0.3218883 | 0.23936823 | -0.1687884 | 0.029151336 | -0.36331676 | 0.19751835 | 0.656042509 | 0.200992382 | 0.296199779 | 0.103612059 |
| 386 | 155 | 5.043425 | -0.3065858 | -0.11587978 | -0.23209172 | -0.06076383 | -0.01937809 | -0.17334835 | 0.548662084 | -0.144320171 | 0.558312171 | 0.093994831 |
| 387 | 217 | 5.379897 | -0.0180294 | -0.11037088 | -0.12123938 | -0.08355302 | 0.130027853 | 0.037849188 | -0.48152319 | -0.011544841 | -0.400889364 | 0.000325059 |
| 388 | 350 | 5.857933 | 0.02160912 | -0.00649058 | 0.07561001 | -0.04364618 | -0.09115649 | 0.052065591 | 0.105136634 | 0.00601644 | -0.032069004 | 0.000466954 |
| 389 | 527 | 6.267201 | -0.0309533 | 0.00777928 | 0.046286424 | 0.027219604 | -0.05779938 | 0.080784975 | 0.14462664 | 0.010670043 | 0.016712333 | 0.000958105 |
| 390 | 680 | 6.522093 | -0.25132 | -0.01114318 | -0.07785491 | 0.016663113 | -0.14565884 | 0.01898375 | 0.224402707 | -0.030121104 | 0.029639009 | 6.32E-02 |
| 391 | 611 | 6.415097 | -0.1953913 | -0.09047521 | 0.025653157 | -0.02802777 | -0.15859697 | -0.00381533 | 0.052732639 | 0.018379265 | -0.083669733 | 0.038177744 |
| 392 | 374 | 5.924256 | -0.053785 | -0.07034085 | 0.053389215 | 0.009235136 | -0.02759825 | -0.01165651 | -0.01059814 | 0.077593397 | 0.051053514 | 2.89E-03 |
| 393 | 185 | 5.220356 | -0.0064841 | -0.01936261 | -0.16736493 | 0.019220118 | 0.199463525 | -0.09468349 | -0.0323792 | -0.046230605 | 0.215537214 | 4.20E-05 |
| 394 | 135 | 4.905275 | 0.24809819 | -0.00233429 | 0.062409505 | -0.06025137 | 0.127771594 | -0.06068427 | -0.2630097 | -0.053864502 | -0.128418346 | 0.06155271 |
| 395 | 125 | 4.828314 | 0.28650766 | 0.08931535 | 0.07347722 | 0.022467422 | 0.14618251 | -0.01900011 | -0.16856741 | -0.146238834 | -0.149623617 | 0.082086637 |
| 396 | 145 | 4.976734 | 0.24255721 | 0.10314276 | 0.631666167 | 0.026451799 | -0.46579992 | 0.136246554 | -0.05277809 | 0.248605582 | -0.406218983 | 0.058833998 |
| 397 | 143 | 4.962845 | -0.1574813 | 0.08732059 | -0.30579386 | 0.22739982 | 0.28839176 | 0.184197402 | 0.378462651 | 0.207914053 | 0.690571062 | 0.024800367 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 398 | 156 | 5.049856 | -0.1638398 | -0.05669328 | -0.29125648 | -0.11008579 | 0.074024181 | -0.17432735 | 0.511659449 | -0.182471945 | 0.577539035 | 0.026843475 |
| 399 | 225 | 5.4161 | 0.05851426 | -0.05898232 | -0.01712791 | -0.10485233 | 0.029772158 | 0.092237283 | -0.48424264 | 0.039841251 | -0.506866513 | 0.003423918 |
| 400 | 280 | 5.63479 | -0.2115868 | 0.02106513 | 0.020528666 | -0.00616605 | -0.2593466 | -0.05899618 | 0.256214676 | 0.054805885 | 0.110670141 | 0.044768954 |
| 401 | 492 | 6.198479 | 0.04145456 | -0.07617123 | -0.02940561 | 0.00739032 | 0.154421719 | -0.07665831 | -0.16387828 | 0.085036815 | 0.152238569 | 0.001718481 |
| 402 | 784 | 6.664409 | -0.0022063 | 0.01492364 | -0.23875404 | -0.01058602 | 0.211038054 | -0.21813218 | -0.21293976 | 0.019982895 | 0.236213376 | 4.87E-06 |
| 403 | 1540 | 7.339538 | 0.68165999 | -0.00079428 | -0.1856217 | -0.08595145 | 0.782124514 | 0.116677606 | -0.60592273 | -0.004016138 | 0.055508041 | 0.464660348 |
| 404 | 702 | 6.553933 | -0.0336375 | 0.2453976 | -0.05109578 | -0.06682381 | -0.29476313 | 0.028227258 | 0.324104461 | -0.012270013 | -0.011155939 | 1.13E-03 |
| 405 | 344 | 5.840642 | -0.0092667 | -0.0121095 | -0.00615992 | -0.01839448 | -0.00939177 | 0.003433808 | 0.07840905 | -0.099666835 | -0.034083368 | 8.59E-05 |
| 406 | 194 | 5.267858 | -0.0231276 | -0.00333602 | 0.235693277 | -0.00221757 | -0.25770245 | -0.03518995 | 0.009538357 | -0.063878175 | -0.27685232 | 0.000534887 |
| 407 | 165 | 5.105945 | 0.09405511 | -0.00832594 | 0.272182272 | 0.08484958 | -0.08495164 | -0.02526225 | -0.09774986 | -0.02000012 | -0.177439376 | 0.008846363 |
| 408 | 166 | 5.111988 | 0.02392587 | 0.03385984 | 0.230429346 | 0.097985618 | -0.14237769 | -0.01357729 | -0.07017292 | 0.143417426 | -0.055555889 | 0.000572447 |
| 409 | 245 | 5.501258 | 0.17921103 | 0.00861331 | -0.14960726 | 0.082954565 | 0.403159534 | 0.160955098 | -0.0377147 | 0.193892002 | 0.398381738 | 3.21E-02 |
| 410 | 361 | 5.888878 | 0.26333516 | 0.06451597 | -0.15564779 | -0.05385861 | 0.300608371 | 0.025614497 | 0.447097494 | -0.183502473 | 0.538588894 | 0.069345406 |
| 411 | 295 | 5.686975 | -0.3617246 | 0.09480066 | 0.055588544 | -0.05603321 | -0.568147 | 0.109825355 | 0.071151381 | 0.097091877 | -0.509729093 | 0.130844679 |
| 412 | 325 | 5.783825 | -0.4730795 | -0.13022085 | -0.20100742 | 0.020011876 | -0.12183938 | -0.14856985 | 0.305070429 | -0.062101242 | 0.269699659 | 0.223804231 |
| 413 | 761 | 6.634633 | 0.22855494 | -0.17030863 | 0.039381832 | -0.07236267 | 0.287119062 | -0.03376447 | -0.41269402 | -0.08069296 | -0.17250345 | 0.052237359 |
| 414 | 1360 | 7.21524 | 0.18068263 | 0.08227978 | -0.00209601 | 0.014177459 | 0.114676317 | 0.015420419 | -0.09379019 | -0.229612824 | -0.224147111 | 0.032646212 |
| 415 | 1180 | 7.07327 | -0.1037216 | 0.06504575 | 0.647576995 | -0.00075456 | -0.81709894 | -0.01363251 | 0.042834496 | 0.122818533 | -0.637813399 | 0.010758177 |
| 416 | 793 | 6.675823 | 0.08573467 | -0.03733979 | -0.03195562 | 0.233127718 | 0.388157796 | 0.03884003 | -0.03786808 | 0.029712903 | 0.341162591 | 0.007350433 |
| 417 | 375 | 5.926926 | -0.0020403 | 0.03086448 | -0.00880338 | -0.01150402 | -0.03560545 | -0.00663778 | 0.107888972 | 0.003614535 | 0.082535842 | 4.16E-06 |
| 418 | 220 | 5.393628 | 0.01994848 | -0.00073452 | -0.02197124 | -0.00316922 | 0.039485019 | -0.0260357 | -0.01843829 | -0.037042053 | 0.010040376 | 0.000397942 |
| 419 | 175 | 5.164786 | 0.03751456 | 0.00718145 | 0.08935235 | -0.00790965 | -0.06692889 | -0.06294752 | -0.07232138 | -0.026591842 | -0.102894591 | 0.001407342 |
| 420 | 168 | 5.123964 | -0.0427963 | 0.01350524 | 0.022729581 | 0.032166846 | -0.04686431 | -0.16214418 | -0.17485421 | -0.014291886 | -0.073866227 | 0.001831526 |
| 421 | 307 | 5.726848 | 0.36027449 | -0.01540668 | 0.170250477 | 0.008182649 | 0.213613346 | -0.02766104 | -0.45040049 | 0.169426419 | -0.039699684 | 0.129797711 |
| 422 | 835 | 6.727432 | 0.93154128 | 0.12969882 | 0.250168401 | 0.061290172 | 0.612964229 | 0.092461687 | -0.07683623 | 0.026962629 | 0.470628941 | 0.867769149 |
| 423 | 604 | 6.403574 | -0.2202991 | 0.33535486 | -0.34363836 | 0.090060624 | -0.12195493 | 0.17559254 | 0.25683802 | 0.115605636 | 0.074896191 | 0.048531671 |
| 424 | 554 | 6.317165 | -0.5882827 | -0.07930766 | -0.44942554 | -0.12370981 | -0.18325934 | -0.17301836 | 0.487757056 | -0.156389312 | 0.321126768 | 0.346076569 |
| 425 | 1650 | 7.408531 | 0.40769631 | -0.21178178 | 0.21712719 | -0.1617932 | 0.240557705 | 0.15882436 | -0.48060656 | -0.035541544 | -0.434414757 | 0.16621628 |
| 426 | 1780 | 7.484369 | -0.2645152 | 0.14677067 | 0.171648496 | 0.078165788 | -0.50476854 | 0.051368765 | 0.441178778 | 0.01623202 | -0.098726511 | 0.069968273 |
| 427 | 1630 | 7.396335 | -0.2016176 | -0.09522546 | -0.09853555 | 0.061793458 | 0.053936912 | 0.137188973 | 0.142691013 | -0.014350009 | 0.045088943 | 0.040649639 |
| 428 | 697 | 6.546785 | -0.407765 | -0.07258232 | 0.081447934 | -0.0354728 | -0.45210339 | 0.00972247 | 0.38108048 | 0.040884242 | -0.039861135 | 0.166272275 |
| 429 | 352 | 5.863631 | -0.112312 | -0.14679539 | -0.00193831 | 0.029321256 | 0.065742961 | -0.02780466 | 0.027006861 | -0.00698714 | 0.113567339 | 0.012613984 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 430 | 227 | 5.42495 | 0.07383385 | -0.04043232 | 0.018951055 | -0.00069779 | 0.094617322 | 0.009384889 | -0.07723516 | -0.027405996 | -0.019408723 | 0.005451437 |
| 431 | 180 | 5.192957 | 0.05224505 | 0.02658019 | 0.035638834 | 0.00682238 | -0.00315159 | 0.032784766 | 0.026069137 | -0.066260542 | -0.076127765 | 0.002729545 |
| 432 | 175 | 5.164786 | -0.0220272 | 0.01880822 | -0.04065651 | 0.01282998 | 0.012651118 | 0.117098894 | 0.091068795 | -0.170678081 | -0.184057062 | 0.000485196 |
| 433 | 173 | 5.153292 | -0.2654108 | -0.00792978 | 0.342260769 | -0.01463635 | -0.61437815 | 0.155885451 | 0.325274705 | -0.029116887 | -0.474105781 | 0.070442899 |
| 434 | 236 | 5.463832 | -0.0238413 | -0.09554789 | 0.884964212 | 0.123213877 | -0.69004377 | -0.07882029 | 0.433015141 | 0.097328092 | -0.080880242 | 0.000568409 |
| 435 | 295 | 5.686975 | 0.01054699 | -0.00858288 | -0.2092841 | 0.318587116 | 0.547001078 | 0.242534263 | -0.21894526 | 0.184834253 | 0.27035581 | 0.000111239 |
| 436 | 298 | 5.697093 | -0.3832018 | 0.00379692 | -0.55886859 | -0.07534228 | 0.096527641 | 0.074680857 | 0.673706286 | -0.18212459 | 0.51342848 | 0.146843589 |
| 437 | 377 | 5.932245 | -0.4056626 | -0.13795263 | 0.387311493 | -0.20119269 | -0.85621418 | 0.024317822 | 0.207446825 | 0.167183537 | -0.50590164 | 0.164562167 |
| 438 | 595 | 6.388561 | -0.1562819 | -0.14603855 | -0.25128941 | 0.139432137 | 0.380478142 | 0.037701318 | 0.067549506 | 0.054072384 | 0.464398713 | 0.024424047 |
| 439 | 475 | 6.163315 | -0.2945472 | -0.0562615 | -0.19153668 | -0.09046419 | -0.13721325 | -0.03827899 | 0.104725884 | 0.144409445 | 0.150201066 | 0.08675808 |
| 440 | 305 | 5.720312 | -0.0179137 | -0.10603701 | -0.38737673 | -0.0689532 | 0.406546856 | -0.09068684 | -0.10633052 | 0.010234179 | 0.401137347 | 0.0003209 |
| 441 | 183 | 5.209486 | 0.19863891 | -0.00644892 | -0.1066964 | -0.13945562 | 0.172328611 | -0.1372756 | -0.25190788 | -0.029268061 | 0.028428275 | 0.039457418 |
| 442 | 151 | 5.01728 | 0.42653771 | 0.07151001 | 0.070142158 | -0.0384107 | 0.246474842 | -0.04366728 | -0.38132112 | 0.009878831 | -0.081300168 | 0.181934419 |
| 443 | 135 | 4.905275 | 0.2979233 | 0.15355358 | 0.049632794 | 0.025251177 | 0.119988108 | 0.005759203 | -0.12129799 | 0.03451028 | 0.027441197 | 0.088758293 |
| 444 | 142 | 4.955827 | 0.14718194 | 0.10725239 | -0.0209258 | 0.017867806 | 0.078723156 | 0.122121045 | 0.015997785 | 0.123261994 | 0.09586189 | 0.021662522 |
| 445 | 156 | 5.049856 | -0.0860982 | 0.0529855 | -0.25214027 | -0.00753329 | 0.105523329 | 0.266443975 | 0.339225124 | 0.164089948 | 0.342394426 | 0.007412892 |
| 446 | 186 | 5.225747 | -0.0975236 | -0.03099534 | -0.02264926 | -0.0907705 | -0.13464954 | 0.066698468 | 0.740122153 | -0.082968729 | 0.455805411 | 0.00951086 |
| 447 | 134 | 4.89784 | -0.5679856 | -0.03510851 | 0.010019638 | -0.00815373 | -0.55105043 | 0.119991014 | 0.185273523 | 0.255299224 | -0.230468692 | 0.322607602 |
| 448 | 140 | 4.941642 | -0.5384391 | -0.2044748 | -0.36404167 | 0.00360707 | 0.033684492 | -0.26356022 | 0.333308373 | 0.078611428 | 0.709164512 | 2.90E-01 |
| 449 | 356 | 5.874931 | 0.24997405 | -0.19383806 | -0.3853795 | -0.131055 | 0.698136607 | -0.22674248 | -0.73211172 | 0.025597708 | 0.218365079 | 0.062487028 |
| 450 | 1230 | 7.114769 | 0.86378193 | 0.08999066 | -0.14846785 | -0.13873662 | 0.783522499 | 0.122263128 | -0.62984023 | 0.039685598 | 0.071104743 | 0.746119216 |
| 451 | 843 | 6.736967 | -0.0679659 | 0.31096149 | -0.27981988 | -0.05344843 | -0.15255589 | 0.036532466 | 0.3396198 | -0.040293672 | 0.110237773 | 4.62E-03 |
| 452 | 500 | 6.214608 | -0.0201064 | -0.02446771 | -0.01701799 | -0.10073516 | -0.07935583 | 0.03859028 | 0.101479073 | -0.095459827 | -0.111926867 | 0.000404266 |
| 453 | 234 | 5.455321 | -0.0608662 | -0.00723829 | 0.188706968 | -0.00612648 | -0.24846136 | -0.02060058 | 0.107195224 | -0.144500634 | -0.265166187 | 0.003704695 |
| 454 | 134 | 4.89784 | -0.0499105 | -0.02191183 | 0.405210826 | 0.067934509 | -0.365275 | -0.06707374 | -0.05722384 | -0.045965554 | -0.40139065 | 0.00249106 |
| 455 | 125 | 4.828314 | 0.16166245 | -0.01796779 | 0.283027136 | 0.145875897 | 0.042478996 | -0.01009254 | -0.18631595 | 0.006062319 | -0.127682094 | 0.026134747 |
| 456 | 132 | 4.882802 | 0.10006746 | 0.05819848 | 0.139822839 | 0.101889769 | 0.003935906 | 0.087609764 | -0.02803484 | 0.128548468 | 0.016839774 | 0.010013496 |
| 457 | 154 | 5.036953 | -0.0359835 | 0.03602428 | -0.08179325 | 0.050336222 | 0.06012173 | 0.226870448 | 0.243360454 | 0.280467342 | 0.357079078 | 1.29E-03 |
| 458 | 187 | 5.231109 | -0.0578906 | -0.01295404 | -0.09264746 | -0.02944557 | 0.018265348 | -0.060406 | 0.630195689 | 0.070208914 | 0.77907595 | 0.00335132 |
| 459 | 168 | 5.123964 | -0.3063116 | -0.02084061 | -0.53958629 | -0.03335308 | 0.220762236 | -0.01575064 | -0.16779444 | 0.126306331 | 0.195024761 | 0.093826782 |
| 460 | 300 | 5.703782 | 0.10847767 | -0.11027217 | -0.5115171 | -0.19425106 | 0.536015873 | -0.13601862 | -0.04375177 | -0.277431809 | 0.350850919 | 0.011767404 |
| 461 | 574 | 6.352629 | 0.17623208 | 0.03905196 | 0.237475352 | -0.18414616 | -0.28444139 | -0.13030328 | -0.37782951 | -0.238676296 | -0.770643913 | 0.031057747 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 462 | 1210 | 7.098376 | 0.30445278 | 0.06344355 | 0.82059283 | 0.085491127 | -0.49409248 | -0.06435828 | -0.36195355 | 0.12869803 | -0.662989711 | 9.27E-02 |
| 463 | 1380 | 7.229839 | 0.25888765 | 0.109603 | -0.06456756 | 0.295413419 | 0.50926563 | 0.011453318 | -0.17877301 | 0.038455228 | 0.357494527 | 0.067022816 |
| 464 | 841 | 6.734592 | 0.12445456 | 0.09319955 | -0.01910105 | -0.02324432 | 0.027111741 | -0.00727222 | 0.031814773 | 0.040621348 | 0.106820077 | 0.015488939 |
| 465 | 447 | 6.102559 | 0.12111104 | 0.04480364 | -0.0578229 | -0.00687638 | 0.127253918 | -0.02746858 | -0.0202006 | -0.021684823 | 0.112837077 | 1.47E-02 |
| 466 | 290 | 5.669881 | 0.14180487 | 0.04359998 | -0.04741499 | -0.02081624 | 0.124803644 | 0.038133711 | -0.07630161 | -0.070603939 | -0.06023562 | 0.020108622 |
| 467 | 223 | 5.407172 | 0.02851539 | 0.05104975 | 0.153579326 | -0.0170694 | -0.19318309 | 0.098242211 | 0.105926976 | -0.010623728 | -0.196122052 | 0.000813127 |
| 468 | 178 | 5.181784 | -0.2298353 | 0.01026554 | 0.095064085 | 0.055288557 | -0.27987641 | 0.114749784 | 0.272895032 | 0.092220804 | -0.029510355 | 0.052824283 |
| 469 | 223 | 5.407172 | -0.0799105 | -0.08274072 | -0.03418428 | 0.034223071 | 0.071237541 | 0.37262904 | 0.3187494 | 0.238810998 | 0.256168899 | 0.006385693 |
| 470 | 182 | 5.204007 | -0.4687786 | -0.02876779 | -0.05499606 | -0.01230634 | -0.3973211 | -0.08918958 | 1.035080667 | -0.063585262 | 0.663363883 | 0.21975338 |
| 471 | 176 | 5.170484 | -0.3663358 | -0.1687603 | -0.290996 | -0.01979858 | 0.073621945 | -0.01408077 | -0.24774883 | -0.016579617 | -0.176625729 | 1.34E-01 |
| 472 | 383 | 5.948035 | 0.27366397 | -0.13188088 | 0.103053785 | -0.10475856 | 0.197732499 | 0.061496244 | -0.03911325 | -0.143177498 | -0.046054492 | 0.074891966 |
| 473 | 426 | 6.054439 | -0.3136024 | 0.09851903 | 0.167420477 | 0.037099362 | -0.54244257 | -0.11106574 | 0.170822901 | -0.137161346 | -0.397715272 | 0.098346479 |
| 474 | 855 | 6.751101 | 0.06697777 | -0.11289687 | 0.289230138 | 0.060271372 | -0.04908412 | -0.04434188 | -0.30851594 | -0.067745563 | -0.381003738 | 0.004486022 |
| 475 | 926 | 6.830874 | 0.11424204 | 0.024112 | 0.245943268 | 0.10412285 | -0.05169037 | 0.025375968 | -0.1231719 | 0.012056124 | -0.188182119 | 0.013051244 |
| 476 | 526 | 6.265301 | 0.00049349 | 0.04112714 | 0.118231836 | 0.088539577 | -0.0703259 | -0.0409813 | 0.070488801 | -0.007654964 | 0.033489234 | 2.44E-07 |
| 477 | 313 | 5.746203 | 0.18560473 | 0.00017766 | 0.11505549 | 0.042563461 | 0.112935043 | -0.00855241 | -0.11383695 | -0.028914296 | -0.021263788 | 0.034449116 |
| 478 | 188 | 5.236442 | 0.0830288 | 0.0668177 | 0.134714628 | 0.041419977 | -0.07708356 | 0.019617981 | -0.0237567 | 0.040140749 | -0.080317488 | 0.006893781 |
| 479 | 143 | 4.962845 | -0.0024055 | 0.02989037 | 0.027089618 | 0.048497266 | -0.01088818 | 0.035516985 | 0.054494392 | 0.103412854 | 0.11150208 | 5.79E-06 |
| 480 | 137 | 4.919981 | -0.0464373 | -0.00086597 | -0.21834357 | 0.009752263 | 0.182524517 | 0.114714127 | 0.098658291 | 0.120789246 | 0.287257928 | 0.002156421 |
| 481 | 155 | 5.043425 | -0.1159728 | -0.01671742 | -0.07591501 | -0.07860369 | -0.10194403 | 0.273421732 | 0.318650352 | 0.392241095 | 0.335525685 | 0.013449684 |
| 482 | 178 | 5.181784 | -0.118237 | -0.0417502 | -0.44533967 | -0.0273294 | 0.341523518 | -0.08241403 | 0.759504811 | -0.093883766 | 1.089558597 | 0.013979977 |
| 483 | 183 | 5.209486 | -0.1690367 | -0.0425653 | -0.34801898 | -0.16032228 | 0.061225295 | 0.078263799 | -0.22892787 | -0.014821863 | -0.260788238 | 0.028573408 |
| 484 | 221 | 5.398163 | -0.2644601 | -0.06085321 | 0.259980767 | -0.12528683 | -0.58887445 | -0.26557027 | 0.217399443 | 0.064732889 | -0.041171841 | 6.99E-02 |
| 485 | 741 | 6.608001 | 0.61671257 | -0.09520562 | -0.2979223 | 0.093593076 | 1.103433567 | 0.069013483 | -0.7376952 | -0.116911303 | 0.17981358 | 0.380334392 |
| 486 | 997 | 6.904751 | -0.0070145 | 0.22201652 | 0.063628886 | -0.10725203 | -0.39991198 | 0.069870095 | 0.19170412 | -0.046675668 | -0.32475362 | 4.92E-05 |
| 487 | 718 | 6.57647 | -0.3249557 | -0.00252523 | 0.108529941 | 0.022906399 | -0.40805397 | -0.05760419 | 0.194083596 | 0.026711546 | -0.129654632 | 0.10559618 |
| 488 | 505 | 6.224558 | 0.05807588 | -0.11698404 | 0.000468816 | 0.039070779 | 0.213661882 | -0.06368671 | -0.16001165 | -0.043138211 | 0.074198738 | 0.003372808 |
| 489 | 324 | 5.780744 | 0.27234615 | 0.02090732 | 0.176324493 | 0.000168774 | 0.075283109 | 0.009201399 | -0.17690754 | -0.009002539 | -0.119828365 | 7.42E-02 |
| 490 | 185 | 5.220356 | 0.06281869 | 0.09804461 | 0.078877358 | 0.063476817 | -0.05062646 | 0.020590537 | 0.025559441 | 0.020650506 | -0.025007052 | 0.003946188 |
| 491 | 143 | 4.962845 | 0.00801983 | 0.02261473 | -0.00228519 | 0.028395849 | 0.016086138 | 0.053305855 | 0.057195935 | 0.0373863 | 0.057362518 | 6.43E-05 |
| 492 | 134 | 4.89784 | -0.0625467 | 0.00288714 | -0.04411542 | -0.00082267 | -0.02214113 | 0.142831573 | 0.14807182 | 0.120751712 | 0.103850833 | 0.003912095 |
| 493 | 146 | 4.983607 | -0.1544868 | -0.02251683 | -0.11017413 | -0.01588155 | -0.03767737 | 0.311467928 | 0.39675437 | 0.287812349 | 0.335421423 | 0.023866165 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 494 | 152 | 5.023881 | -0.2263622 | -0.05561524 | -0.1123251 | -0.03966269 | -0.09808453 | -0.11912621 | 0.865188689 | -0.086751614 | 0.799478749 | 0.051239841 |
| 495 | 155 | 5.043425 | -0.2097962 | -0.08149039 | -0.16058487 | -0.04043704 | -0.00815801 | -0.01570449 | -0.33090613 | 0.082382947 | -0.240976706 | 0.044014457 |
| 496 | 366 | 5.902633 | 0.40157852 | -0.07552664 | -0.25123706 | -0.05781055 | 0.670531668 | 0.118518922 | -0.04362357 | -0.279547655 | 0.228841519 | 1.61E-01 |
| 497 | 376 | 5.929589 | -0.3619916 | 0.14456827 | 0.58587694 | -0.09044534 | -1.18288211 | -0.00449585 | 0.329219227 | 0.072645772 | -0.776521264 | 0.131037894 |
| 498 | 562 | 6.331502 | -0.2427339 | -0.13031696 | -0.00666381 | 0.210915698 | 0.105162562 | -0.03557226 | -0.01248847 | 0.073547468 | 0.201793811 | 0.058919752 |
| 499 | 527 | 6.267201 | -0.1297132 | -0.08738421 | -0.30870788 | -0.00239897 | 0.2639799 | -0.09976644 | -0.09881182 | -0.060635992 | 0.204298523 | 1.68E-02 |
| 500 | 386 | 5.955837 | 0.16015813 | -0.04669676 | 0.055172088 | -0.11113484 | 0.040547961 | -0.13518636 | -0.27712899 | -0.067038645 | -0.168433311 | 0.025650626 |
| 501 | 270 | 5.598422 | 0.38292332 | 0.05765693 | 0.258728838 | 0.019861952 | 0.086399503 | -0.09321402 | -0.37551767 | 0.009685683 | -0.18621846 | 0.146630266 |
| 502 | 214 | 5.365976 | 0.43232951 | 0.13785239 | 0.059677756 | 0.093142382 | 0.327941747 | 0.063783474 | -0.25892785 | 0.021674249 | 0.026904675 | 0.186908809 |
| 503 | 164 | 5.099866 | 0.13317508 | 0.15563863 | 0.007618838 | 0.021483992 | -0.00859839 | 0.164483104 | 0.177176317 | 0.056111427 | 0.060206247 | 0.017735602 |
| 504 | 110 | 4.70048 | -0.3486004 | 0.04794303 | -0.0594194 | 0.002742782 | -0.33438123 | 0.117000231 | 0.456897512 | 0.150349025 | 0.155865074 | 1.22E-01 |
| 505 | 110 | 4.70048 | -0.3366344 | -0.12549614 | -0.14676244 | -0.02139098 | -0.08576682 | 0.149458617 | 0.325000641 | 0.327860977 | 0.417636179 | 0.11332273 |
| 506 | 171 | 5.141664 | 0.1175113 | -0.12118839 | -0.21504408 | -0.05283448 | 0.400909292 | -0.22004883 | 0.415162825 | -0.125396008 | 0.910724936 | 0.013808905 |
| 507 | 232 | 5.446737 | 0.20594274 | 0.04230407 | -0.19930642 | -0.07741587 | 0.285529219 | 0.006073683 | -0.61124674 | -0.016531038 | -0.348322245 | 0.042412412 |
| 508 | 510 | 6.234411 | 0.45585443 | 0.07413939 | 0.381499597 | -0.07175031 | -0.07153486 | 0.11601279 | 0.016871342 | 0.12475676 | -0.04591955 | 0.207803263 |
| 509 | 438 | 6.082219 | -0.4962719 | 0.1641076 | -0.34389199 | 0.137339855 | -0.17914763 | -0.2081689 | 0.32225775 | -0.004732475 | 0.346546555 | 0.246285771 |
| 510 | 1170 | 7.064759 | 0.29517344 | -0.17865787 | -0.23059722 | -0.12380112 | 0.580627411 | -0.02191826 | -0.57824696 | -0.03744448 | -0.013145763 | 0.087127357 |
| 511 | 1137 | 7.036148 | 0.10174065 | 0.10626244 | -0.12322755 | -0.083015 | 0.035690767 | -0.02619816 | -0.06088407 | -0.105017301 | -0.104012443 | 1.04E-02 |
| 512 | 715 | 6.572283 | 0.080636 | 0.03662663 | 0.152150222 | -0.04436192 | -0.15250277 | -0.07586214 | -0.07277266 | -0.142301433 | -0.291714724 | 0.006502165 |
| 513 | 477 | 6.167516 | 0.29068139 | 0.02902896 | 0.36377715 | 0.05477408 | -0.04735064 | 0.03908293 | -0.21072816 | -0.098120026 | -0.395281758 | 0.08449567 |
| 514 | 256 | 5.545177 | -0.0054945 | 0.1046453 | 0.410713039 | 0.130959774 | -0.3898931 | 0.058366722 | 0.108563696 | 0.067140499 | -0.272555629 | 3.02E-05 |
| 515 | 162 | 5.087596 | -0.2147899 | -0.00197803 | 0.126516325 | 0.147856694 | -0.19147152 | -0.04270302 | 0.162129782 | 0.17314011 | 0.186501387 | 0.046134712 |
| 516 | 205 | 5.32301 | 0.18075909 | -0.07732437 | -0.33117037 | 0.045545877 | 0.634799706 | 0.158393604 | -0.11861949 | 0.123158138 | 0.48094475 | 0.032673848 |
| 517 | 224 | 5.411646 | -0.046872 | 0.06507327 | -0.3198027 | -0.11922133 | 0.088636073 | 0.343837228 | 0.439982233 | 0.15732486 | 0.342105938 | 0.002196986 |
| 518 | 230 | 5.438079 | -0.2048592 | -0.01687393 | 0.111635734 | -0.11512897 | -0.41474993 | -0.12829037 | 0.955103411 | -0.231630345 | 0.437013499 | 4.20E-02 |
| 519 | 253 | 5.533389 | -0.1280562 | -0.0737493 | 0.195645602 | 0.040188864 | -0.20976364 | 0.08368521 | -0.35636213 | 0.006393351 | -0.643417624 | 0.016398389 |
| 520 | 407 | 6.008813 | 0.00427941 | -0.04610023 | 0.43306171 | 0.070432417 | -0.31224966 | 0.024568678 | 0.232458916 | 0.122118726 | 0.017759307 | 1.83E-05 |
| 521 | 608 | 6.410175 | -0.0722664 | 0.00154059 | -0.47145828 | 0.155902216 | 0.553553512 | 0.063455992 | 0.068246327 | -0.219125163 | 0.339218684 | 0.005222432 |
| 522 | 697 | 6.546785 | -0.4218057 | -0.0260159 | 0.280414764 | -0.16972498 | -0.84592959 | -0.08405379 | 0.176266644 | -0.023071857 | -0.608681009 | 0.177920088 |
| 523 | 727 | 6.588926 | -0.0853942 | -0.15185007 | 0.096653618 | 0.100949315 | 0.070751601 | -0.12621967 | -0.23348276 | -0.027577007 | -0.064088491 | 0.007292163 |
| 524 | 588 | 6.376727 | 0.26273342 | -0.0307419 | 0.076604203 | 0.034795302 | 0.251666421 | -0.10219587 | -0.35061021 | -0.079854882 | -0.076602797 | 0.069028851 |
| 525 | 394 | 5.976351 | 0.34754385 | 0.09458403 | 0.27614732 | 0.027577513 | 0.004390013 | -0.01652836 | -0.28387742 | 0.041139927 | -0.221819116 | 0.120786729 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 526 | 242 | 5.488938 | 0.1554088 | 0.12511579 | -0.00521981 | 0.099413035 | 0.134925863 | 0.036174835 | -0.04591211 | 0.061438654 | 0.114277575 | 0.024151896 |
| 527 | 177 | 5.17615 | -0.001431 | 0.05594717 | -0.20405043 | -0.00187913 | 0.144793116 | 0.029665297 | 0.100485653 | -0.044950544 | 0.170662929 | 2.05E-06 |
| 528 | 171 | 5.141664 | -0.0252357 | -0.00051516 | 0.171721134 | -0.07345815 | -0.26989982 | 0.104096512 | 0.082403602 | 0.16673011 | -0.124862621 | 0.00063684 |
| 529 | 161 | 5.081404 | -0.2643281 | -0.00908485 | -0.04452842 | 0.061819608 | -0.14889526 | 0.039056446 | 0.289156979 | 0.361933924 | 0.463139193 | 0.069869366 |
| 530 | 403 | 5.998937 | 0.61735484 | -0.09515813 | -0.1946162 | -0.01603023 | 0.89109894 | -0.14082543 | 0.108490129 | -0.135042491 | 1.005372012 | 0.381127 |
| 531 | 518 | 6.249975 | 0.32638469 | 0.22224774 | -0.12165338 | -0.07006183 | 0.155728501 | 0.227754464 | -0.39118176 | 0.088089694 | -0.375118032 | 0.106526967 |
| 532 | 621 | 6.431331 | -0.1287087 | 0.11749849 | 0.004065436 | -0.04379522 | -0.29406786 | 0.119751603 | 0.632651289 | 0.025861766 | 0.244693596 | 0.016565933 |
| 533 | 726 | 6.58755 | -0.3615945 | -0.04633514 | -0.06865308 | 0.001463557 | -0.24514276 | 0.082458119 | 0.332643341 | 0.066795781 | 0.071838239 | 0.130750607 |
| 534 | 831 | 6.72263 | -0.5077051 | -0.13017403 | -0.40071546 | -0.02471511 | -0.00153075 | -0.04650193 | 0.229050329 | -0.088477677 | 0.185543836 | 2.58E-01 |
| 535 | 746 | 6.614726 | -0.269686 | -0.18277385 | -0.08112446 | -0.14425757 | -0.15004526 | -0.16630879 | -0.12917203 | -0.132862815 | -0.245771325 | 7.27E-02 |
| 536 | 688 | 6.533789 | 0.31297736 | -9.71E-02 | 0.249596751 | -0.0292048 | 0.131262767 | -0.06921731 | -0.46196885 | -0.107574599 | -0.369063374 | 0.097954829 |
| 537 | 445 | 6.098074 | 0.31764516 | 0.11267185 | 0.33016666 | 0.08985483 | -0.03533852 | 0.053811247 | -0.1922703 | -0.017398272 | -0.298818332 | 0.100898449 |
| 538 | 225 | 5.4161 | -0.0514301 | 0.11435226 | 0.147638364 | 0.118859998 | -0.1945607 | 0.041322298 | 0.149475687 | 0.038078774 | -0.048328534 | 0.002645052 |
| 539 | 166 | 5.111988 | -0.0643487 | -0.01851483 | -0.00135946 | 0.053149811 | 0.00867538 | 0.048911797 | 0.114784161 | 0.031226628 | 0.105774372 | 0.004140757 |
| 540 | 155 | 5.043425 | -0.0807265 | -0.02316554 | -0.02397391 | -0.00048941 | -0.0340765 | 0.12462425 | 0.135866103 | 0.109575276 | 0.086740633 | 0.006516774 |
| 541 | 155 | 5.043425 | -0.2112835 | -0.02906155 | -0.25111173 | -0.00863061 | 0.060259192 | 0.143173947 | 0.346178473 | 0.041112049 | 0.304375768 | 0.044640712 |
| 542 | 288 | 5.66296 | 0.30282843 | -0.07606206 | 0.5864871 | -0.09040022 | -0.29799683 | -0.16272886 | 0.397705408 | -0.1482373 | 0.114200135 | 0.091705061 |
| 543 | 362 | 5.891644 | 0.18559339 | 0.10901824 | 0.310065457 | 0.211135356 | -0.02235495 | 0.177132254 | -0.45202462 | 0.239741541 | -0.411770277 | 0.034444906 |
| 544 | 535 | 6.282267 | 0.04218347 | 0.06681362 | -0.12227328 | 0.111623565 | 0.209266695 | 0.161406326 | 0.492034038 | 0.126054319 | 0.665948725 | 0.001779445 |
| 545 | 524 | 6.261492 | -0.4613044 | 0.01518605 | -0.34351481 | -0.04401838 | -0.176994 | 0.008004046 | 0.448350907 | 0.086798019 | 0.350150885 | 0.212801724 |
| 546 | 765 | 6.639876 | -0.2814198 | -0.16606957 | -0.48231988 | -0.12366533 | 0.243304369 | -0.02451718 | 0.022233462 | -0.048949403 | 0.24110561 | 0.079197076 |
| 547 | 677 | 6.517671 | -0.198178 | -0.10131111 | -0.2562017 | -1.74E-01 | -0.01430037 | -0.12149496 | -0.06810328 | -0.175061879 | -0.135970563 | 0.039274528 |
| 548 | 565 | 6.336826 | 0.21830824 | -0.07134409 | 0.297328494 | -0.09223261 | -0.09990878 | -0.02397211 | -0.33748601 | -0.072860322 | -0.486282998 | 0.047658487 |
| 549 | 300 | 5.703782 | 0.07598691 | 0.07859097 | 0.301762905 | 0.107038258 | -0.1973287 | -0.0048847 | -0.0665892 | 0.056643418 | -0.202389784 | 0.005774011 |
| 550 | 164 | 5.099866 | -0.0520801 | 0.02735529 | -0.04885857 | 0.108634646 | 0.078057833 | -4.94E-02 | -0.01356861 | 0.043497156 | 0.157342828 | 0.002712336 |
| 551 | 130 | 4.867534 | 0.00948961 | -0.01874883 | -0.06113128 | -0.01758908 | 0.071780636 | -0.13465994 | -0.13710125 | 0.051486102 | 0.120825432 | 9.01E-05 |
| 552 | 195 | 5.273 | 0.42276109 | 0.00341626 | -0.07669021 | -0.02200726 | 0.47402778 | 0.088138865 | -0.37405539 | 0.131183422 | 0.14301695 | 1.79E-01 |
| 553 | 264 | 5.575949 | 0.2820327 | 0.15219399 | -0.20071931 | -0.02760848 | 0.302949545 | 0.33409075 | 0.24483018 | 0.150709418 | 0.364398393 | 0.079542444 |
| 554 | 320 | 5.768321 | 0.03431427 | 0.10153177 | 0.287687012 | -0.07225895 | -0.42716347 | -0.08906442 | 0.928029861 | -0.171293538 | 0.418637271 | 0.001177469 |
| 555 | 320 | 5.768321 | -0.1435842 | 0.01235314 | 0.176313719 | 0.103567324 | -0.22868373 | 0.18618549 | -0.24740117 | 0.186455004 | -0.475815385 | 0.020616423 |
| 556 | 384 | 5.950643 | -0.2833899 | -0.05169031 | 0.040074299 | 0.063472939 | -0.20830098 | -0.03914823 | 0.517181916 | 0.169901396 | 0.517930567 | 0.080309853 |
| 557 | 631 | 6.447306 | -0.0372479 | -0.10202038 | -0.43823915 | 0.014426748 | 0.517438373 | -0.05482973 | -0.10874509 | 0.008425312 | 0.471948323 | 0.001387406 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 558 | 1155 | 7.051856 | 0.1031737 | -0.01340925 | -0.26734876 | -0.1577661 | 0.226165611 | 0.024649608 | -0.1523048 | -0.025807559 | 0.023403644 | 0.010644812 |
| 559 | 924 | 6.828712 | -0.15582 | 0.03714253 | -0.18826912 | -0.09624555 | -0.10093899 | -0.08866963 | 0.068471134 | -0.127889435 | -0.071687664 | 0.02427988 |
| 560 | 678 | 6.519147 | 0.09035526 | -0.05609521 | 0.207392827 | -0.06777688 | -0.12871924 | -0.04500913 | -0.24630452 | -0.025233802 | -0.35524843 | 8.16E-03 |
| 561 | 378 | 5.934894 | 0.07884421 | 0.03252789 | 0.072187566 | 0.074661418 | 0.048790164 | -0.0112831 | -0.12502537 | -0.005141789 | -0.070093895 | 0.006216409 |
| 562 | 227 | 5.42495 | 0.04689217 | 0.02838391 | -0.04947609 | 0.025987524 | 0.093971869 | 0.02495852 | -0.03134194 | -5.20E-02 | -0.014282746 | 0.002198876 |
| 563 | 180 | 5.192957 | 0.04404651 | 0.01688118 | 0.009015126 | -0.01781139 | 0.000338811 | 0.072237828 | 0.069329221 | -0.141747304 | -0.1443171 | 0.001940095 |
| 564 | 135 | 4.905275 | -0.2789128 | 0.01585674 | 0.401623033 | 0.003245445 | -0.69314718 | -0.00596628 | 0.200660634 | 0.092777753 | -0.393742512 | 0.077792378 |
| 565 | 256 | 5.545177 | 0.35989127 | -0.10040863 | 0.267931065 | 0.144584292 | 0.336953121 | 0.414338608 | -0.01657301 | 0.351674473 | 0.257715979 | 0.129521725 |
| 566 | 204 | 5.31812 | -0.3537251 | 0.12956086 | 0.032598553 | 0.096455183 | -0.41942934 | -0.33911432 | 1.150940579 | -0.093752021 | 0.976873537 | 0.125121458 |
| 567 | 390 | 5.966147 | 0.37254523 | -0.12734104 | -0.13640499 | 0.011735479 | 0.648026745 | 0.162449521 | -0.94198423 | 0.195984726 | -0.260422281 | 0.13878995 |
| 568 | 564 | 6.335054 | 0.1005876 | 0.13411628 | -0.26922043 | -0.0491058 | 0.186585956 | 0.052223943 | 0.45124867 | -0.041208666 | 0.544402017 | 0.010117866 |
| 569 | 837 | 6.729824 | -0.0041481 | 0.03621154 | -0.03538551 | -0.09691936 | -0.10189349 | 0.099926031 | 0.145066509 | -0.057715503 | -0.114468516 | 1.72E-05 |
| 570 | 900 | 6.802395 | -0.4227186 | -0.00149332 | 0.098015013 | -0.01273878 | -0.53197907 | -0.06813896 | 0.277572309 | 0.025946956 | -0.160320841 | 0.178691007 |
| 571 | 898 | 6.80017 | -0.1145743 | -0.15217869 | -0.14802902 | 0.035285405 | 0.220918856 | -0.13376736 | -0.18927489 | -0.093336449 | 0.072074878 | 0.013127262 |
| 572 | 725 | 6.586172 | 0.19344758 | -0.04124674 | 0.085837497 | -0.05329045 | 0.09556637 | -0.06411976 | -0.37157601 | -0.047378035 | -0.259267915 | 0.037421966 |
| 573 | 448 | 6.104793 | 0.2165163 | 0.06964113 | 0.074901996 | 0.030901499 | 0.10287467 | 0.044492925 | -0.17811045 | -0.011876945 | -0.131605652 | 0.046879306 |
| 574 | 228 | 5.429346 | -0.0699747 | 0.07794587 | 0.044547563 | 0.026964719 | -0.16550343 | 0.017351675 | 0.123591459 | 0.026272126 | -0.032991515 | 0.004896461 |
| 575 | 178 | 5.181784 | -0.0149527 | -0.0251909 | 0.041844184 | 0.016037123 | -0.01556891 | 0.035691878 | 0.048199098 | 0.076039819 | 0.072978128 | 0.000223585 |
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| 577 | 208 | 5.337538 | -0.0090422 | -0.01999858 | 0.341896705 | -0.09538819 | -0.42632857 | 0.336997579 | 0.309734971 | 0.436145903 | -0.01744527 | 8.18E-05 |
| 578 | 250 | 5.521461 | -0.0513966 | -0.00325521 | -0.33603886 | 0.123082814 | 0.410980289 | -0.22139417 | 0.936104386 | -0.356962445 | 1.211516398 | 0.00264161 |
| 579 | 350 | 5.857933 | 0.14483468 | -0.01850277 | 0.353917971 | -0.12097399 | -0.31155451 | 0.236023533 | -0.6149838 | 0.170999496 | -0.991562348 | 0.020977084 |
| 580 | 424 | 6.049733 | -0.156819 | 0.05214048 | 0.095558222 | 0.127410469 | -0.17710721 | 0.058487686 | 0.655620925 | 0.054972572 | 0.4749986 | 0.024592191 |
| 581 | 522 | 6.257668 | -0.2816322 | -0.05645483 | -0.0039407 | 0.03440096 | -0.18683569 | -0.07188619 | 0.162465793 | 0.105185296 | 0.152701589 | 0.079316683 |
| 582 | 908 | 6.811244 | -0.0205455 | -0.10138758 | -0.40158266 | -0.00141865 | 0.481006098 | -0.08258435 | -0.19968385 | -0.071725223 | 0.292181378 | 0.000422117 |
| 583 | 1004 | 6.911747 | 0.13105545 | -0.00739638 | -0.10884555 | -0.14456976 | 0.102727617 | -0.06824438 | -0.22940098 | -0.140807752 | -0.199236729 | 0.01717553 |
| 584 | 682 | 6.52503 | 0.09742033 | 0.04717996 | 0.183775201 | -0.0391844 | -0.17271923 | -0.0386488 | -0.18956773 | -0.067494487 | -0.391132645 | 0.009490721 |
| 585 | 366 | 5.902633 | 0.03358483 | 0.03507132 | 0.205690481 | 0.066159072 | -0.1410179 | -0.01405634 | -0.10735778 | 0.046834658 | -0.187484685 | 0.001127941 |
| 586 | 208 | 5.337538 | -0.0180817 | 0.01209054 | -0.06647598 | 0.074048573 | 0.11035235 | -0.04052438 | -0.03904538 | 0.018264922 | 0.130096273 | 0.000326947 |
| 587 | 183 | 5.209486 | 0.122727 | -0.0065094 | -0.01420511 | -0.02393135 | 0.119510152 | -0.00622308 | -0.11256773 | 0.037570398 | 0.050735893 | 0.015061915 |
| 588 | 216 | 5.375278 | 0.22013336 | 0.04418172 | -0.05277402 | -0.00511384 | 0.223611826 | 0.219336534 | -0.01728633 | 0.117373252 | 0.104362216 | 0.048458698 |
| 589 | 231 | 5.442418 | -0.0567783 | 0.07924801 | -0.00859013 | -0.01899865 | -0.1464348 | 0.491530836 | 0.609268151 | 0.354734294 | 0.326036812 | 0.003223772 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 590 | 196 | 5.278115 | -0.4144004 | -0.02044018 | -0.04882676 | -0.00309245 | -0.34822589 | -0.20128199 | 1.365363434 | -0.233046493 | 0.985373038 | 0.171727677 |
| 591 | 227 | 5.42495 | -0.1836504 | -0.14918414 | 0.137592944 | -0.01757763 | -0.18963688 | 0.147043687 | -0.55911663 | 0.248445824 | -0.64735137 | 0.033727483 |
| 592 | 321 | 5.771441 | -0.1099348 | -0.06611416 | -0.14897803 | 0.04953346 | 0.154690805 | -0.06541581 | 0.408454687 | 0.061565985 | 0.69012729 | 0.012085669 |
| 593 | 613 | 6.418365 | 0.18549466 | -0.03957654 | -0.26755057 | -0.05363209 | 0.43898968 | 0.010592794 | -0.18171059 | -0.075669669 | 0.171016624 | 0.034408269 |
| 594 | 858 | 6.754604 | -0.0737596 | 0.06677808 | -0.01951822 | -0.0963182 | -0.21733763 | -0.06465057 | 0.029424428 | -0.086930897 | -0.210193525 | 5.44E-03 |
| 595 | 932 | 6.837333 | 0.08720158 | -0.02655344 | 0.124502674 | -0.00702656 | -0.01777421 | -0.0277206 | -0.17958492 | -0.071836192 | -0.241474713 | 0.007604116 |
| 596 | 612 | 6.416732 | 0.04523803 | 0.03139257 | 0.092549314 | 0.044820963 | -0.03388289 | 0.047977469 | -0.07700167 | -0.040682948 | -0.199544978 | 0.00204648 |
| 597 | 235 | 5.459586 | -0.3198769 | 0.01628569 | 0.031905584 | 0.033317753 | -0.33475044 | -0.10326765 | 0.133270749 | -0.014796145 | -0.113008189 | 0.102321245 |
| 598 | 173 | 5.153292 | 0.11498205 | -0.11515569 | -0.01717758 | 0.01148601 | 0.258801334 | -0.0296101 | -0.28685459 | -0.042657247 | -0.041100403 | 0.013220872 |
| 599 | 161 | 5.081404 | 0.22033281 | 0.04139354 | 0.116590645 | -0.00618393 | 0.056164697 | 0.085856167 | -0.08225027 | -0.006550609 | -0.118492352 | 0.048546547 |
| 600 | 130 | 4.867534 | -0.1331883 | 0.07931981 | 0.209126695 | 0.041972632 | -0.37966217 | 0.107903881 | 0.238489352 | 0.230880563 | -0.018196135 | 0.017739122 |
| 601 | 174 | 5.159055 | 0.04720879 | -0.04794779 | -0.05393936 | 0.07528561 | 0.224381546 | 0.400180532 | 0.299733002 | 0.51740088 | 0.641334896 | 0.00222867 |
| 602 | 240 | 5.480639 | 0.12861965 | 0.01699517 | -0.39368036 | -0.01941817 | 0.485886675 | -0.05160118 | 1.111612589 | -0.211875775 | 1.437224667 | 0.016543013 |
| 603 | 167 | 5.117994 | -0.4959204 | 0.04630307 | -0.17446791 | -0.14172493 | -0.50948047 | 0.090509573 | -0.14333661 | 0.154782829 | -0.588543821 | 0.245937024 |
| 604 | 288 | 5.66296 | -0.0216854 | -0.17853134 | -0.1044381 | -0.06280845 | 0.198475562 | -0.04892001 | 0.251415479 | -0.068858751 | 0.429952302 | 0.000470258 |
| 605 | 545 | 6.300786 | 0.19691239 | -0.00780675 | 0.176219926 | -0.03759771 | -0.0090985 | 0.057437199 | -0.13588892 | 0.01115031 | -0.191274308 | 0.03877449 |
| 606 | 671 | 6.508769 | -0.1908781 | 0.07088846 | -0.07007159 | 0.063439173 | -0.12825582 | -0.06773436 | 0.159547775 | -0.068053231 | 0.030973083 | 0.036434457 |
| 607 | 653 | 6.481577 | -0.0705696 | -0.06871612 | 0.082841504 | -0.02522577 | -0.10992072 | -0.13821455 | -0.188151 | -0.029179581 | -0.189036754 | 0.004980064 |
| 608 | 570 | 6.345636 | 0.27240791 | -0.02540505 | 0.04297613 | 0.029822942 | 0.284659764 | 0.032287448 | -0.38392931 | 0.050502599 | -0.081054391 | 0.074206068 |
| 609 | 265 | 5.57973 | -0.0300474 | 0.09806685 | -0.30388308 | 0.015471407 | 0.191240234 | 0.031939801 | 0.089687357 | -0.108702792 | 0.140284998 | 0.000902846 |
| 610 | 131 | 4.875197 | -0.1904248 | -0.01081706 | 0.109232948 | -0.10939791 | -0.39823858 | -0.03873324 | 0.088721669 | -0.031168525 | -0.301952199 | 0.036261601 |
| 611 | 105 | 4.65396 | -0.0479104 | -0.06855293 | 0.209316169 | 0.039323861 | -0.14934974 | -0.07998793 | -0.10759233 | 0.090374912 | -0.086579236 | 0.002295403 |
| 612 | 105 | 4.65396 | -0.0052605 | -0.01724773 | -0.12652888 | 0.075353821 | 0.213869915 | -0.14577717 | -0.22218869 | 0.113583032 | 0.251041423 | 2.77E-05 |
| 613 | 335 | 5.814131 | 0.9571543 | -0.00189379 | 0.044848354 | -0.0455504 | 0.868649333 | 0.569446997 | -0.40493658 | 0.421242665 | 0.315508423 | 0.91614435 |
| 614 | 228 | 5.429346 | -0.2557497 | 0.34457555 | 0.122188663 | 0.016145407 | -0.70636853 | -0.34900686 | 1.581797215 | -0.05431703 | 1.170118514 | 0.065407921 |
| 615 | 352 | 5.863631 | 0.18974848 | -0.0920699 | -0.47112436 | 0.043987919 | 0.796930658 | 0.07362104 | -0.96946349 | 0.095273234 | -0.15088064 | 0.036004485 |
| 616 | 780 | 6.659294 | 0.46800915 | 0.06830945 | -0.02060115 | -0.16960477 | 0.250696076 | 0.139056343 | 0.204502889 | -0.051494749 | 0.264647873 | 0.219032562 |
| 617 | 852 | 6.747587 | -0.1865662 | 0.16848329 | 0.187066771 | -0.00741641 | -0.54953271 | 0.04023609 | 0.386267619 | 0.060460209 | -0.143040969 | 0.034806958 |
| 618 | 1252 | 7.132498 | -0.1389144 | -0.06716384 | -0.18133421 | 0.067344038 | 0.176927683 | 0.049450248 | 0.111766917 | -0.071299326 | 0.167945026 | 0.019297214 |
| 619 | 1055 | 6.961296 | -0.1957795 | -0.05000919 | -0.06704109 | -0.06528032 | -0.1440095 | 0.045916985 | 0.137361799 | -0.145489 | -0.198053684 | 0.038329597 |
| 620 | 545 | 6.300786 | -0.3121278 | -0.07048061 | 0.258787512 | -0.02413479 | -0.52456948 | 0.041100597 | 0.127547181 | 0.033986788 | -0.404136111 | 0.097423753 |
| 621 | 226 | 5.420535 | -0.3484188 | -0.112366 | -0.02854503 | 0.093163504 | -0.11434426 | -0.06096284 | 0.114168326 | 0.033620843 | 0.094407744 | 0.121395659 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 622 | 148 | 4.997212 | -0.0148483 | -0.12543077 | -0.18090355 | -0.01027621 | 0.281209777 | -0.0222945 | -0.16934121 | -0.040771832 | 0.09339123 | 0.000220473 |
| 623 | 130 | 4.867534 | 0.10582419 | -0.0053454 | -0.04551484 | -0.06512528 | 0.09155915 | 0.058687259 | -0.06192916 | -0.08419782 | -0.113255088 | 0.011198758 |
| 624 | 114 | 4.736198 | -0.0818514 | 0.03809671 | -0.00499749 | -0.01638534 | -0.131336 | 0.112117345 | 0.163020164 | -0.153449651 | -0.233882834 | 0.006699658 |
| 625 | 133 | 4.890349 | -0.1243903 | -0.02946652 | 0.909296583 | -0.0017991 | -1.0060195 | 0.33108449 | 0.311437071 | 0.599417892 | -0.42624903 | 0.015472957 |
| 626 | 204 | 5.31812 | 0.19746624 | -0.04478052 | -0.24296224 | 0.32734677 | 0.812555768 | -0.30019043 | 0.919679139 | -0.367375639 | 1.6650497 | 0.038992915 |
| 627 | 307 | 5.726848 | 0.31325751 | 0.07108785 | 0.180261053 | -0.08746641 | -0.02555779 | 0.238563616 | -0.83386231 | 0.077495832 | -1.020487886 | 0.098130268 |
| 628 | 437 | 6.079933 | 0.04991012 | 0.1127727 | 0.44460869 | 0.064893979 | -0.4425773 | 0.151208312 | 0.66267671 | 0.146375098 | 0.215266199 | 0.00249102 |
| 629 | 455 | 6.120297 | -0.3672578 | 0.01796764 | -0.17723792 | 0.160059128 | -0.04792838 | 0.007850993 | 0.42002309 | 0.042353779 | 0.406597494 | 0.134878281 |
| 630 | 701 | 6.552508 | -0.1530764 | -0.1322128 | -0.13196869 | -0.06380565 | 0.047299443 | 0.003511262 | 0.021808313 | 0.052052892 | 0.117649386 | 0.023432384 |
| 631 | 586 | 6.37332 | -0.2015759 | -0.0551075 | -0.18599049 | -0.04750873 | -0.00798659 | -0.09449079 | 0.009753505 | 0.048333669 | 0.144591367 | 0.040632826 |
| 632 | 425 | 6.052089 | 0.0371475 | -0.07256731 | -0.29652139 | -0.06695658 | 0.339279631 | -0.01419118 | -0.26247441 | 0.043263787 | 0.134260191 | 0.001379937 |
| 633 | 230 | 5.438079 | 0.05536388 | 0.0133731 | -0.33099786 | -0.1067477 | 0.266240935 | 0.042472401 | -0.03941994 | -0.064171407 | 0.120177185 | 0.003065159 |
| 634 | 102 | 4.624973 | -0.2647995 | 0.019931 | -0.01410592 | -0.11915923 | -0.38978377 | -0.11701886 | 0.117978891 | -0.023467892 | -0.178253907 | 0.070118754 |
| 635 | 91 | 4.51086 | 0.02584782 | -0.09532781 | 0.100532977 | -0.00507813 | 0.015564517 | -0.18252323 | -0.3250524 | 0.061776062 | -0.065188588 | 0.00066811 |
| 636 | 130 | 4.867534 | 0.38336542 | 0.00930521 | -0.07775887 | 0.036191872 | 0.488010946 | -0.07257994 | -0.50700897 | 0.118018258 | 0.171600173 | 0.146969045 |
| 637 | 288 | 5.66296 | 0.68910927 | 0.13801155 | -0.11817082 | -0.02799319 | 0.64127535 | 0.460345893 | -0.20161095 | 0.34850999 | 0.327828496 | 0.474871585 |
| 638 | 365 | 5.899897 | 0.28737977 | 0.24807934 | 0.187592925 | -0.0425415 | -0.19083399 | -0.19616863 | 1.278738591 | -0.315989928 | 0.968083305 | 0.082587131 |
| 639 | 365 | 5.899897 | -0.0752099 | 0.10345672 | 0.297594636 | 0.067533453 | -0.40872775 | 0.17522877 | -0.54491287 | 0.251119596 | -0.877749799 | 0.005656522 |
| 640 | 580 | 6.363028 | 0.0232503 | -0.02707555 | 0.047414612 | 0.107134069 | 0.110045302 | 0.058404097 | 0.486746582 | 0.159166645 | 0.697554432 | 0.000540576 |
| 641 | 756 | 6.628041 | -0.132945 | 0.00837011 | -0.3488949 | 0.01706926 | 0.224649049 | -0.04698271 | 0.162233602 | 0.008264203 | 0.442129568 | 0.017674373 |
| 642 | 1215 | 7.102499 | -0.0254331 | -0.0478602 | -0.14542258 | -0.12560216 | 0.042247512 | -0.10752008 | -0.13050754 | 0.003696065 | 0.022956119 | 0.000646843 |
| 643 | 1726 | 7.453562 | 0.38194976 | -0.00915592 | -0.19149706 | -0.05235213 | 0.530250613 | 0.121852885 | -0.2986669 | -0.099463985 | 0.010266847 | 0.14588562 |
| 644 | 670 | 6.507278 | -0.3833226 | 0.13750191 | 0.035290129 | -0.06893894 | -0.62505354 | -0.02522254 | 0.338480235 | -0.014938083 | -0.276288849 | 0.14693618 |
| 645 | 356 | 5.874931 | -0.116442 | -0.13799612 | 0.052595686 | 0.012704446 | -0.01833712 | -0.00219726 | -0.07006261 | 0.04470779 | -0.041494676 | 0.01355874 |
| 646 | 198 | 5.288267 | -0.0859703 | -0.04191912 | -0.25155949 | 0.018934447 | 0.226442795 | -0.02702677 | -0.00610351 | -0.123177751 | 0.124188306 | 0.007390885 |
| 647 | 144 | 4.969813 | -0.1201729 | -0.03094929 | 0.024555427 | -0.09056142 | -0.20434042 | -0.12938409 | -0.07507437 | -0.192129716 | -0.342160418 | 0.01444152 |
| 648 | 182 | 5.204007 | 0.1896134 | -0.04326223 | 0.364197149 | 0.008839954 | -0.12248156 | -0.02458809 | -0.35940025 | -0.076399939 | -0.533693657 | 0.035953243 |
| 649 | 264 | 5.575949 | 0.16832004 | 0.06826083 | 0.654653806 | 0.131110973 | -0.42348361 | 0.205012813 | -0.06830025 | 0.484574624 | -0.212222054 | 0.028331637 |
| 650 | 798 | 6.682109 | 0.96715325 | 0.06059522 | 0.273010779 | 0.23567537 | 0.869222621 | -0.11383126 | 0.569480035 | -0.206493299 | 1.346040622 | 0.9353854 |
| 651 | 693 | 6.54103 | 0.03736333 | 0.34817517 | -0.07144936 | 0.09828388 | -0.1410786 | 0.300767279 | -0.31619796 | 0.184451336 | -0.573592496 | 0.001396018 |
| 652 | 695 | 6.543912 | -0.3989886 | 0.0134508 | 0.022087782 | -0.02572177 | -0.4602489 | -0.07567107 | 0.835464663 | 0.061477997 | 0.512364823 | 0.159191866 |
| 653 | 1635 | 7.399398 | 0.31258773 | -0.14363588 | -0.12629775 | 0.007951602 | 0.590472965 | 0.160047853 | -0.21019741 | -0.049455489 | 0.170772213 | 0.097711091 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 654 | 1690 | 7.432484 | -0.3075349 | 0.11253158 | -0.02416145 | -0.04546719 | -0.44137226 | 0.027402438 | 0.444577369 | -0.113179034 | -0.137376359 | 0.094577734 |
| 655 | 1558 | 7.351158 | -0.1715503 | -0.11071258 | 0.362852273 | -0.00869812 | -0.4323881 | 0.086382187 | 0.076117883 | 0.128266194 | -0.314386206 | 0.029429498 |
| 656 | 700 | 6.55108 | -0.4103351 | -0.0617581 | -0.36415643 | 0.130626818 | 0.146206268 | 0.003311762 | 0.239950519 | -0.026550041 | 0.356294984 | 0.168374875 |
| 657 | 404 | 6.001415 | -0.0445627 | -0.14772063 | -0.1106199 | -0.13109631 | 0.082681524 | 0.163318068 | 0.009199339 | -0.002312909 | -0.073750114 | 0.001985834 |
| 658 | 173 | 5.153292 | -0.3193507 | -0.01604257 | -0.08167175 | -0.03982316 | -0.26145958 | 0.170177231 | 0.453661301 | -0.028449234 | -0.006424748 | 0.101984891 |
| 659 | 96 | 4.564348 | -0.4702183 | -0.11496626 | -0.11416423 | -0.02940183 | -0.27048967 | 0.145056731 | 0.472714532 | -0.13619378 | -0.079025651 | 0.221105286 |
| 660 | 66 | 4.189655 | -0.5569336 | -0.1692786 | 0.180132734 | -0.04109912 | -0.60888684 | 0.146482382 | 0.402935364 | -0.0258822 | -0.378316055 | 0.310175015 |
| 661 | 83 | 4.418841 | -0.2481964 | -0.20049609 | 0.159904042 | 0.064847784 | -0.14275655 | 0.551836916 | 0.406895504 | 0.215802961 | -0.071895001 | 0.061601444 |
| 662 | 103 | 4.634729 | -0.1183917 | -0.0893507 | 0.918795583 | 0.057565455 | -0.89027111 | -0.07666584 | 1.532880323 | -0.119822383 | 0.599452669 | 1.40E-02 |
| 663 | 82 | 4.406719 | -0.4248234 | -0.04262101 | 0.035495161 | 0.33076641 | -0.08693114 | 0.349545271 | -0.21296067 | 0.316597135 | -0.332839953 | 0.180474919 |
| 664 | 61 | 4.110874 | -0.843481 | -0.15293642 | -0.37903913 | 0.012778258 | -0.29872723 | -0.28685839 | 0.970959086 | -0.079653756 | 0.879436487 | 0.711460258 |
| 665 | 242 | 5.488938 | 0.65233688 | -0.30365317 | 0.296958347 | -0.13645409 | 0.522577624 | 0.115480632 | -0.79682885 | 0.168471424 | -0.221260432 | 0.425543409 |
| 666 | 313 | 5.746203 | 0.05995783 | 0.23484128 | -0.29215819 | 0.106905005 | 0.22417974 | 0.105827766 | 0.320779532 | 0.028844672 | 0.467976177 | 0.003594941 |
| 667 | 243 | 5.493061 | -0.2080272 | 0.02158482 | -0.16297276 | -0.10517695 | -0.17181617 | 0.132954381 | 0.293966017 | 0.090928618 | 0.080124088 | 0.043275301 |
| 668 | 98 | 4.584967 | -0.514054 | -0.07488978 | -0.38981832 | -0.05867019 | -0.10801607 | 0.012208224 | 0.369317726 | 0.003486065 | 0.252579494 | 0.264251494 |
| 669 | 37 | 3.610918 | -0.5114435 | -0.18505943 | -0.04233456 | -0.1403346 | -0.42438411 | -0.22824213 | 0.033911734 | 0.171913756 | 0.009683515 | 0.261574457 |
| 670 | 34 | 3.526361 | 0.29130348 | -0.18411966 | -0.3033832 | -0.01524044 | 0.763565895 | -0.16884432 | -0.63400593 | 0.179133928 | 0.477538211 | 0.084857716 |
| 671 | 35 | 3.555348 | 0.38531072 | 0.10486925 | -0.44670742 | -0.10921795 | 0.61793094 | -0.195984 | -0.46901199 | 0.152691296 | 0.497594244 | 0.148464352 |
| 672 | 45 | 3.806662 | 0.39644751 | 0.13871186 | -0.5290869 | -0.16081467 | 0.626007878 | -0.18834263 | -0.5444 | 0.154191981 | 0.424142489 | 1.57E-01 |
| 673 | 122 | 4.804021 | 0.86557851 | 0.1427211 | -0.23578656 | -0.19047129 | 0.768172689 | 0.397568633 | -0.52317396 | 0.580880964 | 0.428311057 | 0.749226163 |
| 674 | 271 | 5.602119 | 0.86622872 | 0.31160826 | -0.1124721 | -0.08488316 | 0.582209396 | -0.00769241 | 1.104357314 | -0.080700887 | 1.613558235 | 0.750352203 |
| 675 | 145 | 4.976734 | -0.4486253 | 0.31184234 | -0.40358223 | -0.04048996 | -0.39737534 | 0.173368374 | -0.02136781 | 0.367942391 | -0.224169131 | 0.201264631 |
| 676 | 261 | 5.56452 | 0.06610957 | -0.1615051 | -0.80130698 | -0.1452896 | 0.883632048 | 0.041192473 | 0.481578816 | -0.301956195 | 1.022062196 | 0.004370475 |
| 677 | 335 | 5.814131 | -0.1964637 | 0.02379944 | 0.619720039 | -0.28847051 | -1.12845374 | -0.05370443 | 0.114423536 | 0.12155856 | -0.838767209 | 0.038598001 |
| 678 | 666 | 6.50129 | 0.19302745 | -0.07072695 | 0.056959937 | 0.223099214 | 0.429893674 | 0.054449677 | -0.14917898 | 0.111397649 | 0.337662665 | 0.037259597 |
| 679 | 553 | 6.315358 | -0.0814314 | 0.06948988 | -0.19762581 | 0.020505577 | 0.067210078 | 0.048973249 | 0.151249104 | 0.13995198 | 0.309437913 | 0.006631077 |
| 680 | 322 | 5.774552 | -0.0792338 | -0.02931531 | -0.48835128 | -0.07114529 | 0.367287509 | 0.127419574 | 0.136036803 | 0.012850762 | 0.388755501 | 0.006277994 |
| 681 | 117 | 4.762174 | -0.3769171 | -0.02852417 | -0.48587133 | -0.17580646 | -0.03832805 | 0.039663776 | 0.353943261 | -0.240254878 | 0.035696562 | 0.142066483 |
| 682 | 56 | 4.025352 | -0.336303 | -0.13569015 | 0.276738304 | -0.17491368 | -0.65226486 | -0.0524439 | 0.110177155 | -0.17773086 | -0.667374661 | 0.113099723 |
| 683 | 50 | 3.912023 | 0.00303409 | -0.12106909 | 0.366045185 | 0.099625789 | -0.14231622 | -0.00059584 | -0.1456775 | -0.206298946 | -0.493696833 | 9.21E-06 |
| 684 | 48 | 3.871201 | -0.0461953 | 0.00109227 | 0.376625131 | 0.131776267 | -0.29213642 | 0.081005711 | -0.0016551 | -0.198255397 | -0.573052628 | 0.002134005 |
| 685 | 55 | 4.007333 | -0.1911421 | -0.0166303 | 0.822299588 | 0.135585047 | -0.86122638 | 0.332992215 | 0.225015863 | 0.418493298 | -0.550709435 | 0.036535319 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 686 | 82 | 4.406719 | 0.05936655 | -0.06881117 | 0.822917288 | 0.296027852 | -0.39871171 | -0.644312 | 0.924978374 | -0.008097276 | 1.162481383 | 0.003524387 |
| 687 | 276 | 5.620401 | 1.13799443 | 0.02137196 | -0.426194 | 0.296250224 | 1.839066697 | 0.254296603 | -1.78975555 | 0.182493025 | -0.022492433 | 1.295031315 |
| 688 | 406 | 6.006353 | 0.42407755 | 0.40967799 | 0.062804091 | -0.15342984 | -0.20183437 | 0.040980511 | 0.706379454 | 0.043360498 | 0.50692507 | 0.179841773 |
| 689 | 622 | 6.43294 | 0.1203947 | 0.15266792 | -0.18664055 | 0.022609473 | 0.176976809 | 0.113834753 | 0.113834753 | -0.056530982 | 0.120445827 | 0.014494885 |
| 690 | 998 | 6.905753 | 0.07956282 | 0.04334209 | 0.183376078 | -0.0671906 | -0.21434596 | 0.316207646 | 0.316207646 | 0.05731545 | -0.157030505 | 0.006330242 |
| 691 | 411 | 6.018593 | -0.815961 | 0.02864261 | -0.07735985 | 0.066015388 | -0.70122839 | 0.069467385 | 0.878354572 | 0.051550789 | 0.159209583 | 0.665792386 |
| 692 | 184 | 5.214936 | -0.6040195 | -0.29374597 | -0.0752721 | -0.02784955 | -0.262851 | -0.07895681 | 0.192964957 | 0.134125867 | 0.143196635 | 0.364839587 |
| 693 | 116 | 4.75359 | 0.00261175 | -0.21744703 | -0.35807122 | -0.02709796 | 0.551032044 | 0.000887056 | -0.21932448 | 0.041751343 | 0.372571853 | 6.82E-06 |
| 694 | 66 | 4.189655 | -0.0167552 | 0.00094023 | -0.31948787 | -0.12890564 | 0.172886795 | 0.004170783 | 0.002464046 | -0.055204105 | 0.115975952 | 0.000280737 |
| 695 | 50 | 3.912023 | -0.0524369 | -0.00603187 | 0.002882381 | -0.11501563 | -0.16430305 | 0 | 0.011585509 | -0.000627195 | -0.153344737 | 0.00274963 |
| 696 | 44 | 3.78419 | -0.1508118 | -0.01887729 | -0.04388552 | 0.001037657 | -0.08701138 | 0 | 0 | 0.085269169 | -0.001742208 | 0.022744213 |
| 697 | 45 | 3.806662 | -0.3337378 | -0.05429226 | -0.18158504 | -0.01579879 | -0.11365932 | 0 | 0 | 0.350518121 | 0.236858803 | 0.11138094 |
| 698 | 350 | 5.857933 | 1.65350782 | -0.12014562 | 0.056398223 | -0.06537061 | 1.651884603 | 0 | 0 | -0.678223156 | 0.973661447 | 2.734088112 |
| 699 | 137 | 4.919981 | -0.4955797 | 0.59526282 | 1.081094705 | 0.02030336 | -2.15163385 | 0 | 0 | 0.267680635 | -1.883953212 | 0.245599226 |
| 700 | 406 | 6.006353 | 0.53569084 | -0.17840869 | 0.402873677 | 0.389194094 | 0.70041994 | 0 | 0 | 0.04313738 | 0.74355732 | 0.286964672 |
| 701 | 622 | 6.43294 | 0.16218915 | 0.1928487 | 0.114374969 | 0.145034524 | 0 | 0 | 0 | 0.119826055 | 0.119826055 | 0.026305319 |
| 702 | 998 | 6.905753 | 0.09279778 | 0.05838809 | 0.075584675 | 0.041174989 | 0 | 0 | 0 | 0.332850154 | 0.332850154 | 0.008611428 |
| 703 | 963 | 6.870053 | 0.08249395 | 0.0334072 | -0.77516297 | 0.027210483 | 0.851460197 | 0 | 0 | 0.073123563 | 0.92458376 | 0.006805251 |
| 704 | 574 | 6.352629 | 0.02117138 | 0.02969782 | -0.57381855 | -0.27905867 | 0.286233441 | 0 | 0 | -0.083112433 | 0.203121008 | 4.48E-04 |
| 705 | 287 | 5.659482 | -0.0151241 | 0.0076217 | 0.002481161 | -0.20657468 | -0.23180161 | 0 | 0 | 0.000933744 | -0.23086787 | 0.000228738 |
| 706 | 163 | 5.09375 | -0.0240519 | -0.00544467 | -0.01591745 | 0.000893218 | -0.00179657 | 0 | 0 | 0.004390298 | 0.002593732 | 0.000578494 |
| 707 | 125 | 4.828314 | -0.0405482 | -0.00865868 | -0.04981506 | -0.00573028 | 0.012195273 | 0 | 0 | 0 | 0.012195273 | 0.001644156 |
| | | | | | | | | | | | SS= | 60.58038048 |

# الخلاصة

ان توليد السلاسل الزمنية لجريان الانهار من السلاسل الزمنية التاريخية مهم في عمليات التصميم والتخطيط في منظومة الموارد المائية. تناولت هذه الدراسة طريقتان لتوليد السلاسل الزمنية للجريان الشهري لنهر الزاب الكبير احد روافد نهر دجلة. الطريقة الاولى تم استخدام نموذج بارما (PARMA) . حيث تم تطبيق نموذج بارما لاول مرة في العراق في هذه الدراسة حسب علم المؤلف على بيانات الجريان الشهري لنهر الزاب الكبير للفترة من شهر تشرين الاول (October) 1933 لغاية شهرايلول (September) 1992. عند بناء نموذج بارما تم استخدام تحويل بوكس- كوكس ( Box-Cox) كافضل تحويل لكل فصل لجعل البيانات طبيعية (Normality) ثم نمذجتها (Standardization) بواسطة طرح المعدل للبيانات الطبيعية والقسمة على الانحراف المعياري. استخدمت دالتي الارتباط الذاتي الدورية (PeACF) والارتباط الذاتي الجزئي الدورية (PePACF) لتحديد نوع النموذج لكل فصل. كما استخدمت دالة الاحتمال الشرطية (Conditional Maximum Likelihood) لتخمين معاملات النموذج استنادا الى اقل مجموع مربعات. حيث تم اقتراح ثمانية نماذج من هذا النوع لكل فصل وهي (PARMA(1,0), PARMA(2,0), PARMA(0,1) PARMA(0,2) PARMA(1,1) PARMA(2,2) PARMA(1,2) PARMA(2,1)) . تم اختيار النموذج PARMA(1,0) كافضل نموذج ولكل فصل بواسطة استخدام معياري (AIC ) و (SIC) حيث اعطى هذا النموذج اقل قيمة لمعياري (AIC) و (SIC) من باقي النماذج الاخرى . كما تم استخدام اختبار عجز ملائمة بورتمانتيو (Portmanteau) كفحص للدقة. استخدم هذا النموذج لايجاد القيم المستقبلية للجريان الشهري لنهر الزاب الكبير ولعشر سنوات للفتره من 1993 الى 2002.

اما الطريقة الثانية فهي تطبيق نماذج الانحدار الذاتي – التكاملي- المتوسط المتحرك – الموسمية التضاعفية (SARIMA) لنفس بيانات الجريان الشهري لنهر الزاب الكبير للفترة من 1933 الى 1992. عند بناء هذا النموذج تم استخدام اللوغارتم الطبيعي لجعل البيانات طبيعية ومن ثم استخدام الفرق البسيط والفصلي. واستخدمت دالتي الارتباط الذاتي (ACF) والارتباط الذاتي الجزئي (PACF) لتحديد نوع النموذج. كما استخدمت دالة الاحتمال الغير شرطية (unconditional Maximum Likelihood) لاختيار افضل معاملات مخمنة للنماذج استناداً الى اقل مجموع مربعات. تم اقتراح اربعة نماذج من هذا النوع وهي SARIMA(0,1,1)(0,1,1)$_{12}$, SARIMA(0,1,2)(0,1,1)$_{12}$ SARIMA(1,1,0)(1,1,0)$_{12}$ , SARIMA(2,1,0)(1,1,0)$_{12}$. تم اختيار نموذج SARIMA(0,1,1)(0,1,1)$_{12}$ بواسطة استخدام معياري (AIC) و(SIC) حيث اعطى هذا النموذج اقل قيمة لمعياري (AIC ) و (SIC) من باقي النماذج الاخرى. كما تم استخدام اختبار عجز ملائمة بورتمانتيو (Portmanteau) كفحص للدقة. استخدام هذا النموذج لايجاد القيم المستقبلية للجريان الشهري لعشر سنوات من 1993 الى 2002.

تم مقارنة القيم المستقبلية للجريان الشهري لكلا الطريقتين مع القيم التاريخية للفترة من 1993 الى 2002 بواسطة استخدام اقل خطأ تنبؤ ولوحظ ان نموذج SARIMA(0,1,1)(0,1,1)$_{12}$ هو افضل من نموذج PARMA(1,0) في تمثيل البيانات الاصلية لنهر الزاب الكبير. لذلك تم استخدام نموذج SARIMA(0,1,1)(0,1,1)$_{12}$ لايجاد القيم المستقبلية للجريان الشهري لعشر سنوات للفترة من 2003 الى 2012.

**وزارة التعليم العالي والبحث العلمي**

**جامعة بابل**

**كلية الهندسة**

**قسم الهندسة المدنية**

# النماذج التصادفية للجريان الشهري لنهر الزاب الكبير

رسالة

مقدمة الى كلية الهندسة – جامعة بابل

كجزء من متطلبات نيل شهادة الماجستير

في علوم هندسة الموارد المائية

من قبل

**أنيس كاظم ادريس السعدي**

(دبلوم عالي هندسة مدنية)

باشراف

**أ.م.د. صلاح توفيق علي البزاز**

**أ.م. كاظم نايف كاظم الطائي**

شباط 2010 م